Localization gameS in graphs

Nicolas Nisse

Université Côte d'Azur, Inria, CNRS, I3S, France

Graph Searching Online seminar, July 10th, 2020

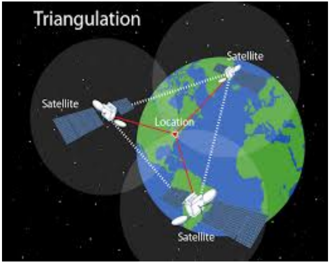
based on joint works with Júlio Araújo, Julien Bensmail, Bartłomiej Bosek, Victor Campos, Przemysław Gordinowicz, Jarosław Grytczuk, Frédéric Havet, Karol Maia, Dorian Mazauric, Fionn Mc Inerney, Stéphane Pérennes, Ana Shirley, Joanna Sokół and Małgorzata Śleszyńska-Nowak

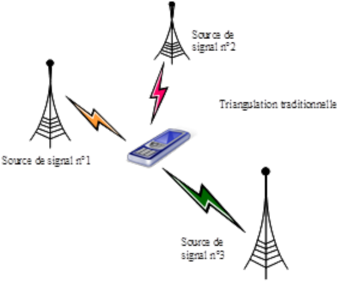
Aim of the talk: present the numerous variants of the problem, give (most of) the known results (almost without proof) and hope you will be interested to work on the numerous open questions...

Outline

1. Metric dimension and Centroidal dimension
2. Sequential localization of an **immobile** target
3. Sequential localization of a **mobile** target
4. Metric dimension in oriented graphs

Metric Dimension of graphs





Precisely locate using few information

Fix any three points A, B and C in the plane. For any point v, $(dist(A, v), dist(B, v), dist(C, v))$ is sufficient to locate v ! !

How to generalize to graph metric ?

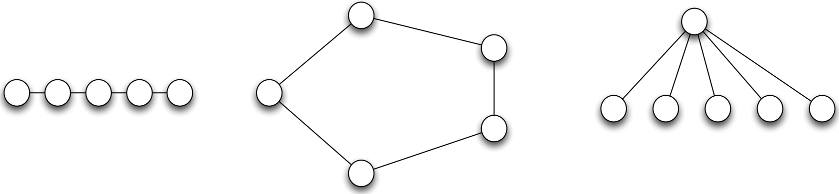
Metric Dimension of graphs

[Harary,Melter 76, Slater 75]

A target is **hidden** at some (**unknown**) vertex t of a graph $G = (V, E)$
Probing a vertex $v \in V(G) \Rightarrow$ the (**absolute**) distance $dist_G(t, v)$.

Resolving set : set of vertices to probe s.t. the target is uniquely located

Set $R = \{v_1, \cdots, v_i\} \subseteq V$ s.t. $(dist_G(v, v_i))_{j \leq i}$ pairwise distinct $\forall v \in V$.



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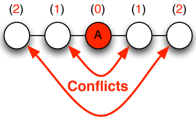

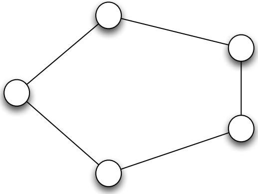

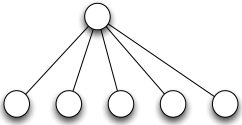

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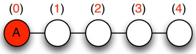

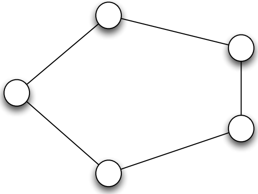

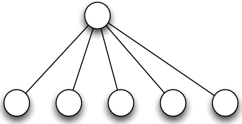

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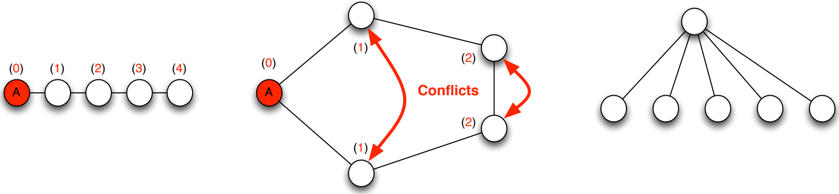

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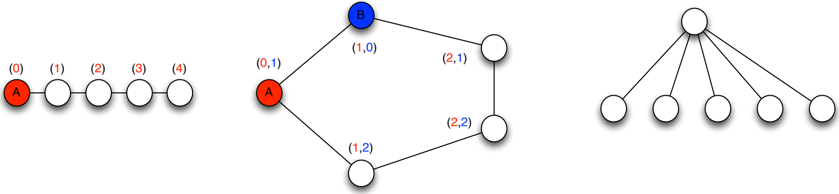

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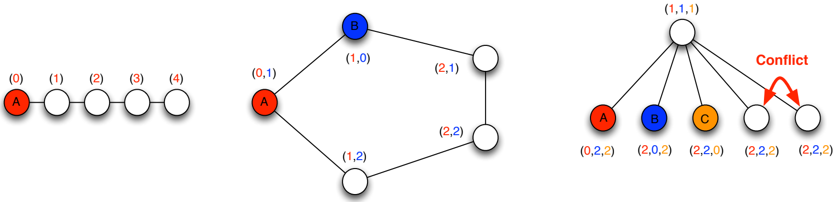

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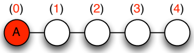

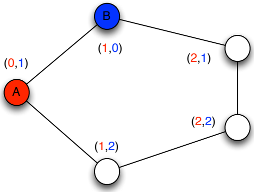

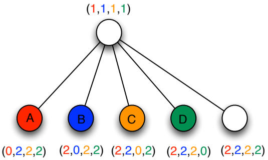

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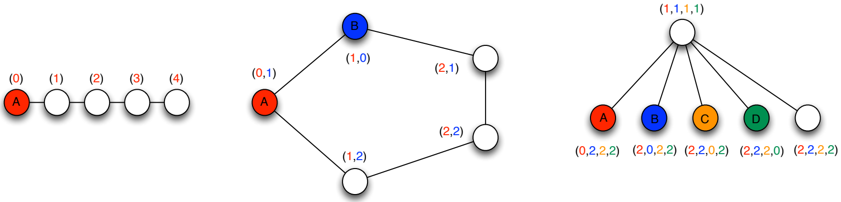


Metric Dimension $MD(G)$: min. size of a **resolving set** in G. $(MD(G) < |V(G)|)$

example : for any tree T, $MD(T) = \#leaves - \#$"branching nodes"

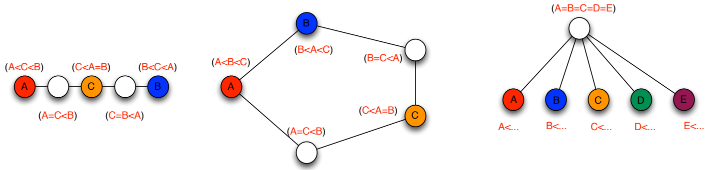
Computing $MD(G)$ [Harary, Melter 76, Slater 75] is NP-c in planar graphs [Díaz *et al.* 17], W[2]-hard [Hartung,Nichterlein 13], FPT in tree-length [Belmonte *et al.* 17]...

variant : Centroidal Dimension

[Foucaud,Klasing,Slater 14]

Probing **two** (or more) vertices $A, B \in V(G) \Rightarrow$ the **relative** "positions" of A and B w.r.t. the target t, i.e., $A > B$ if $dist_G(A,t) > dist_G(B,t)$, $A = B$ if $dist_G(A,t) = dist_G(B,t)$ and $A < B$ otw.

Centroidal Basis : set of vertices to probe s.t. the target is uniquely located



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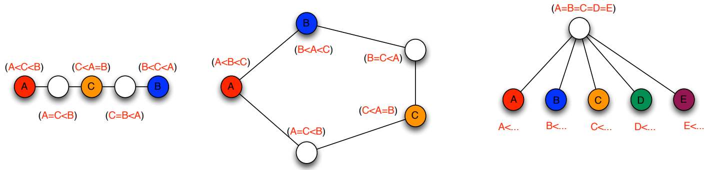


Centroidal Dimension $CD(G)$: min. size of a **basis** in G. $(2 \leq CD(G) < |V(G)|)$

$MD(G) \leq CD(G)$ for every graph G

$(1+o(1))\frac{\log n}{\log\log n} \leq CD(G) < n$ in any n-node graph G.

Computing $CD(G)$ is NP-complete, cannot be approximated up to a factor $o(\log n)$ and there exists a $O(\sqrt{n \log n})$-approximation algorithm.

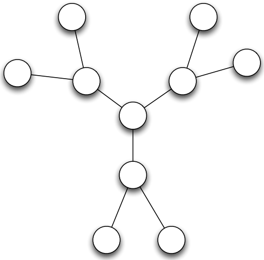
Finally $CD(G) = \Theta(\sqrt{n})$ if G is a n-node path or cycle. [Foucaud,Klasing,Slater 14]

A lot is Open : Exact value of $CD(P_n)$ unknown ? ? and so any other graph class would be interesting...

Outline

Sequential Metric Dimension

Sequentiel variant : Seager (2013) : Probe only ONE vertex per turn.
look for the unknown location of an **immobile** target



Each turn brings some new information (absolute distances)

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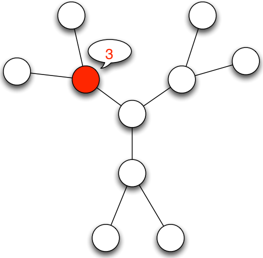

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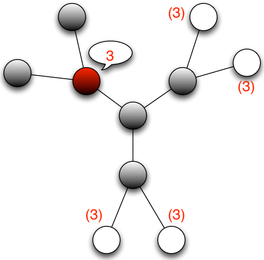

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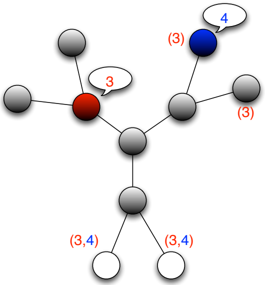

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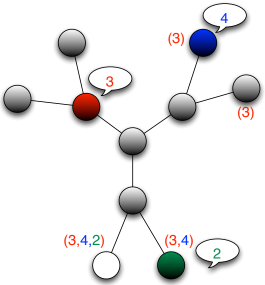

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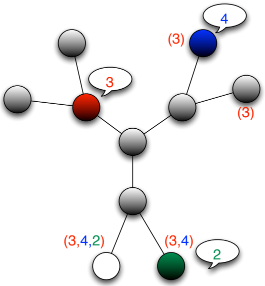

Target found in $< n$ turns in any n-node graph :
Probe each vertex (but one) one by one

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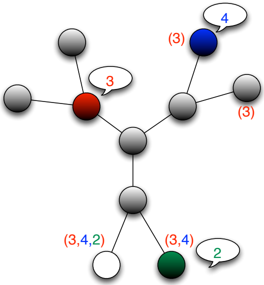

Target found in $< n$ turns in any n-node graph :
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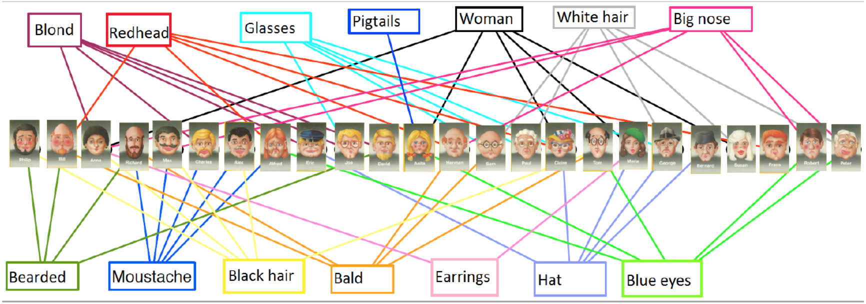
Goal : Minimize # of turns to locate an **immobile** target hidden in G.

Sequential Metric Dimension & Game of Guess Who ?



Bipartite graph : characters / characteristics
+ One universal vertex not depicted on the figure

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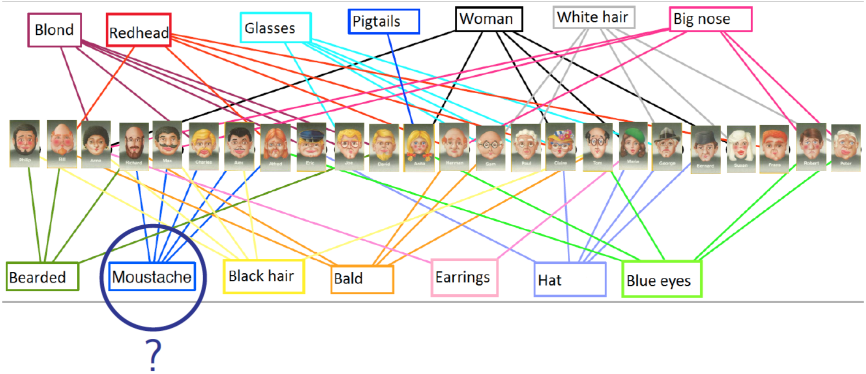

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Always better to probe a "characteristic-vertex"

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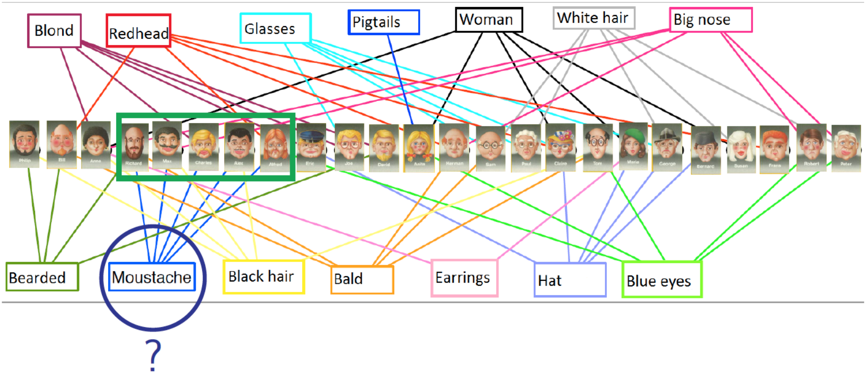

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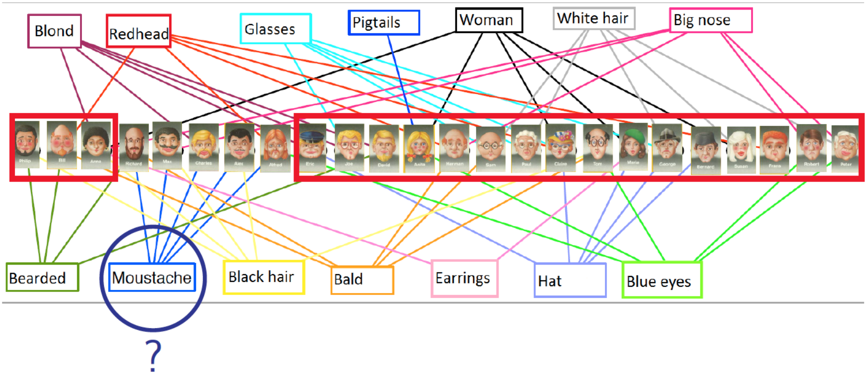

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Sequential Metric Dimension

[Bensmail, Mazauric, Mc Inerney, N., Pérennes, 2018]

What if more than one vertex can be probed per turn ?

Sequential Metric Dimension of G

Given k, ℓ, G, is it possible to locate the **immobile** target in G in at most ℓ turns by probing (**absolute distances**) at most $k \geq 1$ vertices each turn.

$\lambda_k(G)$: min. # turns to locate an immobile target, probing k vertices per turn.

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Remark : for any $G, k \geq 1$, $\lambda_k(G) \leq \lceil \frac{MD(G)}{k} \rceil$

(at each turn, probe k vertices of an optimal resolving set)

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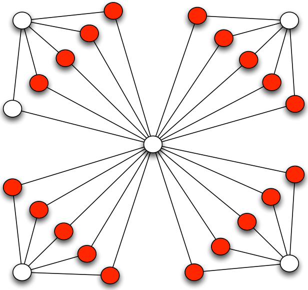

Metric Dimension $MD(G) = 19$

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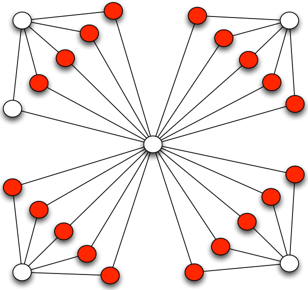

Metric Dimension $MD(G) = 19$

$\lambda_4(G) \leq \lceil \frac{19}{4} \rceil = 5$.
But...

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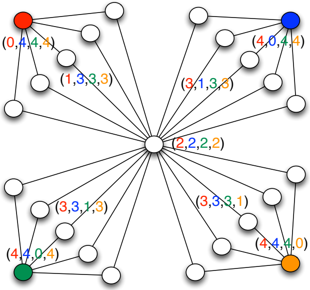

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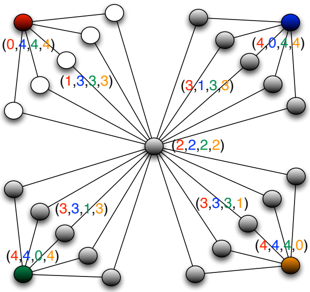

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But...
In one turn, only five locations remain possible

Sequential Metric Dimension

[Bensmail, Mazauric, Mc Inerney, N., Pérennes, 2018]

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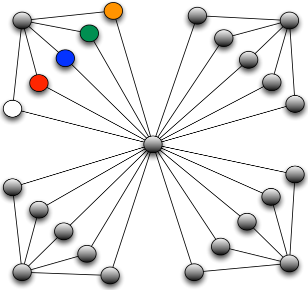

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$\lambda_4(G) \leq \lceil \frac{19}{4} \rceil = 5.$
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$\lambda_4(G) = 2 < \lceil \frac{19}{4} \rceil.$

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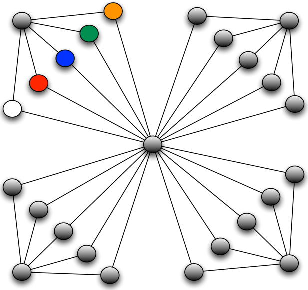

Metric Dimension $MD(G) = 19$
$\lambda_4(G) = 2 < \lceil \frac{19}{4} \rceil$.

Lemma : for any $k \geq 1$

$\lambda_k(G)$ may be arbitrary smaller than $\lceil \frac{MD(G)}{k} \rceil$

Sequential Metric Dimension

[Bensmail, Mazauric, Mc Inerney, N., Pérennes, 2018]

$\lambda_k(G)$: min. # turns to locate an immobile target, probing k vertices per turn.

In general graphs :

Computational complexity *[Bensmail et al. 2018]*

- Let $k \geq 1$ be a **fixed** integer.
  The problem that takes any graph G with **diameter** 2 and an integer $\ell \geq 1$ as inputs and decides if $\lambda_k(G) \leq \ell$ is NP-complete.
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Previous results extend (because of diameter 2) to the case of **relative** distances

Polynomial-time algorithm *[Bensmail et al. 2018]*

- Let $k, \ell \geq 1$ be two **fixed** integers. The problem of deciding if $\lambda_k(G) \leq \ell$, for any n-node graph G, can be solved in time $n^{O(k\ell)}$.

Open : FPT algorithm in $k + \ell$? What about particular graph classes (planar...) ?

Sequential Metric Dimension

[Bensmail, Mazauric, Mc Inerney, N., Pérennes, 2018]

$\lambda_k(G)$: min. # turns to locate an immobile target, probing k vertices per turn.

In TREES :

Computational complexity

[Bensmail et al. 2018]

- The problem that takes any tree T and two integers $k, \ell \geq 1$ as inputs and decides if $\lambda_k(T) \leq \ell$ is NP-complete.

Polynomial-time algorithms

[Bensmail et al. 2018]

- There exists a polynomial-time $+1$-approximation to compute $\lambda_k(T)$.

  Precisely, there exists an algorithm that computes, in time $O(n \log n)$, a localization strategy using ℓ turns, probing k vertices per turn in any n-node tree T, with $\lambda_k(T) \leq \ell \leq \lambda_k(T) + 1$.
- Let $k \geq 1$ be a **fixed** integer. The problem of deciding if $\lambda_k(T) \leq \ell$, for any n-node tree T and any $\ell \geq 1$, can be solved in time $O(n^{k+1} \log n)$.

Open : FPT algorithm in k for trees ? exact or approximation algorithms in other graph classes ?

$\lambda_k(T)$ in Trees : very sketchy [Bensmail, Mazauric, Mc Inerney, N., Pérennes, 2018]

Probing k vertices per steps, find the target in the min. $\lambda_k(T)$ number of steps.

Deciding the first k vertices to be probed (first step) *[Bensmail et al. 2018]*

NP-complete : Reduction from `Hitting Set`

Probing k vertices per steps, find the target in the min. $\lambda_k^X(T)$ number of steps, given that the first k vertices (set X) to be probed (first step) are given.

If first step given : dynamic programming *[Bensmail et al. 2018]*

$\lambda_k^X(T)$ can be computed in time $O(n \log n)$ in n-node trees T.

Polynomial-time algorithms *[Bensmail et al. 2018]*

- There exists a polynomial-time $+1$-approximation to compute $\lambda_k(T)$.
- Let $k \geq 1$ be a **fixed** integer. The problem of deciding if $\lambda_k(T) \leq \ell$, for any n-node tree T and any $\ell \geq 1$, can be solved in time $O(n^{k+1} \log n)$.
  (try all $O(n^k)$ possible first steps)

Outline

Sequential localization against a moving target

The game(s) in a graph G

Initialization : Target chooses an (unknown) vertex $t \in V(G)$
Each turn :

- Player probes $k \geq 1$ vertices $\Rightarrow$ absolute/relative distances to Target.
- Then, Target may move to some neighbor of its current location.

Goal : locate the target after a finite number of steps.

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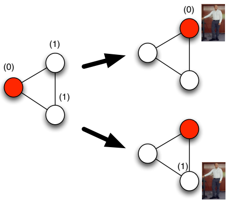
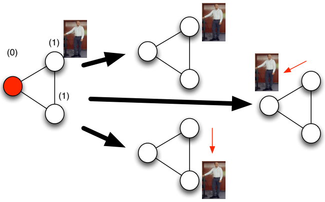
If $G = K_3$ (triangle) and $k = 1$, the target cannot be located.



If the target cannot move in K_3

In ≤ 2 steps, the target is located.



If the target can move (along one edge) after every probe. No way to locate the target in K_3 !

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If $G = K_3$ (triangle) and $k = 1$, the target cannot be located.

Minimize k such that Target can be located in finite number of steps whatever be Target's strategy

$\zeta(G)$ = min. k with **absolute** distances
$\zeta^*(G)$ = min. k with **relative** distances

$\zeta(G) \leq \zeta^*(G)$ for all graphs G

Sequential localization against a moving target

$\zeta(G)$: min. number of probes (**absolute** distances) per turn to locate the target moving in G ?
$\zeta^*(G)$: min. number of probes (**relative** distances) per turn to locate the target moving in G ?

Sequential localization against a moving target

$\zeta(G)$: min. number of probes (**absolute** distances) per turn to locate the target moving in G ?
$\zeta^*(G)$: min. number of probes (**relative** distances) per turn to locate the target moving in G ?

Localization of a moving target in a graph G : *[Seager 13]*

- $\zeta(T) \leq 2$ for any **tree** T ; Characterization for $\zeta(T) = 2$; *[Seager 14]*
- Deciding if $\zeta(G) \leq k$ (resp., $\zeta^*(G) \leq k$) is NP-hard ; *[Bosek et al. 18]*
- $\zeta^*(T) \leq 2$ for any **tree** T ; *[Bosek et al. 18]*
- $\zeta(G)$ unbounded in the class of graphs obtained from a tree plus one vertex (i.e., with **treewidth** ≤ 2) ; *[Bosek et al. 18]*
- $\zeta(G) \leq k$ (resp., $\zeta^*(G) \leq k+1$) for any graph G with **pathwidth** k *[Bosek et al. 18]*
- For any **outerplanar** graph G ($\Rightarrow tw(G) \leq 2$), $\zeta^*(G) \leq 3$ *[Bosek et al. 18]* and $\zeta(G) \leq 2$ *[Bonato,Kinnersley 18]*.

Open : $\zeta^*(G) \leq 2$ for any outerplanar G ? what about other graph classes ?
Tradeoff between # vertices to be probed at each step and number of steps ?

Outline

Metric Dimension in Oriented Graphs

Orientation of G **:** each edge $\{u, v\}$ becomes exactly one arc among uv or vu.

Probing a vertex $v \in V(G) \Rightarrow$ the distance $dist_G(v, t)$ FROM v TO t.

Resolving set : set of vertices to probe s.t. the target is uniquely located

Set $R = \{v_1, \cdots, v_i\} \subseteq V$ s.t. $(dist_D(v_i, v))_{j \leq i}$ pairwise distinct $\forall v \in V$.

Remark : *A priori*, $dist_D(v, t)$ may be ∞.

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$MD(D)$: min. size of a resolving set in a oriented graph D.

Computation of $MD(D)$

- upper bounds *[Chartrand et al. 00]*
- NP-complete in strong oriented graphs *[Rajan et al. 14]* and DAGs *[Araújo et al. 20]*
- complete graphs *[Lozano 13]*, Cayley digraphs *[Fehr et al. 06]*, de Bruijn and Kautz *[Rajan et al. 14]*, poly-time algorithm for oriented trees *[Araújo et al. 20]*

Worst/Best Oriented Metric Dimension (WOMD/BOMD)

Given a class $\mathcal{G}$ of undirected n-node graphs,

- $WOMD_{(s)}(\mathcal{G}) = \max_{D \text{ (strong) orientation of } G \in \mathcal{G}} MD(D)$
- $BOMD_{(s)}(\mathcal{G}) = \min_{D \text{ (strong) orientation of } G \in \mathcal{G}} MD(D)$

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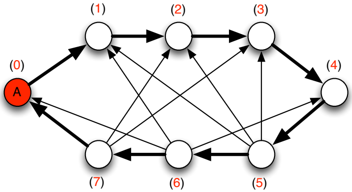
Very few previous work *[Chartrand et al. 01]*

- tournaments : $WOMD(K_n) = n/2$
- $\mathcal{H}$, class of Hamiltonian graphs : $BOMD(\mathcal{H}) = 1$.

Every Hamiltonian graph has an orientation D with $MD(D) = 1$.



Some recent results

$\mathcal{G}_\Delta$: class of n-node graphs with maximum degree $\leq \Delta$. *[Bensmail, Mc Inerney, N. 19]*

- $\frac{2n}{5} \leq WOMD_s(\mathcal{G}_3) \leq \frac{n}{2}$
- $\frac{n}{2} \leq WOMD_s(\mathcal{G}_4) \leq \frac{6n}{7}$
- $\lim_{\Delta \to \infty} WOMD_s(\mathcal{G}_\Delta) = n$

Grids : class of cartesian n-node grids. *[Bensmail, Mc Inerney, N. 19]*

- $\frac{n}{2} \leq WOMD_s(Grids) \leq \frac{2n}{3}$

Trees *[Araújo et al. 20]*

- $WOMD(T) \in \{\alpha(T) - 1, \alpha(T)\}$ for any tree T. α independence number
- $WOMD(T)$ and $BOMT(T)$ can be computed in linear time
  (surprisingly technical) dynamic programming

Open : Close the gaps in grids. Other graph classes...
None is known about sequential metric dimension in oriented graphs.

Conclusion : Recap on some open problems

Centroidal dimension — $CD(G)$

- Exact value for paths unknown. Other graph classes...

Sequential metric dimension, immobile target — $\lambda_k(G) \leq \ell$?

- FPT in $k + \ell$?
- What about particular graph classes (planar...) ? (when k or ℓ is fixed)
- FPT in trees ? exact or approximation algorithms in general graphs ?

Sequential metric dimension, mobile target — $\zeta(G), \zeta^*(G)$

- $\zeta^*(G) \leq 2$ for any outerplanar G ? Other graph classes ?
- Tradeoff between # probed vertices and number of steps ?

Oriented graphs — WOMD/BOMD...

- Close the gaps in grids, bounded max. degree. Other graph classes...
- None is known about sequential metric dimension in oriented graphs.

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Thank you ! !

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~~May 11-15th, 2020~~ → (hopefully) May 15-21th, 2021...

GRASTA 2020

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May 11st-15th, 2020, Porquerolles, France

`http://www-sop.inria.fr/coati/events/grasta2020/`

if you are not already in the mailing list and want to attend, send me an email.