Recovery of disrupted airline operations using k-Maximum Matching in graphs

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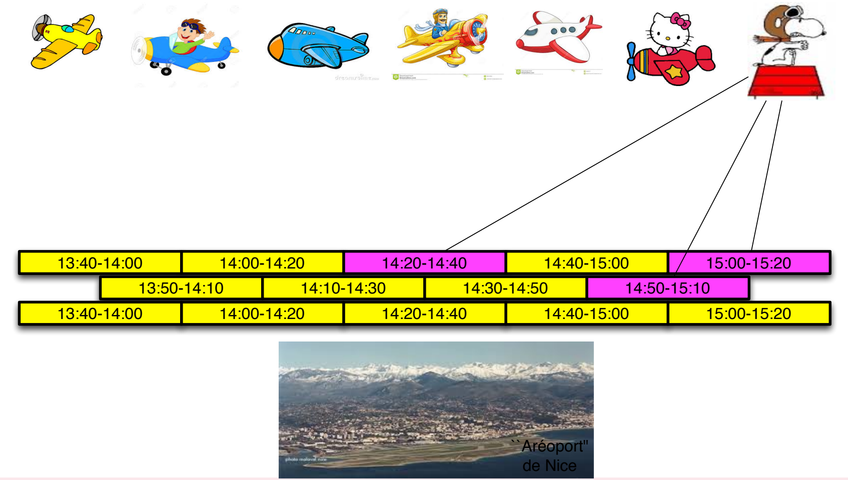
Assignment of slots for landing to aircrafts

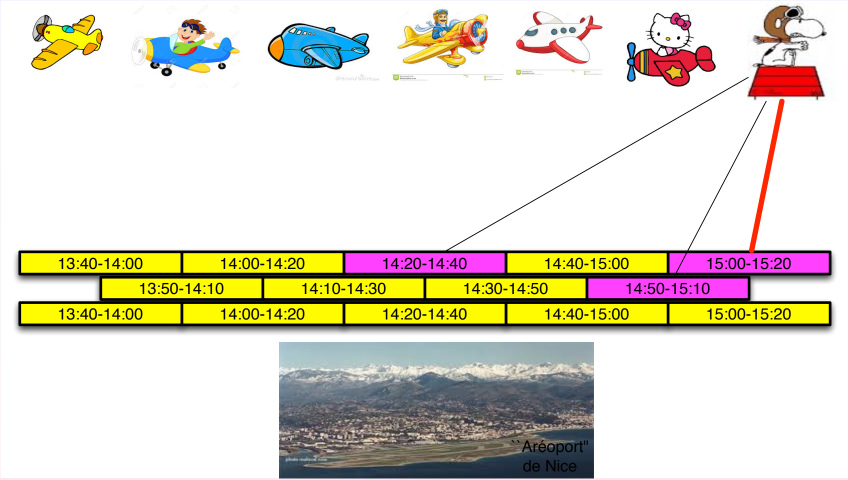
Aircrafts arriving at some airport

Assignment of slots for landing to aircrafts



Each aircraft has a set of available and compatible slots depending on the tracks, the schedules, the companies...

Assignment of slots for landing to aircrafts



Initially, one compatible slot is assigned to each aircraft.

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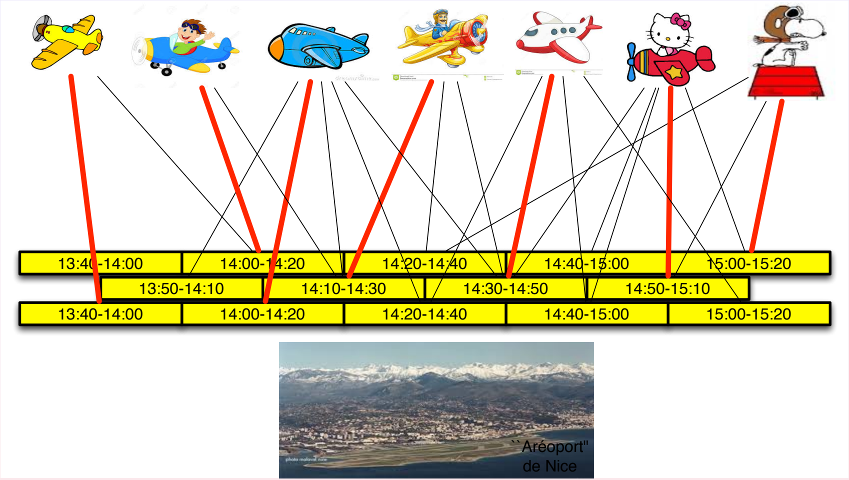

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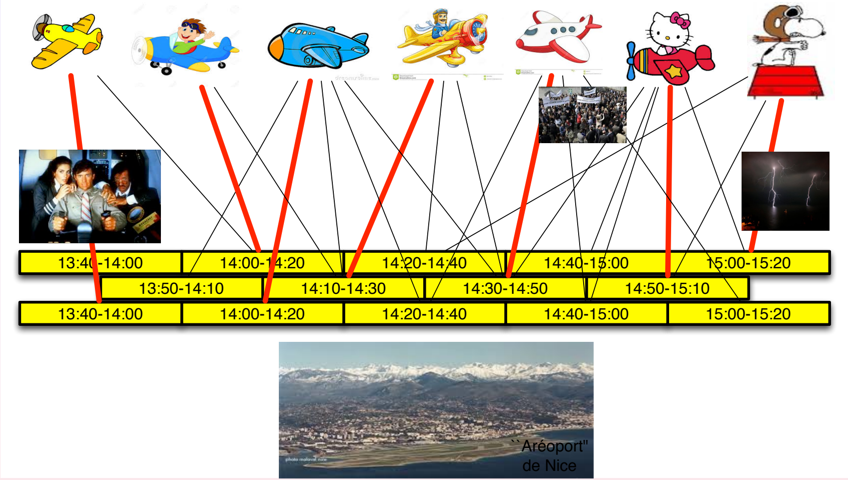
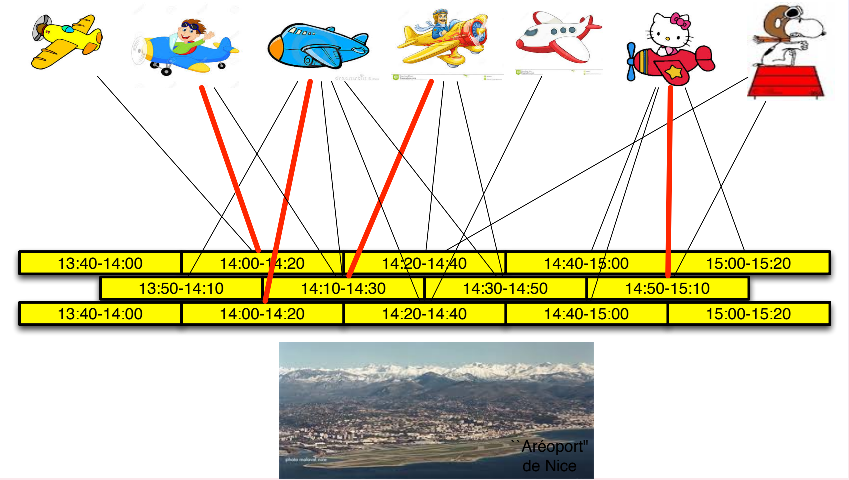


Imponderable problems may happen (no refund...)

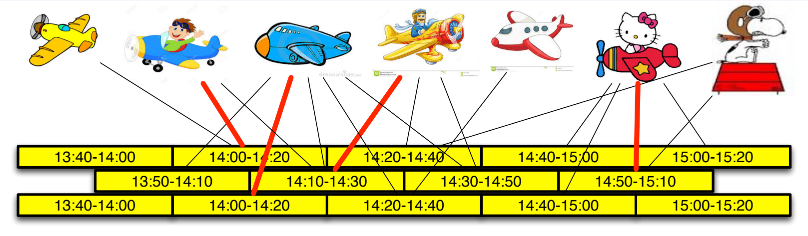
Assignment of slots for landing to aircrafts



How to return to a normal situation?

i.e., maximize # of aircrafts having a slot for landing!!

A simple matching problem in bipartite graphs?



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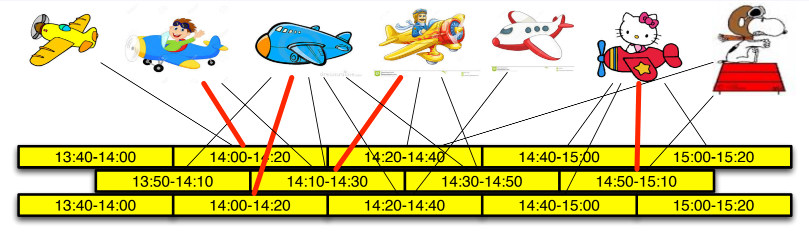

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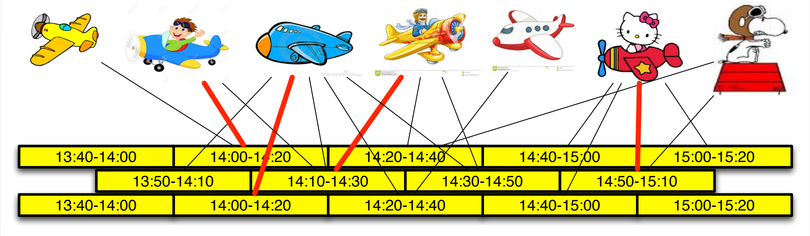

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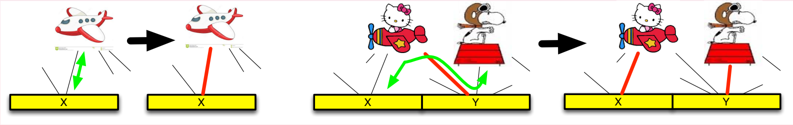

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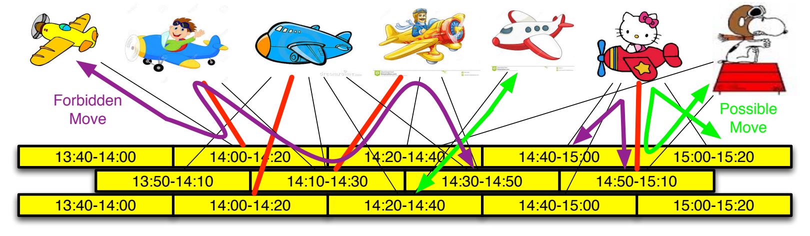

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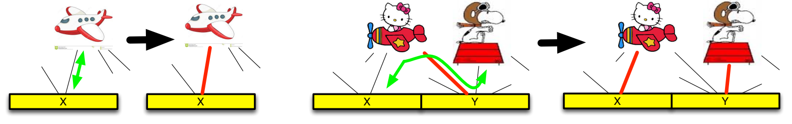
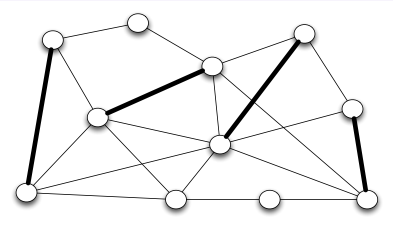


Reminder on Matchings in Graphs

Let $G = (V, E)$ be a graph.
A matching $M \subseteq E$ is a set of pairwise disjoint edges



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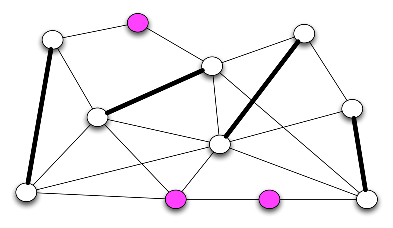

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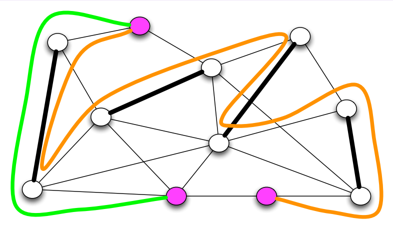

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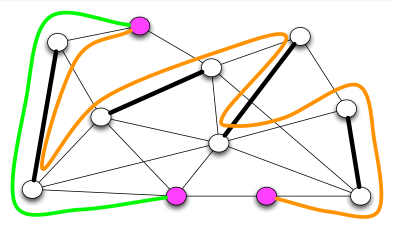

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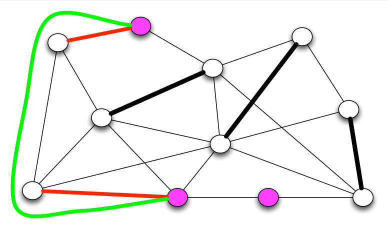

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M maximum matching ($|M| = \mu(G)$) iff no M-augmenting path.

$\Rightarrow$ the order in which the augmenting paths are augmented is not important

Computing a maximum matching

Maximum Matching in Bipartite Graphs (flow problem)

"easy" [Hungarian method, Kuhn 1955]

Maximum Matching $\mu(G)$

Polynomial [Edmonds 1965]

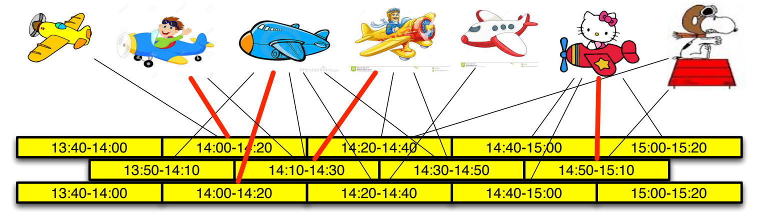
finding an augmenting path in polynomial time + Berge's theorem

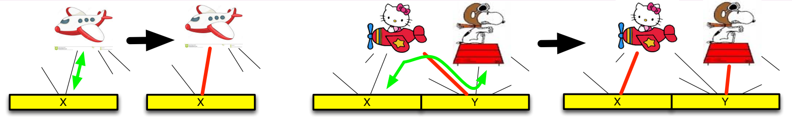
Augment "greedily" paths of length $\leq 2k-3$

$(1-\frac{1}{k})$-Approximation for $\mu(G)$ [Hopcroft,Kraft 1973]

Applications in wireless networks

Let's go back to slots assignment





Let G be a graph, M be a (partial) matching and $k \in \mathbb{N}$ odd

Let $\mu_k(G, M)$ be the maximum size of a matching that can be obtained from M by augmenting only paths of lenghts $\leq k$. here $k = 3$

Our problem

Given a graph G, a matching M and $k \in \mathbb{N}$ odd

Compute a matching of size $\mu_k(G, M)$ that can be obtained from M by augmenting only paths of lenghts $\leq k$.

Goal: algorithm that computes a sequence $(P_1, \cdots, P_r)$ such that:

- $\forall i \leq r$, P_i a path of length $\leq k$ in G
- $\forall i \leq r$, after augmenting $P_1, \cdots, P_{i-1}$ starting from M
  P_i is augmenting
- and r is maximum (w.r.t. these constraints)
  $(r_{max} + |M| = \mu_k(G, M))$

Easy Results

Problem: Given a graph G, a matching M and $k \in \mathbb{N}$ odd

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Case $M = \emptyset$. For any odd $k \geq 0$, $\mu_k(G,\emptyset) = \mu(G)$

Compute a maximum matching, augment its edges one by one.

Case $k = 1$. For any matching M, $\mu_1(G,M) = \mu(G \setminus V(M)) + |M|$.

The edges of M cannot be "modified"
Compute a max. matching in $G \setminus V(M)$, augment these edges 1 by 1

Our contributions

$k = 3$

Let G be a graph and M be a matching

Computing a matching of size $\mu_3(G, M)$, obtained from M by augmenting paths of length ≤ 3, is in P.

$k \geq 5$

Let G be a planar bipartite graph of max. degree 3 and M be a matching

Computing a matching of size $\mu_k(G, M)$, obtained from M by augmenting paths of length at most $k \geq 5$, is NP-complete.

$G = T$ is a tree.

Computing $\mu_k(T, M)$ (k odd) can be done in polynomial-time in trees T

- with bounded max. degree Δ (dynamic prog., FPT in $k + \Delta$)
- or with vertices of degree ≥ 3 pairwise at distance $> k$.

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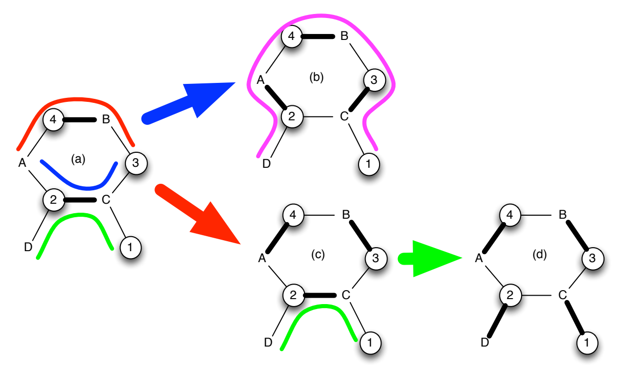
Why bounding the length of the augmenting paths makes the problem harder? ex: $k = 3$



$(a) \to (b)$ not optimal, but $(a) \to (c) \to (d)$ optimal

$\Rightarrow$ The result is impacted by the order in which paths are augmented.

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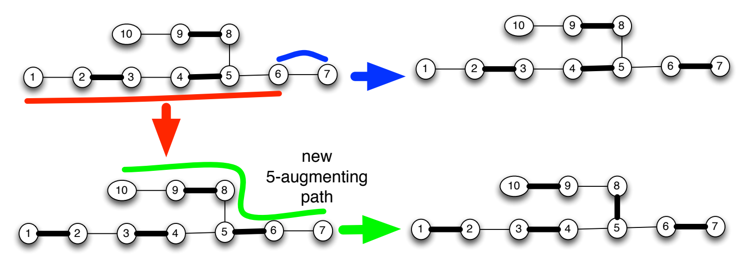

$\Rightarrow$ The order in which paths are augmented impacts the creation of new augmenting paths that are necessary to reach the optimum

Our contributions: ideas of the proofs

$k = 3$

$\mu_3(G, M)$ can be computed in polynomial time

- possible to focus only on 3-augmenting paths initially present;
- after "reducing" the graph, augmentations may be in any order.

$k \geq 5$

$\mu_5(G, M)$ is NP-hard even if G planar bipartite of max. degree 3.

Reduction from 3-SAT where "creating" new augmenting paths corresponds to a choice in the assignment.

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Question: Complexity of $\mu_k(T, M)$ in trees T?

What we know:

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Going further: $\Rightarrow$ a new problem

New Problem: Given a graph G, a matching M and $k \in \mathbb{N}$ odd

Compute a matching of size $\mu_{=k}(G, M)$ that can be obtained from M by augmenting only paths of lenghts $= k$.

Our results

- $\forall G$ and matching M, deciding if $\mu_{=3}(G, M) \leq q$ is NP-complete
- given a tree T, M a matching and $q, k \in \mathbb{N}$ as inputs, deciding if $\mu_{=k}(T, M) \leq q$ is NP-complete

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Conclusion

Firstly: Learn how to communicate with companies

After many months of discussion, hypotheses are changing...

Actual (?) constraints of the system/of Airline Operation Controllers

- Switch alternating cycles of size 4 and augment paths of length 1
- two classes of slots : the ones of the company and the others

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Muchas Gracias / Muito Obrigado / Gramàci ... !