Localization GameS in Graphs

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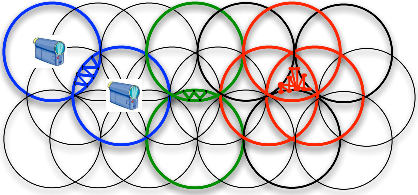
COATI seminar, March 27th, 2018

Motivation / First Game

Several antennas (cameras), with some radius of detection, are spread at Inria

A camera returns whether it sees the target  or not.
Determine the location of a "target" depending on the answers of all cameras ?



Example in the plane : A target seen by both blue (resp., green, resp., by red and not **black**) cameras must be in the blue (resp., green, red) area.

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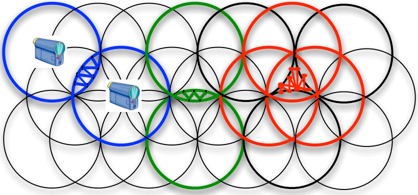


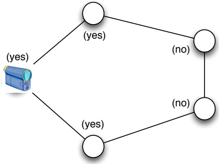
Example in the plane : A target seen by both blue (resp., green, resp., by red and not **black**) cameras must be in the blue (resp., green, red) area.

Question : How to place a minimum amount of cameras to detect the location of any "intruder" (according to some precision) ?

Localization in Graphs / Identifying Code

Metric space : a graph $G = (V, E)$

Camera at $v \in V$: says if  is at $N_G[v]$ (closed neighborhood of v) or not.



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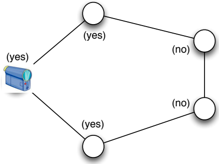
Identifying Code : vertices to place cameras s.t., the target is always located

Set $I = \{v_1, \cdots, v_i\} \subseteq V$ s.t. $(|N_G[v_j] \cap \{v\}|)_{j \leq i}$ pairwise distinct $\forall v \in V$.



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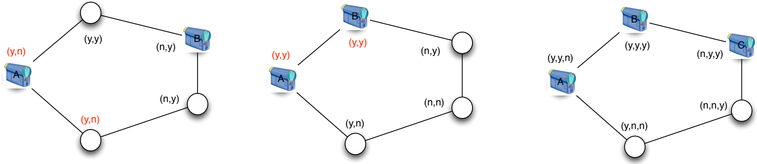

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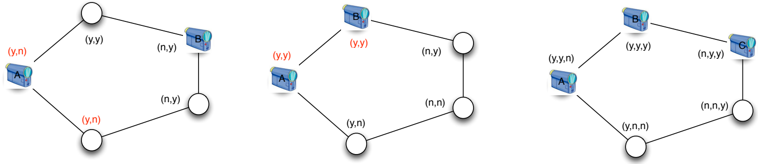


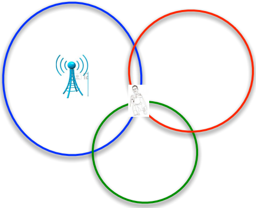
IC introduced in [Karpovsky,Chakrabarty,Levitin 98]. Computing a smallest IC is NP-complete [Charon,Hudry,Lobstein 03]. IC has been studied a lot [Foucaud,Mertzios,Naserasr,Parreau,Valicov 17 ; Dantas,Havet,Sampaio 17...].

Here, we consider "more powerful cameras".

2^{nd} Game : Exact distances (in the plane)

An antenna gives the (exact) distance to the target

Determine the location of the target depending on the answers of all antennas ?



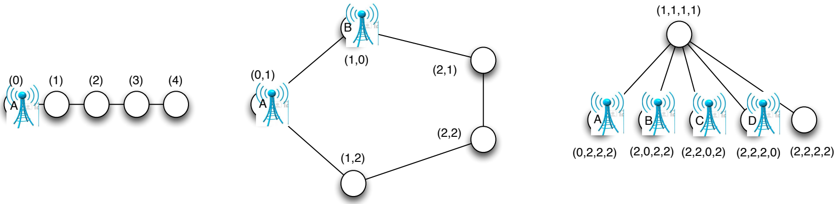
Triangulation

In the plane, 3 antennas (non-colinear) are sufficient to locate any target !

What if the metric space is a graph ?

2^{nd} Game : Exact distances in Graphs

An antenna gives the (exact) graph's distance to the target in $G = (V, E)$.



Resolving set : vertices to place antennas s.t., the target is always located

Set $R = \{v_1, \cdots, v_i\} \subseteq V$ s.t. $(dist_G(v, v_i))_{j \leq i}$ pairwise distinct $\forall v \in V$.

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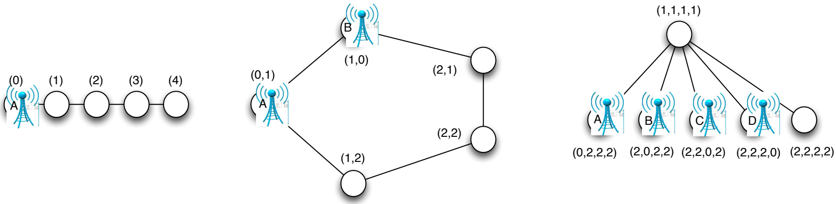

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Set $R = \{v_1, \cdots, v_i\} \subseteq V$ s.t. $(dist_G(v, v_i))_{j \leq i}$ pairwise distinct $\forall v \in V$.

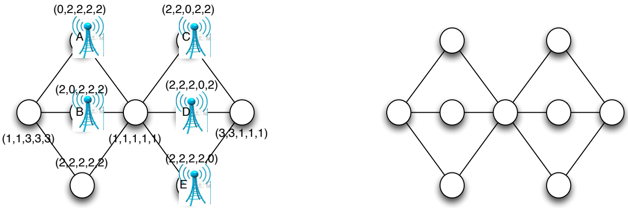
Metric Dimension $MD(G)$: min. size of a resolving set in G. $(MD(G) \leq |V|)$

Computing $MD(G)$ [Harary,Melter 76, Slater 75] is NP-c in planar graphs [Díaz *et al.* 17], W[2]-hard [Hartung,Nichterlein 13], FPT in tree-length [Belmonte *et al.* 17]...

3^{rd} Game : Sequential + Exact distances in Graphs

An antenna gives the (exact) graph's distance to the target in $G = (V, E)$.

Every vertex contains an antenna, but only $k \in \mathbb{N}^*$ vertices can be probed at each step.



$MD(G) = 5$: to locate the target in 1 step, 5 vertices must be probed.

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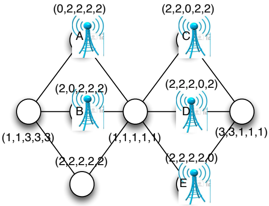

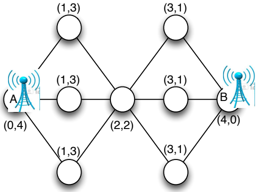

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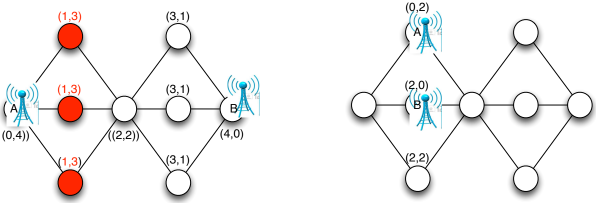

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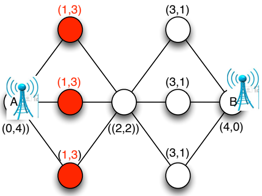

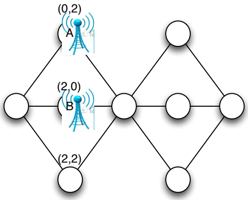

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Remark : If k vertices can be probed per step. $\lceil MD(G)/k \rceil$ steps are sufficient !

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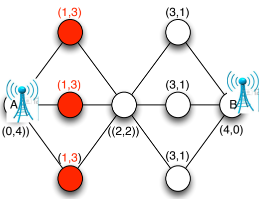

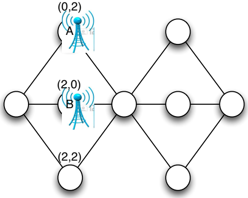

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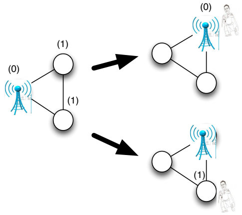
TEASER !!! this guy  will tell you everything you should know about :
[Bensmail, Mazauric, Mc Inerney, N., Pérennes 2018]

$\lambda_k(G) =$ min. # of steps probing $\leq k$ vertices per step to locate the target.
$\kappa_\ell(G) =$ min. # of vertices probed per step during at $\leq \ell$ steps to locate the target.

4^{th} Game : Finally our topic
Sequential + Exact distances in Graphs + moving target

Assume **a single vertex can be probed** at each step

If the target cannot move in C_3



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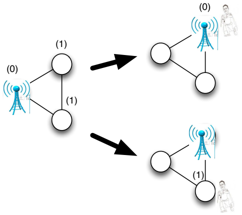

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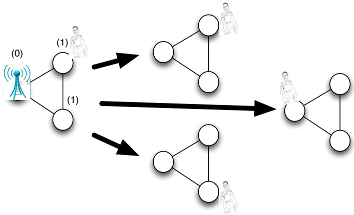

No way to locate the target in C_3 !

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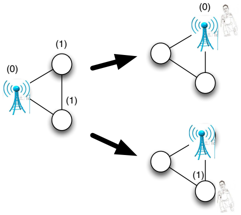

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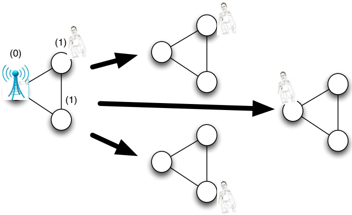

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Question : Is it possible to eventually locate a moving target probing $\leq k$ vertices per step ?

Locating a moving target [Seager 2012]

Let $k \in \mathbb{N}^*$ and G be a graph. Two-Player turn-by-turn game (Player 2=target).

Initially, Player 2 *hides* the target at some vertex. Then, at every step :

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Let $\zeta(G)$ be the minimum k such that Player 1 can win in G, probing $\leq k$ vertices per step, whatever Player 2 does.

- $\forall$ graph G, $\zeta(G) \leq MD(G)$ (probing $MD(G)$ vertices, $P1$ wins in one step)
- $\forall$ graph G with max. degree Δ, $\zeta(G) \leq \lfloor \frac{(\Delta+1)^2}{4} \rfloor + 1$ [Haslegrave,Johnson,Koch 17]
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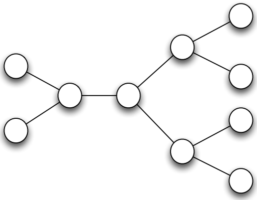

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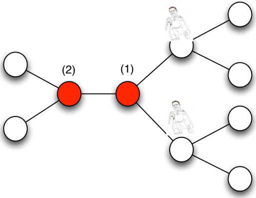

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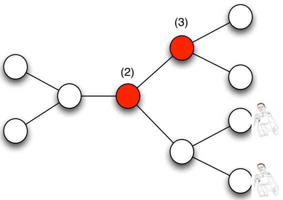

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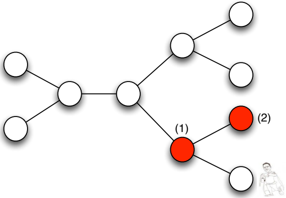


Our results (1/2) [Bosek,Gordinowicz,Grytczuk,N.,Sokó,Śleszyńska-Nowak 2017]

Complexity

Given G, k, deciding if $\zeta(G) \leq k$ is NP-hard in the class of graphs with diameter 2.

Classes of graphs G with bounded $\zeta(G)$

- for any graph G, $\zeta(G) \leq pw(G)$ where $pw(G)$ is the *pathwidth* of G. (equality in interval graphs)
- Let $R^2 = (V, E)$ be the graph with $V = \mathbb{R}^2$ and $\{u, v\} \in E$ iff $dist_{\mathbb{R}^2}(u, v) = 1$. Then, $\zeta(R^2) = 2$. Moreover,
  - probing 2 vertices/step locates the target in 2 steps ;
  - probing 3 vertices/step locates the target in 1 step ;
  - $\forall \epsilon > 0$, probing 1 vertex/step locates the target up to distance ϵ.
- for any outer-planar n-node graph G, $\zeta(G) \leq 3$ (in $O(n^2)$ steps)

Given a graph G, let $\dot{G}$ be the graph obtained from G by adding a universal vertex.

Unbounded ζ

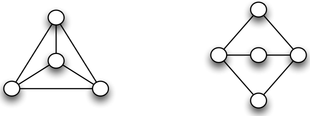
There is a family $(T_k)_{k\in\mathbb{N}}$ of trees such that $\zeta(\dot{T}_k) \geq k$ for all $k \in \mathbb{N}$.

($\dot{T}_k$ is planar and with treewidth 2)

For any outer-planar graph G, $\zeta(G) \leq 3$ (in $O(n^2)$ steps)

Outer-planar : G can be drawn s.t. planar + vertices on the outer-face ($tw(G) \leq 2$).

Theorem [folklore ?] : G outerplanar iff G has no K_4 nor $K_{2,3}$ as minor



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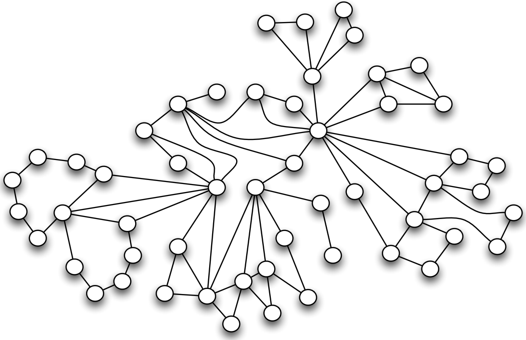

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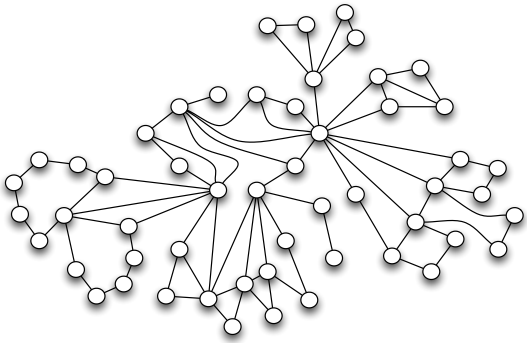

The strategy is recursive using separators of size ≤ 2.

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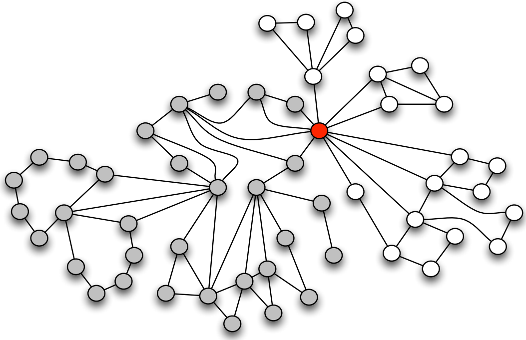

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First Configuration (separator of size 1)

One vertex v has just been probed. The target is known to be in R (white vertices), a union of connected components of $G \setminus \{v\}$.

$\Rightarrow$ check the components of R one by one, until finding the one hiding the target.

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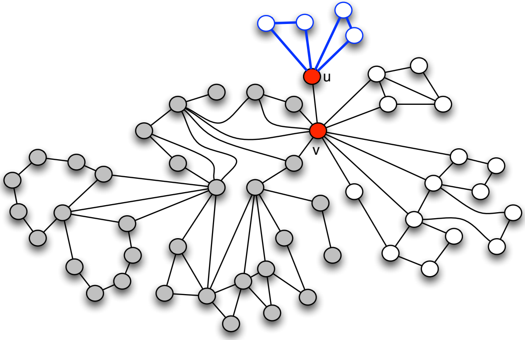

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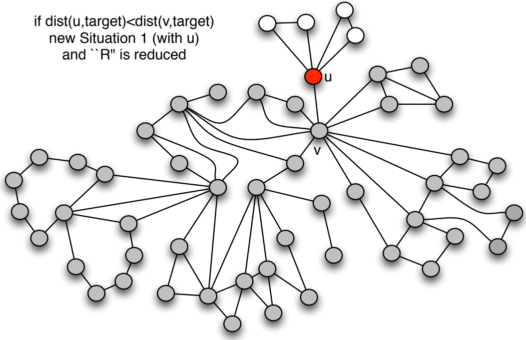

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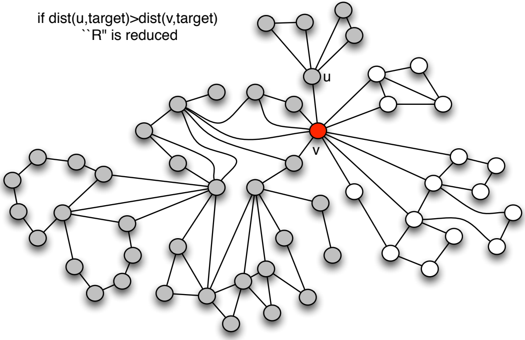

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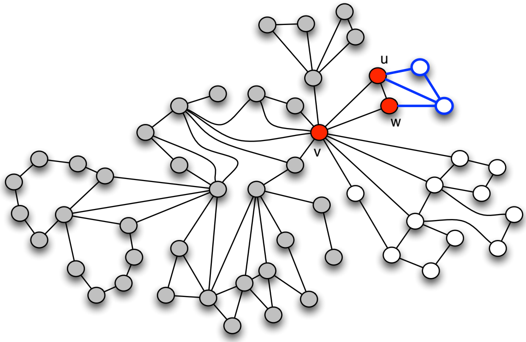

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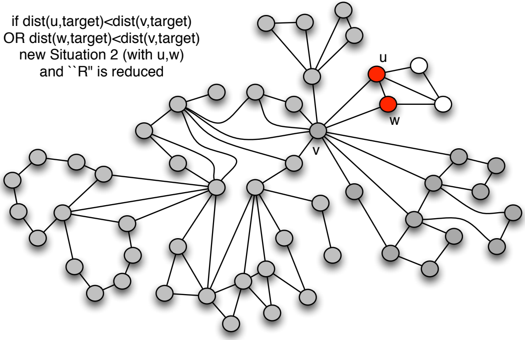

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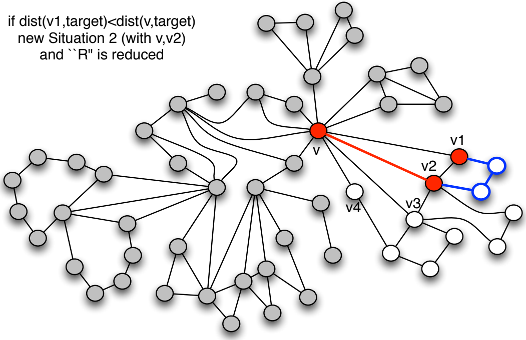

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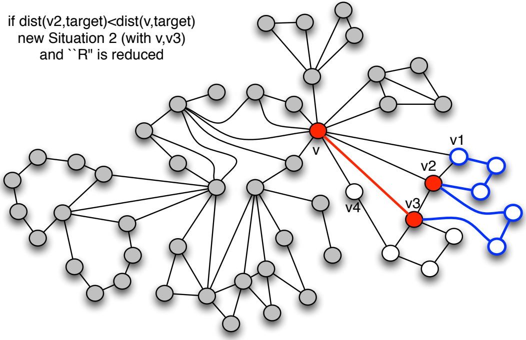

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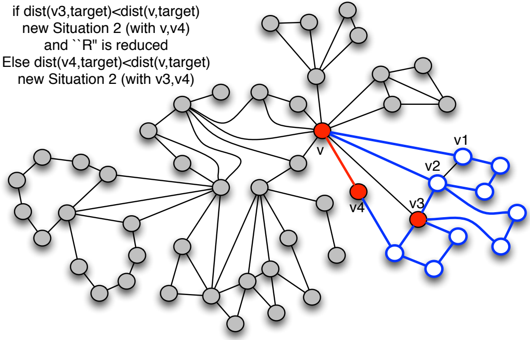

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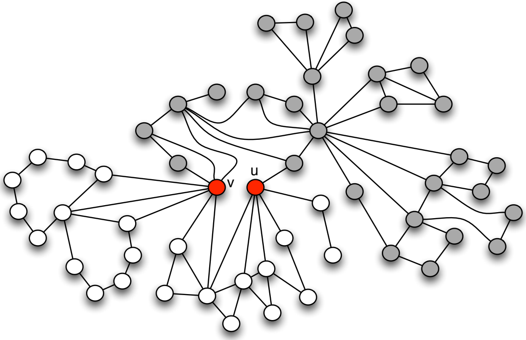

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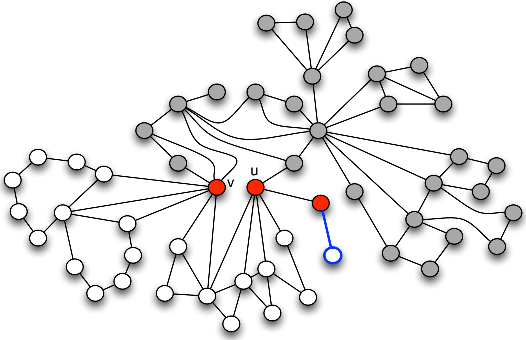

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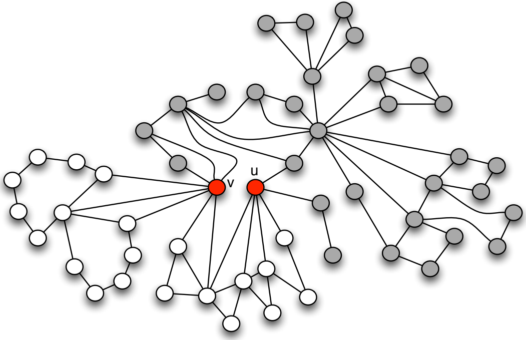

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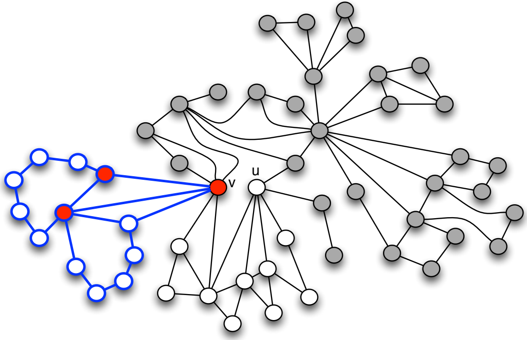

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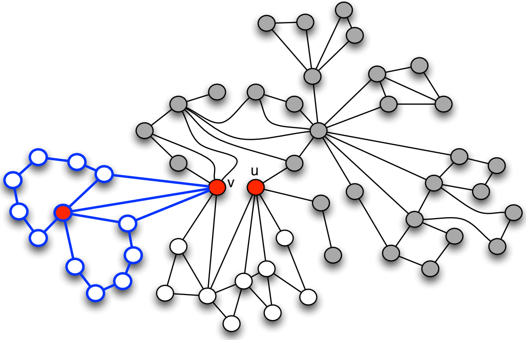

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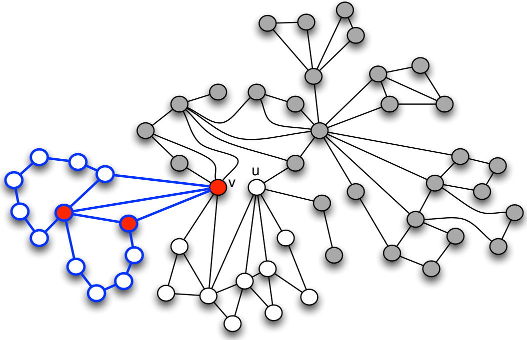

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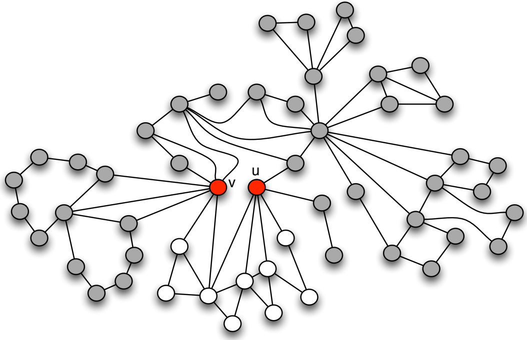

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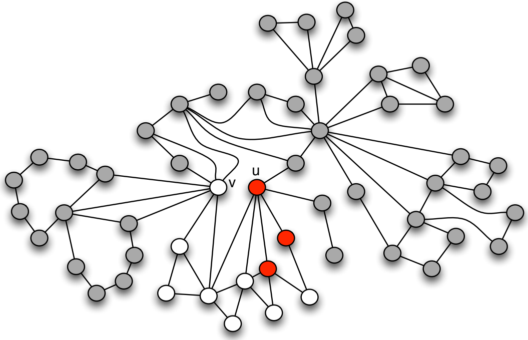

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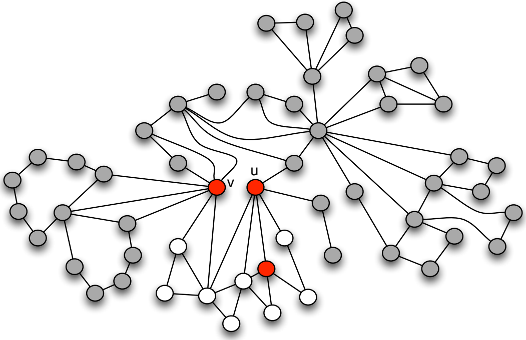

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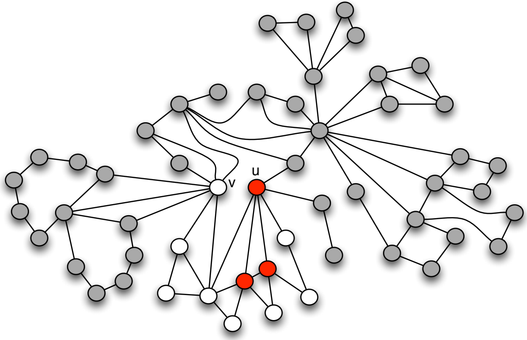

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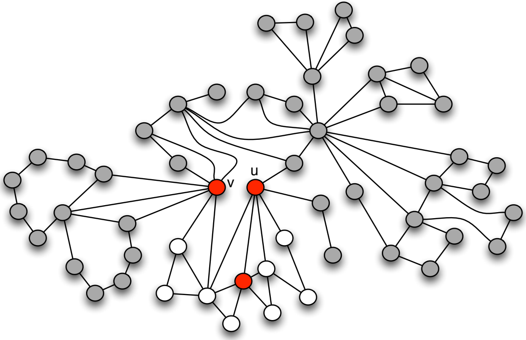

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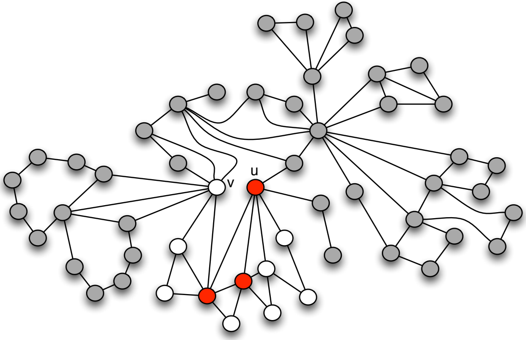

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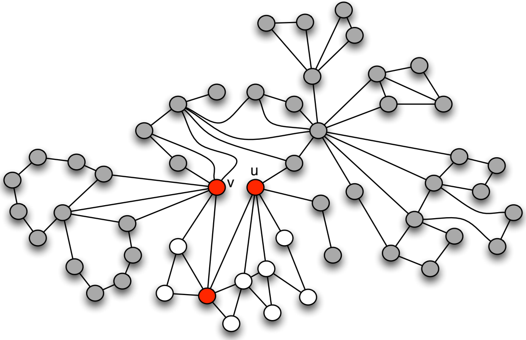

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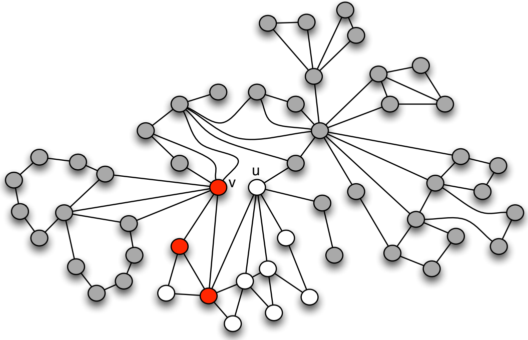

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Key point : change of point of view ! ! A new game :

Let $k \in \mathbb{N}^*$ and G be a graph. Two-Player turn-by-turn game (Player 2=target).

Initially, Player 2 *hides* the target at some vertex. Then, at every step :

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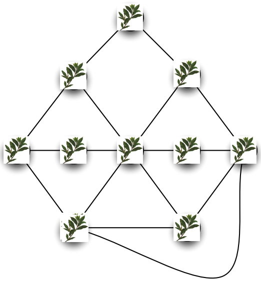

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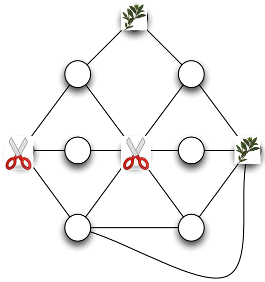

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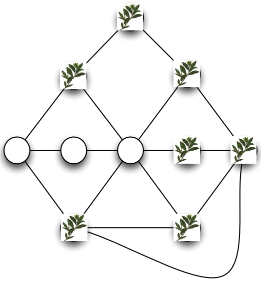

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Claim 1 (it is sufficient to prove that B unbounded in trees)

For any graph G, $B(G) \leq \zeta(\dot{G})$ ($\dot{G}$ obtained from G by adding a universal vertex)

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Claim 2 (B is unbounded in trees)

Let T_k be the tree obtained from the complete $(12k+1)$-ary rooted tree with height $6k$ by subdividing each edge twice. Then, $B(T_k) > k$.

The proof of Claim 2 uses the fact that any ("almost" balanced) bi-coloration of T_k leaves a "large" bi-colored matching.

Weaker antennas : Relative distances [Foucaud,Klasing,Slater 2014]

Assume that probing one vertex (antenna) does not give the distance to the target (unless the target occupies a probed vertex), but *"relatively" probing* 2 vertices determines which one is closer to the target (or whether they are at equal distance).

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Let us consider the Localization game with relative probes.

Initially, Player 2 *hides* the target at some vertex. Then, at every step :

- Player 1 *relatively probes* k vertices (receiving the relative distances between the target and the probed vertices). If the target is located, Player 1 wins !
- Otherwise, Player 2 may move the target to a neighboor of its current location.

Let $\zeta^*(G)$ be the minimum k such that Player 1 can win in G, relatively probing $\leq k$ vertices per step, whatever Player 2 does.

Our results (2/2) [Bosek,Gordinowicz,Grytczuk,N.,Sokó,Śleszyńska-Nowak 2017]

For any graph G, $\zeta(G) \leq \zeta^*(G)$ (by definition)

Complexity (similar proof as for ζ)

Given G, k, deciding if $\zeta^*(G) \leq k$ is NP-hard in the class of graphs with diameter 2.

Classes of graphs G with bounded $\zeta^*(G)$

- for any graph G, $\zeta^*(G) \leq pw(G) + 1$ where $pw(G)$ is the *pathwidth* of G.
- Let $R^2 = (V, E)$ be the graph with $V = \mathbb{R}^2$ and $\{u, v\} \in E$ iff $dist_{\mathbb{R}^2}(u, v) = 1$. Then, $\forall \epsilon > 0$, probing 2 vertices/step locates the target up to distance $2\sqrt{2} + \epsilon$.
- for any outer-planar graph G, $\zeta^*(G) \leq 3$ (the proof above (for ζ) actually holds for ζ^*)

Conclusion / Further Work

Outer-planar graphs — Prove or disprove

For every outer-planar graph G, $\zeta(G) \leq 2$? $\zeta^*(G) \leq 2$?
What if, in the "relative variant", the target is not detected in a probed vertex ?

Complexity of deciding if $\zeta(G)$ (resp. $\zeta^*(G)$) $\leq k$

PSPACE-hard ? EXP-TIME-complete ?

Computation / Bounds / Complexity

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Given k, ℓ, can an (immobile) target be located by (relatively) probing $\leq k$ vertices, in $\leq \ell$ steps ?

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