Jeux des gendarmes et du voleur dans les graphes.

Mineurs de graphes, stratégies connexes, et approche distribuée

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Soutenance de thèse
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Outline

Motivation: Layout problems

Numerical analysis

Reorder rows and columns of sparse matrix,
Choleski factorisation, Gaussian elimination.

VLSI design

Circuits must be laid out in order to minimize physical and cost constraints.

Relationship with several graph's parameters:
bandwidth, cutwidth, profile, minimum fill-in, etc.

Motivation: Computational complexity

Models of computation: Pebble games

A DAG represents a computation circuit

Model for the allocation of registers in a processor.

Tradeoff space/time complexity

- Number of pebbles = space complexity
- Number of *moves* = time complexity

Kirousis and Papadimitriou 86

The smallest number of searchers to clear G is equal to the smallest number of pebbles among acyclic orientations of G

Motivation: Graph minors theory

Minor of a graph G: graph obtained by a sequence of vertex or edge deletions and edge contraction.

Wagner's conjecture

Graphs are WQO for the minors' relation.

Graph Minors, [JCTB, 1983-]

Robertson and Seymour prove the Wagner's conjecture.

- Any minor closed class of graphs admits a finite obstruction set
- Tree-like decompositions of graphs excluding a minor.

General problem

Context

A fugitive is running in a graph.
A team of searchers is aiming at capturing the fugitive.

Goal

To design a strategy that capture **any** fugitive using the **fewest searchers as possible**.

Variants of graph searching games

Invisible searchers

Random walk, Aleliunas *et al.* [FOCS 79]. Graph's exploration
Minimize the capture time using only one searcher.

Visible searchers

- **playing rules**: turn by turn, or simultaneous moves;
- **way to capture the fugitive**: same location, domination;
- **fugitive/searchers'moves**: move along edges or/and jump from a vertex to another one;
- **fugitive/searchers' velocity**;
- **fugitive's visibility**.

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Taxonomy of graph searching games

	fugitive's characteristics			
	bounded speed		arbitrary fast	
	visible	invisible	visible	invisible
turn by turn game	Cops and Robber	X	X	?
simultaneous moves	?	X	Visible graph searching	Graph searching

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Table: Classification of the graph searching games

Search Strategy, Parson. [GTC,1978]
Model of Kirousis and Papadimitriou. [TCS,86]

Sequence of two basic operations,. . .

1. Place a searcher at a vertex of the graph;
2. Remove a searcher from a vertex of the graph.

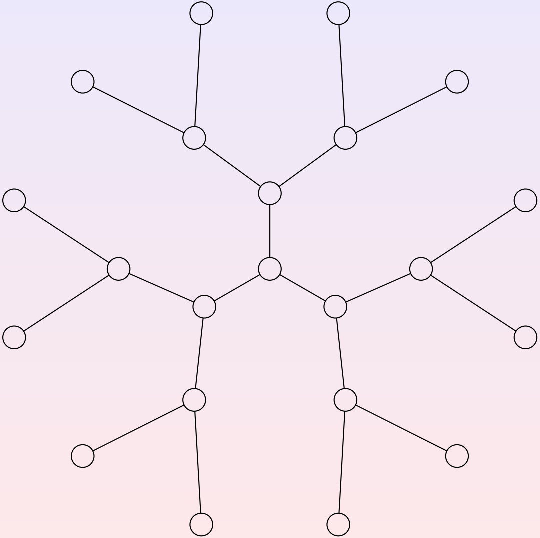
. . . that must result in catching the fugitive

The fugitive is caugth when it occupies the same vertex as a searcher and it cannot move away.

The node-search number

Let $\mathbf{s}(G)$ be the smallest number of searchers needed to catch an invisible fugitive in a graph G.

Simple example: A ternary tree



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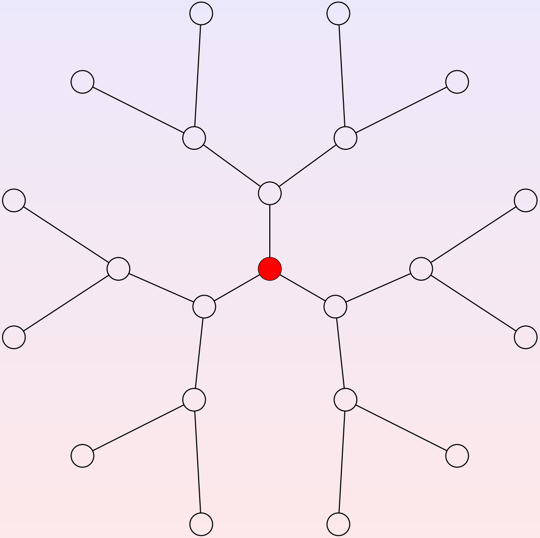

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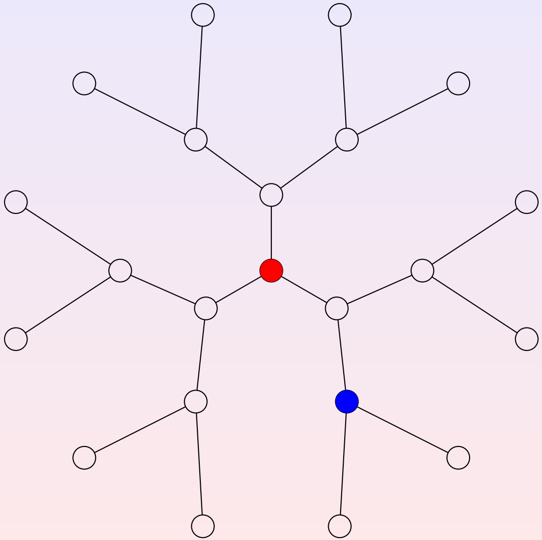

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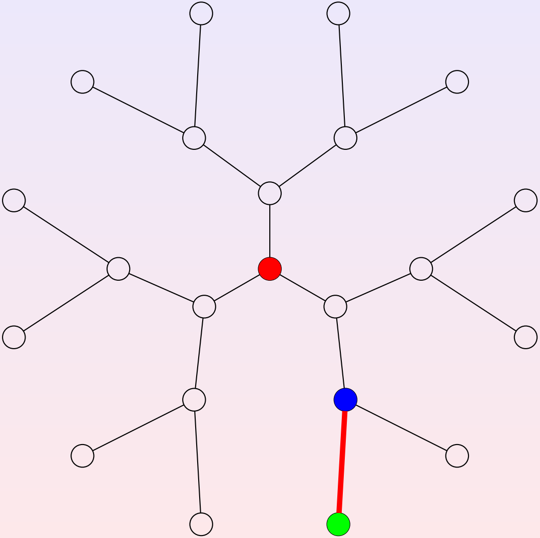

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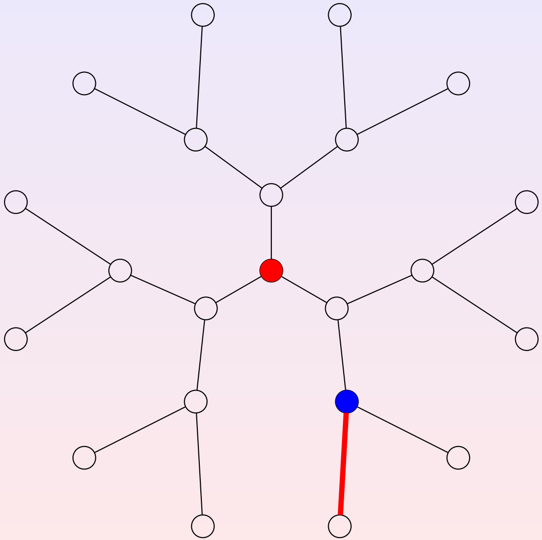

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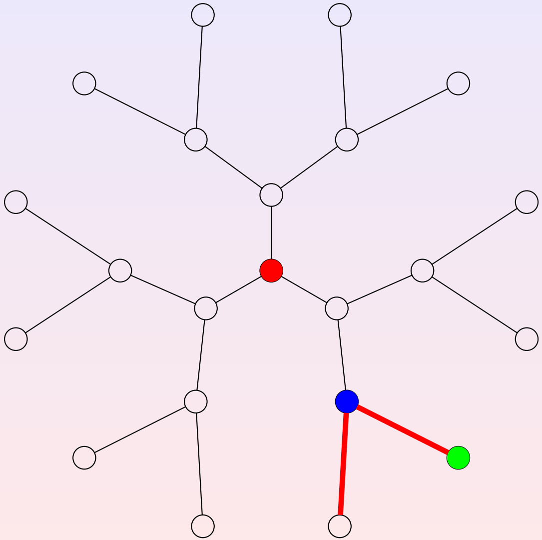

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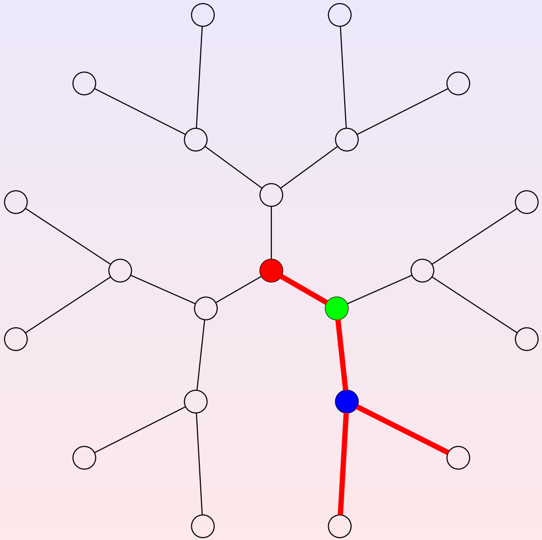

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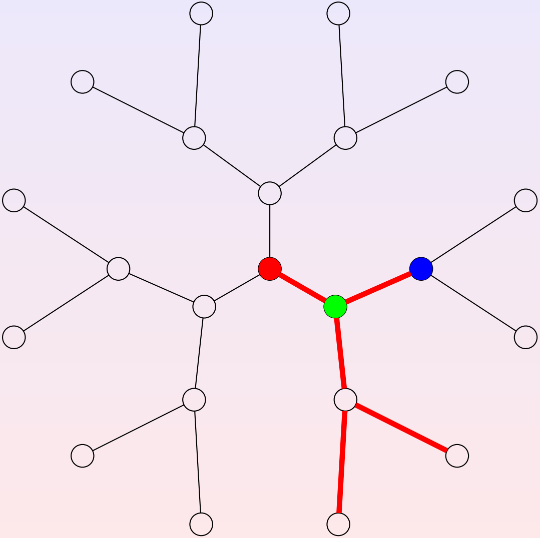

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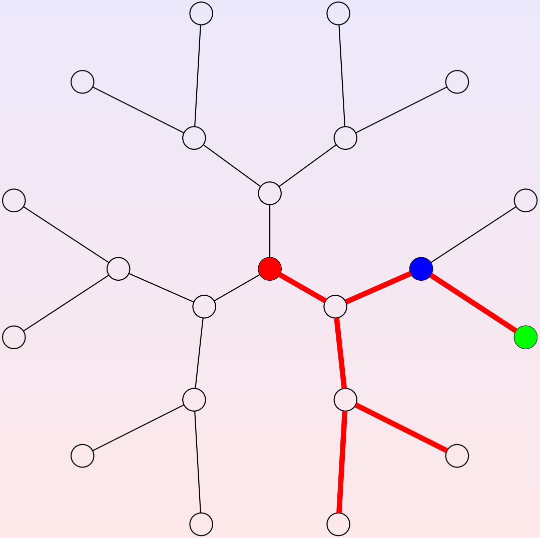
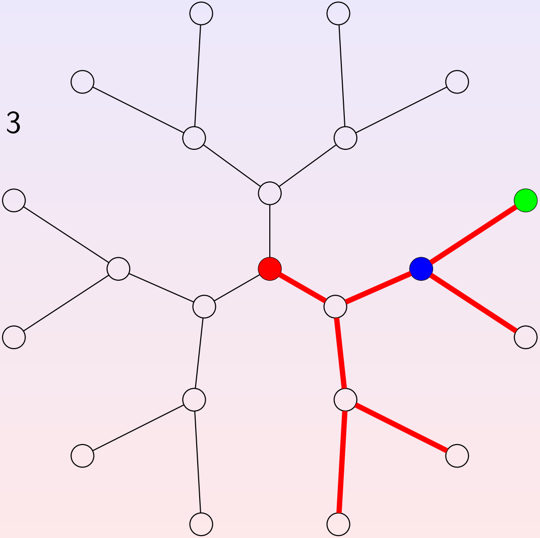


Simple example: A ternary tree

$\mathbf{s}(T) = 3$



Visibility of the fugitive

Visible fugitive

The fugitive is visible if, at every step, searchers know its position.
Let $\mathbf{vs}(G)$ be the visible search number of the graph G.

Obviously, for any graph G, $\mathbf{vs}(G) \leq \mathbf{s}(G)$.

In trees

For any n-nodes tree T, $\mathbf{s}(T) \leq 1 + \log_3(n-1)$ (tight)
Megiddo *et al.* [JACM 88]

For any tree T (with at least 2 vertices), $\mathbf{vs}(T) = 2$.

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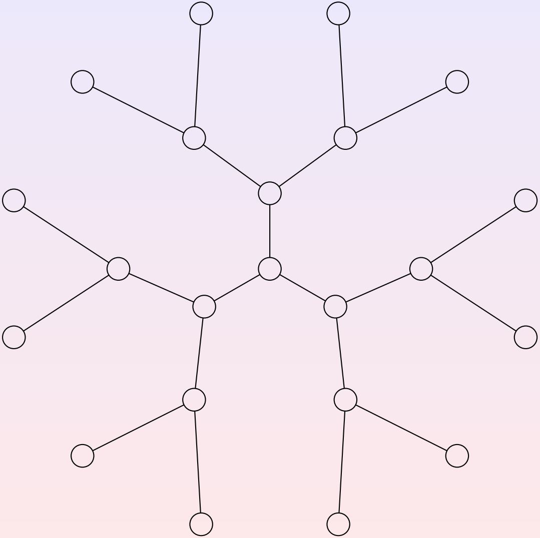
Visible graph searching in a tree



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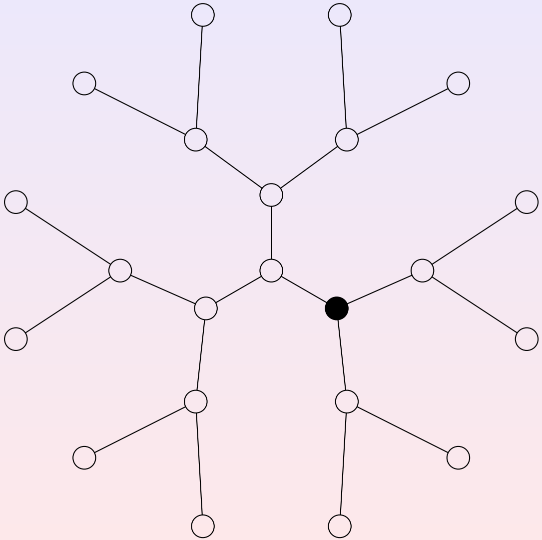

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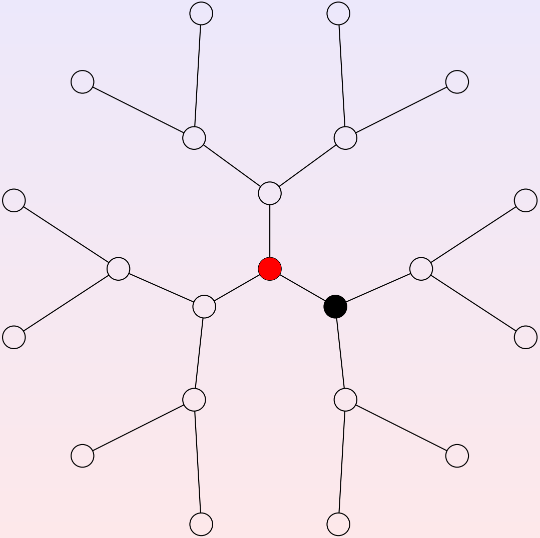

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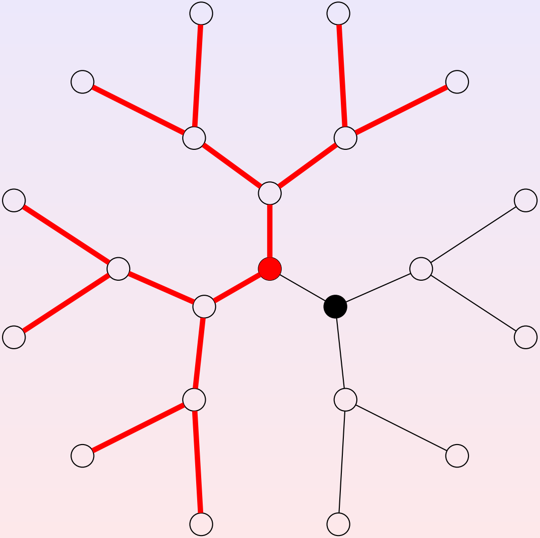

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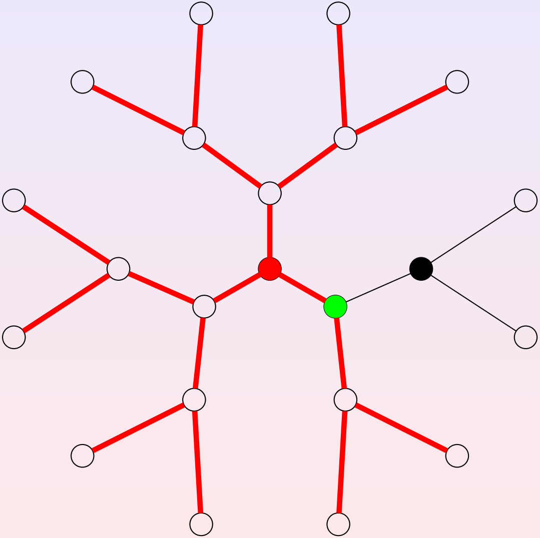

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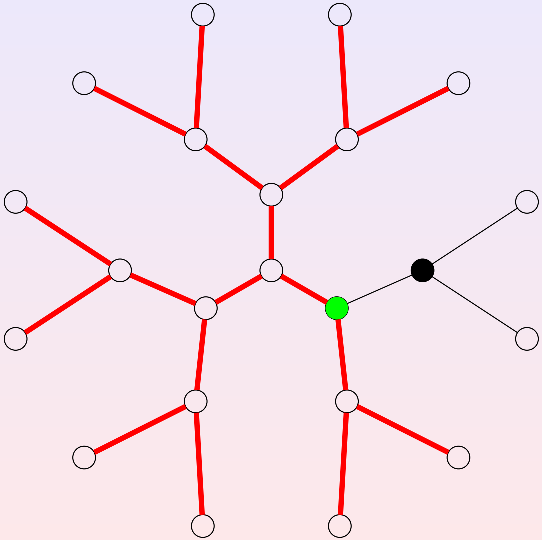
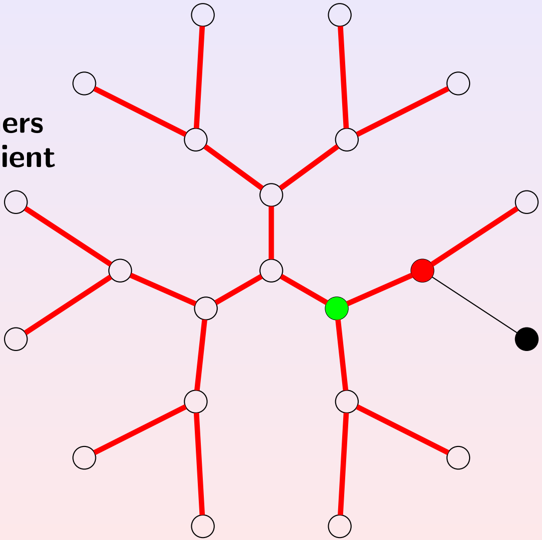


Visible graph searching in a tree

2 searchers are sufficient



NP-hardness

The following problems are NP-hard

Input: a graph G, an integer $k > 0$, — Megiddo *et al.*, [JACM 88]
Output: $\mathbf{s}(G) \leq k$?

Input: a graph G , an integer $k > 0$, — Seymour and Thomas [JCTB 93]
Output: $\mathbf{vs}(G) \leq k$?

Remark: linear in the class of trees, Skodinis [JAlg 03]

NP-membership? Certificate?

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Monotonicity and NP-completeness

A vertex v is recontaminated if the fugitive can move to v after v has been occupied by a searcher.

Monotonicity

A search strategy is monotone if no recontamination ever occurs. That is, a vertex is occupied by a searcher only once.

Recontamination does not help

Threre always exists an optimal monotone search strategy.

invisible fugitive:	LaPaugh, [JACM 93] Bienstock and Seymour [JAlg 91]
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Corollary: The above problems belong to NP.

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Search numbers and graphs'decompositions

Thanks to the monotonicity, we get:

Search number and Pathwidth (pw)

For any graph G, $\mathbf{s}(G) = \mathbf{pw}(G) + 1$,
Kinnersley [IPL 92],
Ellis, Sudborough, and Turner [Inf.Comp.94]

Visible search number and Treewidth (tw)

For any graph G, $\mathbf{vs}(G) = \mathbf{tw}(G) + 1$,
Seymour and Thomas [JCTB 93]

Outline

Non-deterministic Graph Searching

Invisible fugitive

An Oracle permanently knows the position of the fugitive

One extra operation is allowed

Searchers can perform a query to the oracle:
"What is the current position of the fugitive?"

Sequence of three basic operations

1. Place a searcher at a vertex of the graph;
2. Remove a searcher from a vertex of the graph;
3. Perform a query to the Oracle.

Tradeoff number of searchers / number of queries

q-limited (non-deterministic) search number, $s_q(G)$

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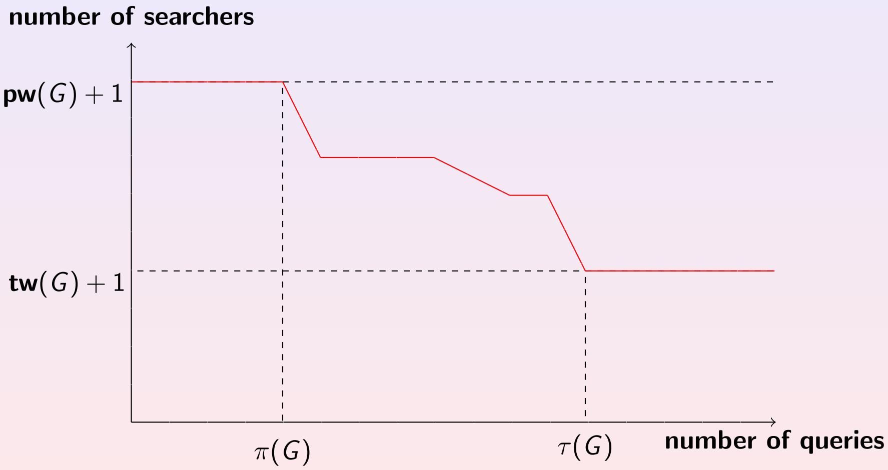
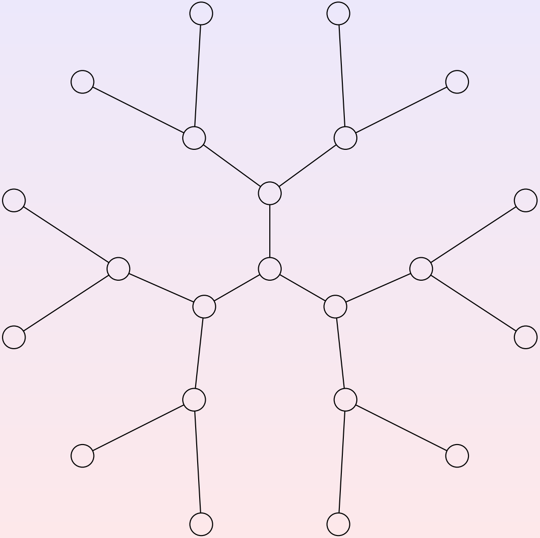
Controlled Amount of Nondeterminism



Still the same ternary tree

$s_0(T)=3$



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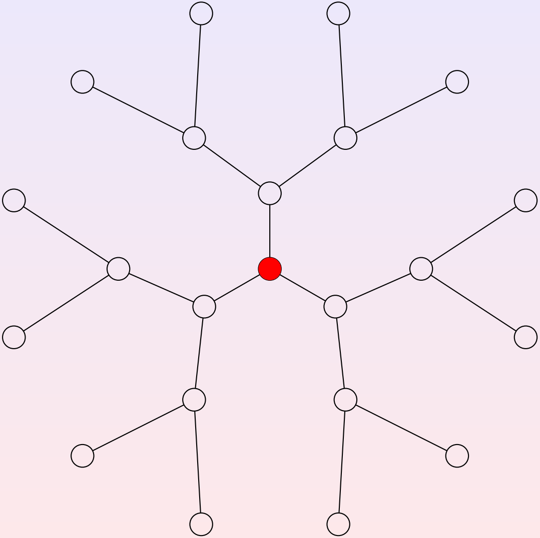

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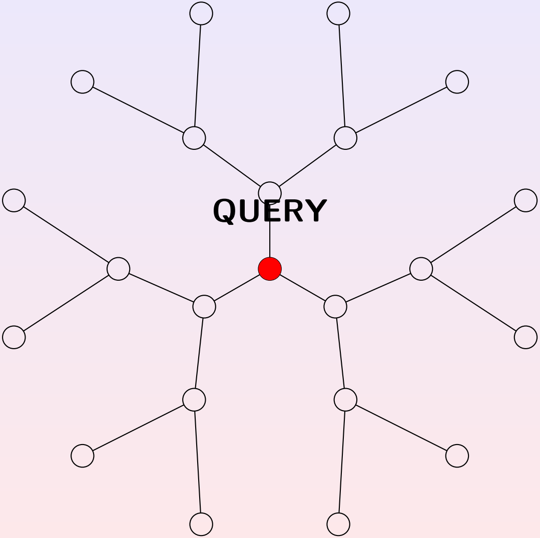

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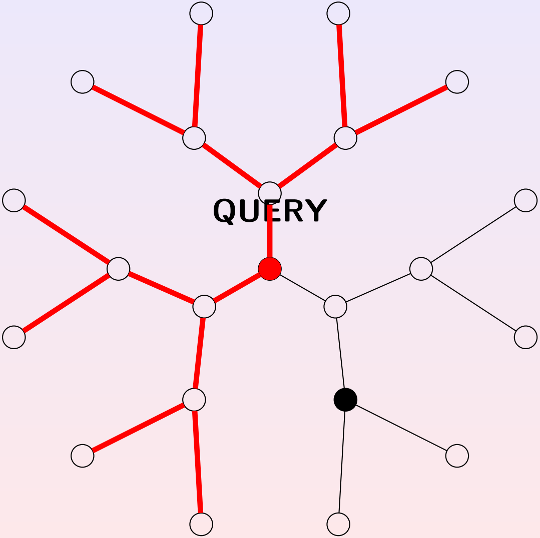

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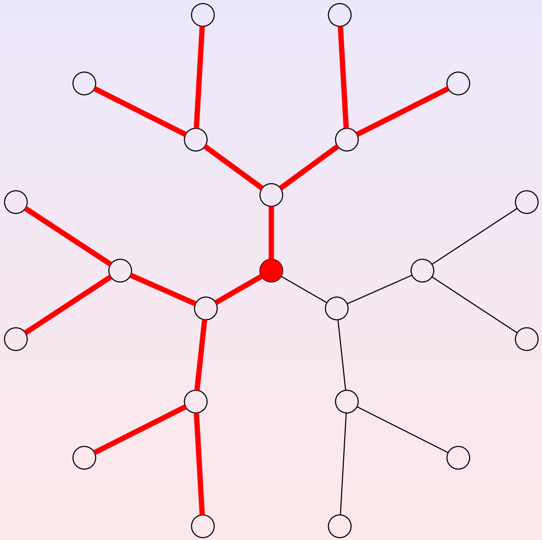

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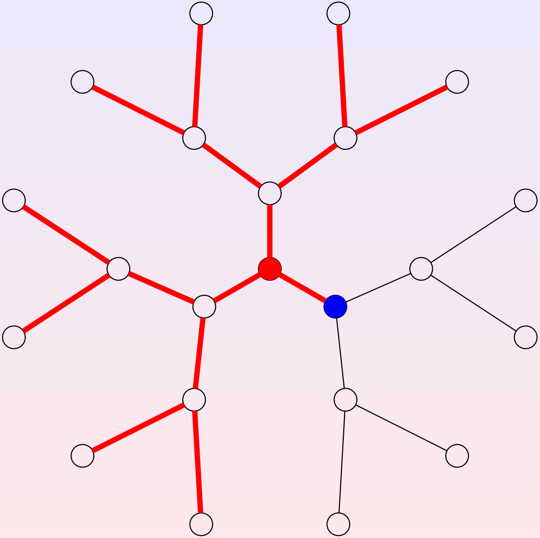

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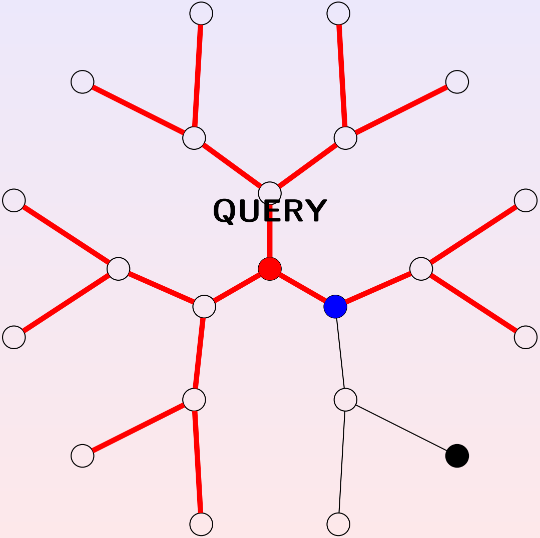
2 queries



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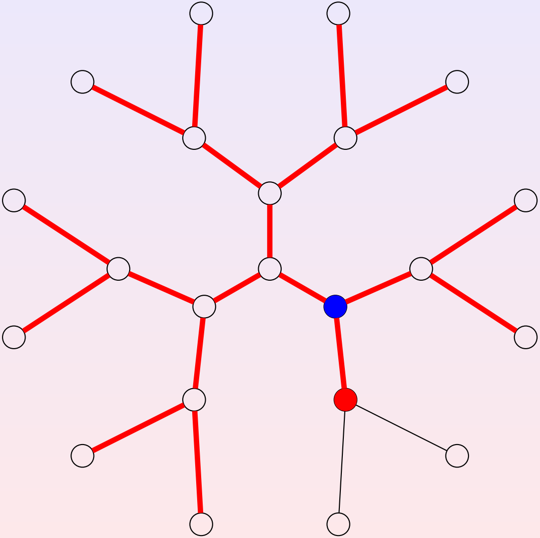

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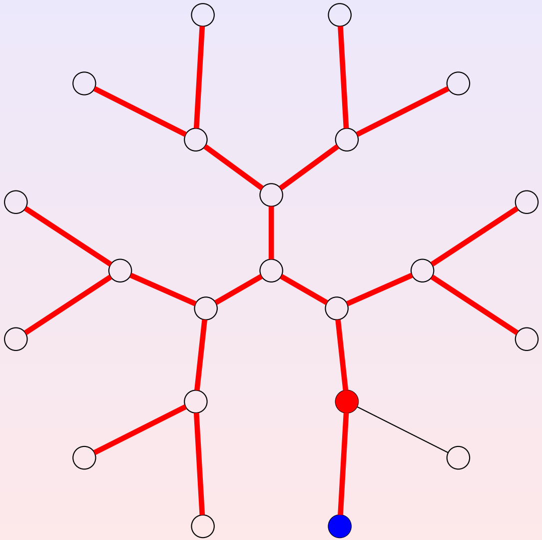

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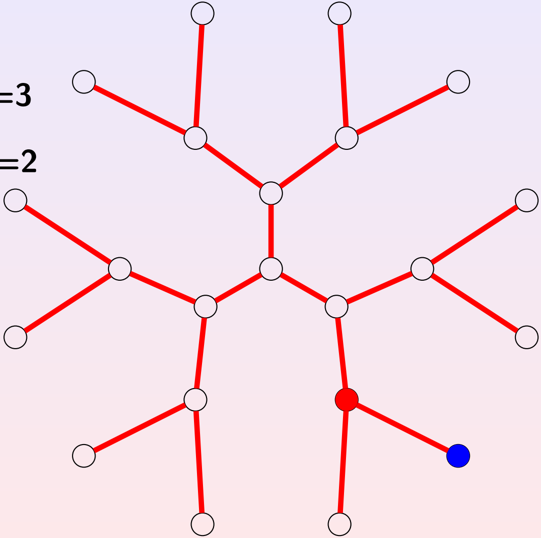
$s_2(T)=s_\infty(T)=2$



Results

In collaboration with F. Mazoit

For any $q \geq 0$, recontamination does not help to catch a fugitive in G performing at most q queries.

- Constructive proof;
- Generalize the existing proofs ($q = 0$ and $q = \infty$).

In collaboration with F.V. Fomin and P. Fraigniaud

- Equivalence between non-deterministic graph searching and branched tree-decomposition;
- Exponential exact algorithm computing $\mathbf{s}_q(G)$ in time $O^*(2^n)$;
- $\mathbf{s}_q(G) \leq 2\, \mathbf{s}_{q+1}(G)$ (almost tight).

Monotonicity: Search-tree

Auxiliary structure inspired by the tree-labelling [Robertson and Seymour, Graph Minor X]:

Search-tree = A tree T labelled with subsets of $E(G)$

For any vertex $v \in V(T)$ incident to $e_1, \ldots, e_p$:

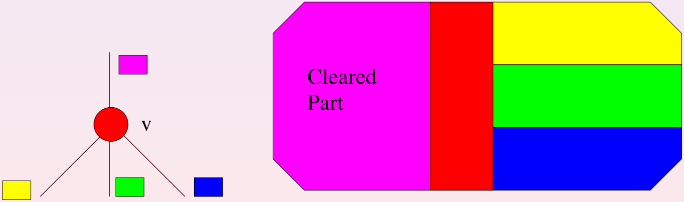
- label of v: $\ell(v) \subseteq E(G)$
- label of e_i: $\ell_v(e_i) \subseteq E(G)$

Any edge has two labels: one for each extremity.

Monotonicity: Search-tree

Non-deterministic search strategy ⇒ Search-tree

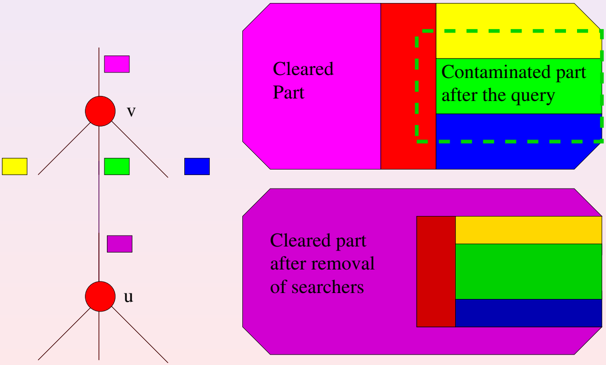
- placement of searchers ⇒ vertex of T
- query ⇒ fork (vertex of T with more than one child)
- removal of searchers ⇒ edge of T



Monotonicity: Search-tree

Two Properties

1. $\{\ell(v), \ell_v(e_1), \ell_v(e_2), \ldots, \ell_v(e_p)\}$ partition of $E(G)$;
2. $\forall e = \{u, v\} \in E(T)$, $\ell_v(e)$ and $\ell_u(e)$ are disjoint.



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Sketch of the proof

- (possibly non monotone) strategy $\Rightarrow$ Search-tree
- weight function over the search-trees
- minimal search-tree $\Rightarrow$ monotone strategy
- local optimization without increasing neither the number of searcher, nor the number of queries.

Outline

Connected Graph Searching

Limits of the Parson's model

- Searchers cannot move at will in a real network;
- Secured communications.

Connected Search Strategy, Barrière *et al.*, [SPAA 02]

At any step, the cleared part of the graph must induce a connected subgraph.
Let $\mathbf{cs}(G)$ be the connected search number of the graph G.

Two main questions

What is the cost of connectivity? ratio $\mathbf{cs}/\mathbf{s}$?
Monotonicity property of connected graph searching?

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The cost of connectedness

In terms of number of searchers

For any tree T, $\mathbf{s}(T) \leq \mathbf{cs}(T) \leq 2\,\mathbf{s}(T) - 2$. (tight)
Barrière, Flocchini, Fraigniaud, and Thilikos [WG 03]

For any connected graph G, $\mathbf{cs}(G) \leq \mathbf{s}(G)\ (2 + \log |E(G)|)$.
Fomin, Fraigniaud, and Thilikos [Tech. Rep. 04]

About monotonicity

Recontamination does not help in trees.
Barrière, Flocchini, Fraigniaud, and Santoro [SPAA 02]

Recontamination helps in general.
Alspach, Dyer, and Yang [ISAAC 04]

Results: Case of a invisible fugitive

Using the concept of *connected* tree-decomposition.

In collaboration with P. Fraigniaud

For any n-node connected graph G, $\mathbf{cs}(G)/\mathbf{s}(G) \leq \log n$.

Graphs with bounded chordality k

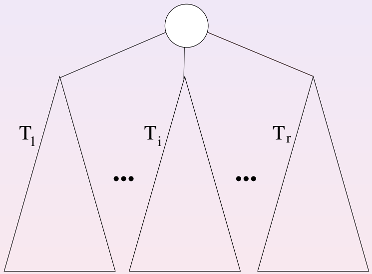
(T,X) an optimal tree-decomposition of G

$\mathbf{cs}(G) \leq (\mathbf{tw}(G)\lfloor k/2 \rfloor + 1)\mathbf{cs}(T)$.

$\Rightarrow \mathbf{cs}(G)/\mathbf{s}(G) \leq 2\,(\mathbf{tw}(G)+1)$ if G chordal

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



For any $1 \leq i \leq r$, $G[T_i]$ is a connected subgraph with at most $n/2$ vertices.

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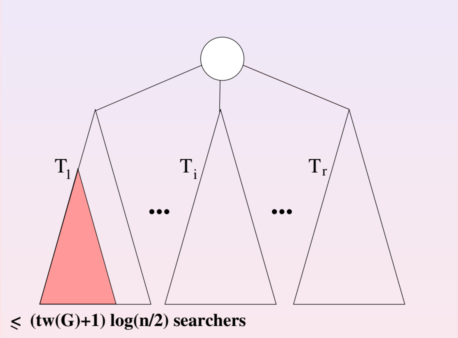

There is a connected search strategy for $G[T_1]$, using at most $(\mathbf{tw}(G)+1)\log(n/2)$ searchers.

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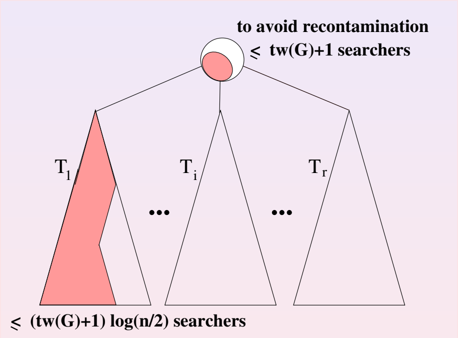

At most $\mathbf{tw}(G)+1$ searchers are required to protect $G[T_1]$ from recontamination from the remaining part of G.

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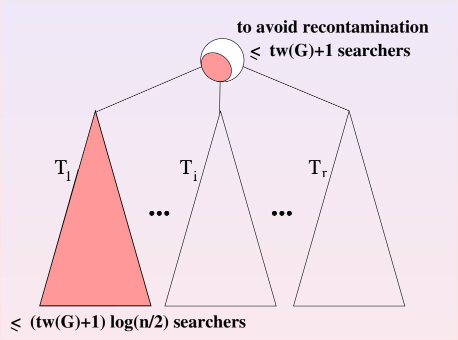

Then, we can terminate the clearing of $G[T_1]$.

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

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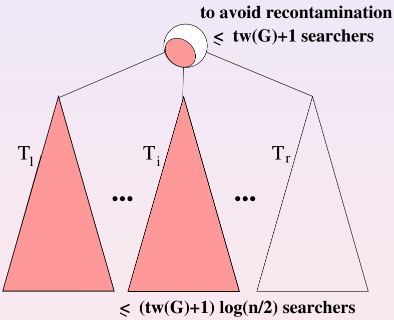
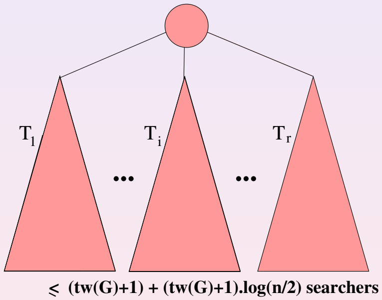


The $(\mathbf{tw}(G) + 1) \log(n/2)$ searchers can be used to clear another subgraph $G[T_i]$, and so on...

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



Connected search strategy using at most $(\mathbf{tw}(G)+1)\log n$ searchers. Thus, $\mathbf{cs}(G) \leq \mathbf{s}(G)\log n$

Results: Case of a visible fugitive

In collaboration with P. Fraigniaud

For any n-node graph G, $\mathbf{cvs}(G)/\mathbf{vs}(G) \leq \log n$
tight for monotone strategies: $\mathbf{mcvs}(G)/\mathbf{vs}(G) \geq \Omega(\log n)$.

In collaboration with P. Fraigniaud

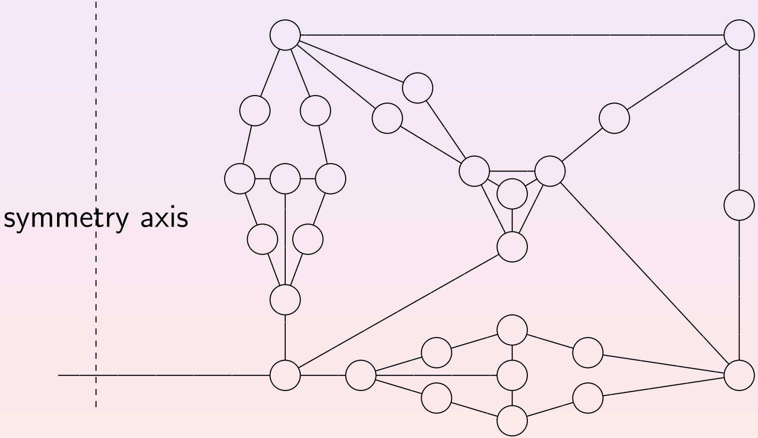
In visible connected graph searching, recontamination helps

For any $k \geq 4$, there exists a graph G such that $\mathbf{cvs}(G) = 4k+1$ and any monotone connected visible search strategy uses at least $4k+2$ searchers.

Non-monotonicity

Recontamination helps in visible connected graph searching

Let G be the graph below: **mcvs**$(G) >$ **cvs**$(G) = 4$.



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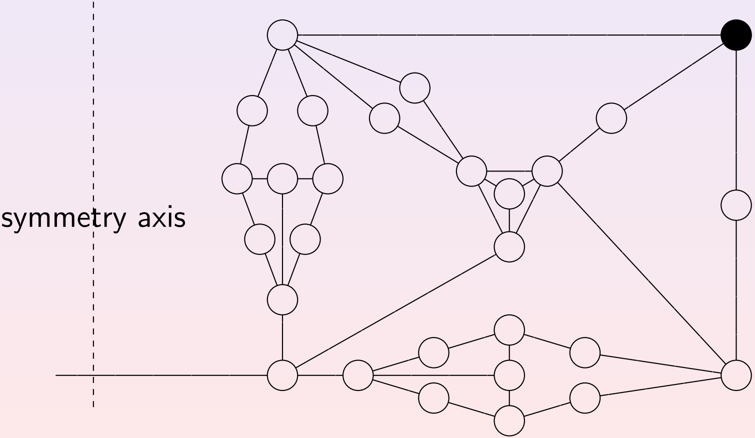

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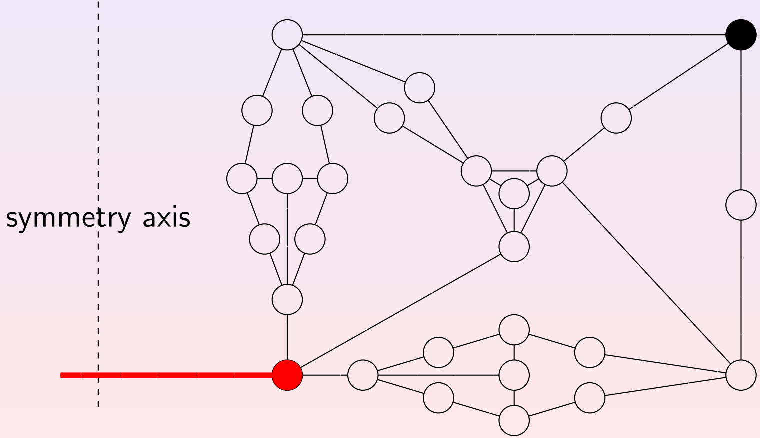

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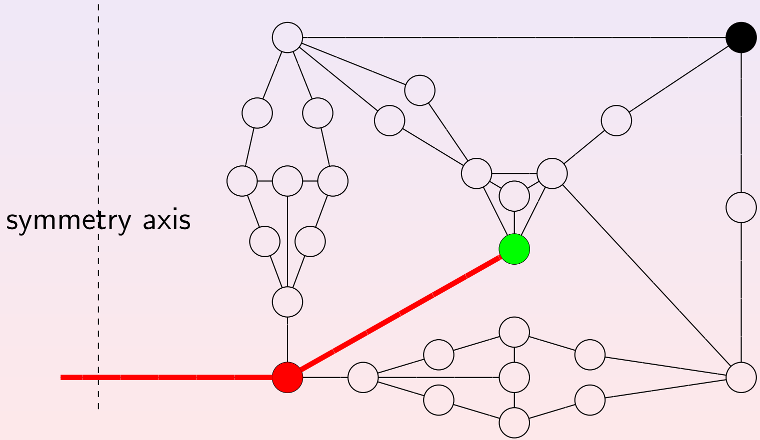

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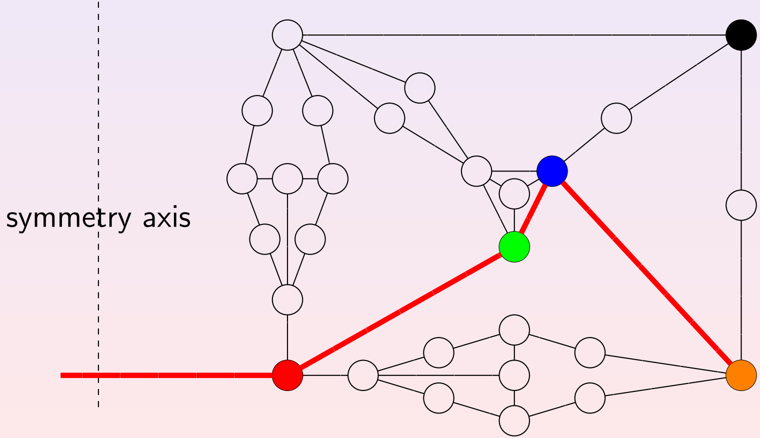

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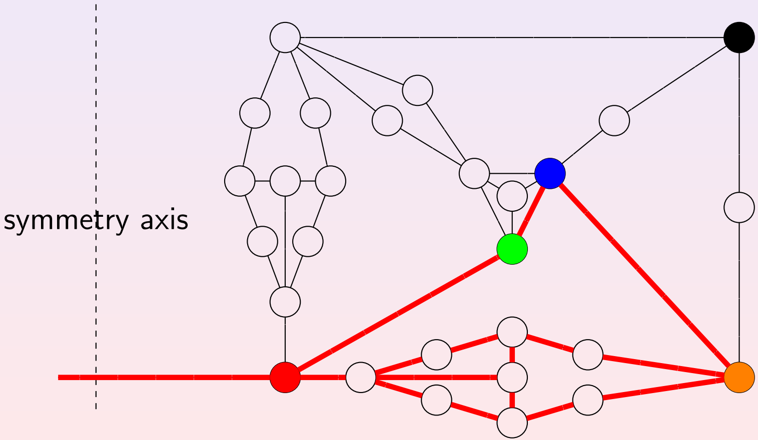

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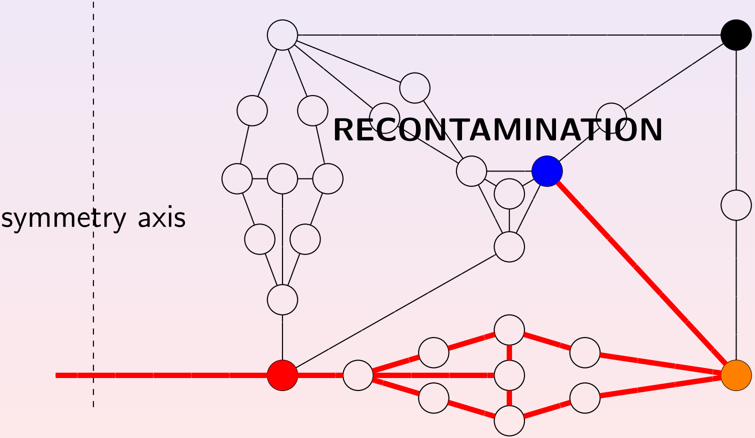

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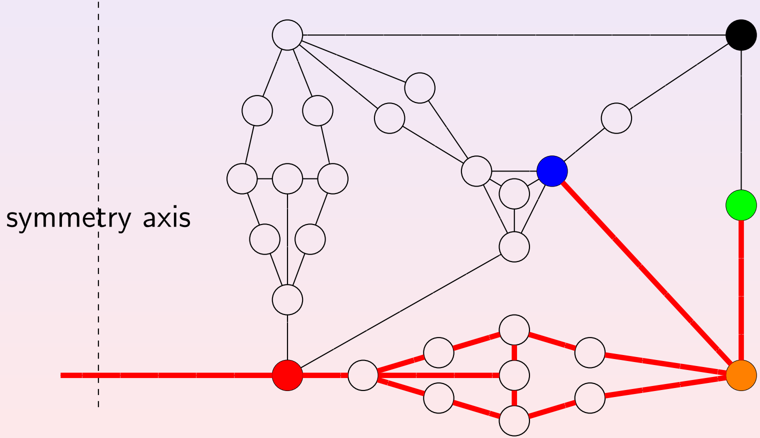

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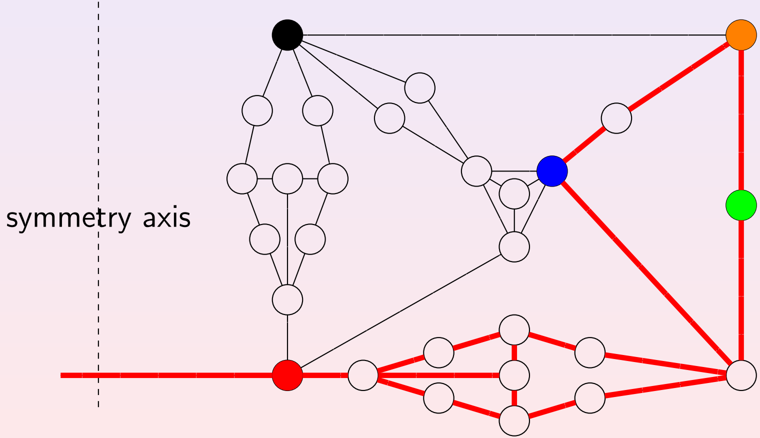

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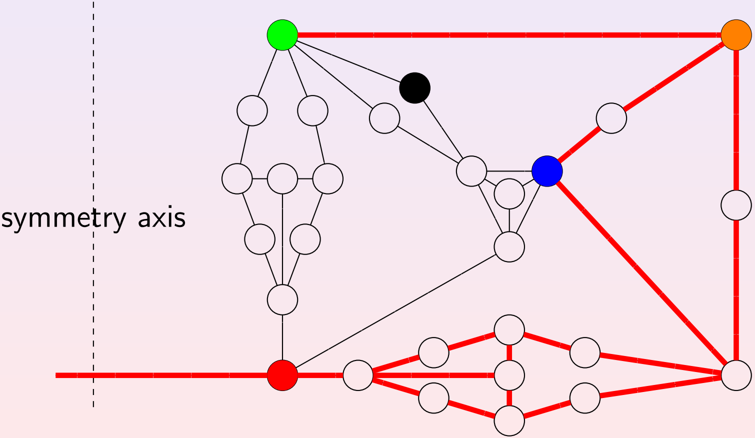

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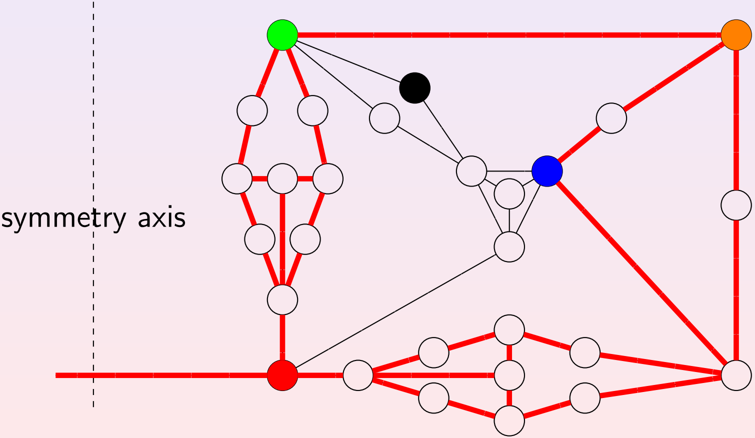

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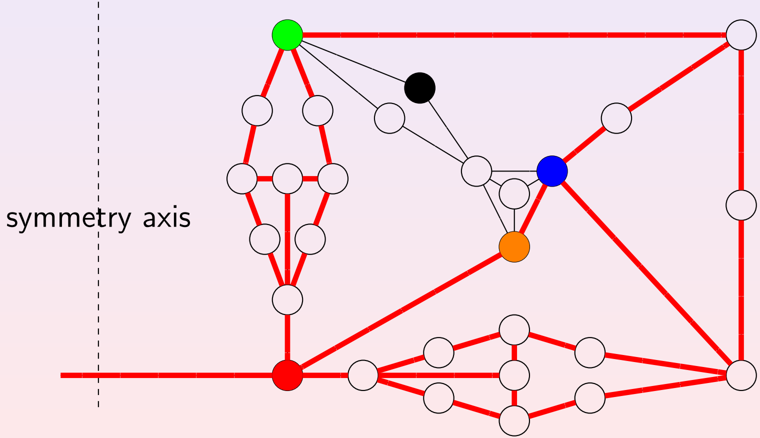

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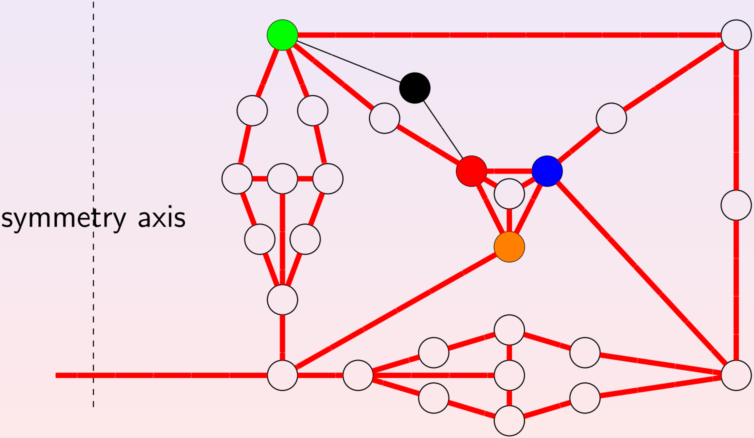

Outline

Graph searching in a distributed way

Distributed search problem

To design a *distributed protocol* that enables the *minimum number* of searchers to clear the network.
The searchers must compute themselves a strategy.

We consider connected search strategies.

mcs refers to the smallest number of searchers required to catch an invisible fugitive in a monotone connected way.

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Distributed graph searching: model

The searchers

- autonomous mobile computing entities with distinct IDs;
- automata with $O(\log n)$ bits of memory;
- their decision is computed locally.

The network

- undirected connected graph;
- local orientation of the edges;
- whiteboards on vertices (zone of local memory);
- asynchronous environment.

Distributed graph searching: related work

The searchers have a prior knowledge of the topology.

Protocols to clear specific topologies

- **Tree**. Barrière *et al.*, [SPAA 02]
- **Mesh**. Flocchini, Luccio, and Song. [CIC 05]
- **Hypercube**. Flocchini, Huang, and Luccio. [IPDPS 05]
- **Tori**. Flocchini, Luccio, and Song. [IPDPS 06]
- **Sierpinski's graph**. Luccio. [FUN 07]

A monotone connected and strategy is performed using **mcs** $+ 1$ searchers.

Remark:

The extra searcher is due to the asynchronicity of the network and it is necessary [CIC 05].

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Results

In collaboration with L. Blin, P. Fraigniaud and S. Vial

Distributed protocol that enable **mcs**$(G)+1$ searchers to clear an unknown graph G in a connected way

Drawback: the strategy is not monotone and may be performed in expentional time.

In collaboration with D. Soguet

$\Theta(n \log n)$ bits of information must be provided to the searchers to clear a unknown graph in a monotone connected way.

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Outline

1. Introduction
2. Non-deterministic Graph Searching
3. Connected Graph Searching
4. Distributed Graph Searching
5. Conclusion and Further Works

Summary of the results

Non-deterministic graph searching

A unified approach of visible and invisible graph searching
Unified proof of monotonicity.

Connected graph searching

Upper bounds for the ratio cs/s
Case of a visible fugitive

Distributed graph searching

Distributed protocol to clear an unknown graph
Amount of information required for monotonicity

Open Problems

Non-deterministic graph searching

Explicit FPT Algorithm?

Polynomial-time algorithm in trees?

Connected graph searching

cs/s ?

FPT Algorithm?

NP-membership?

Distributed graph searching

Tradeoff between amount of information and number of searchers?

Further Works

Directed graph decompositions...

Directed treewidth. [Johnson *et al.*, 95]
DAG-width. [Obdrzalek, and Berwanger *et al.* 06]
Kelly-width. [Hunter and Kreutzer, 07]

... and related directed graph searching games

Monotonicity ? [Barat 06, Adler 07]
Open problem: Is the graph searching game corresponding to DAG-width (resp., Kelly-width) monotone ?

Matroid decompositions

Matroid's treewidth, [Hlineny and Whittle, 06]
Matroid's branchwidth [Mazoit and Thomassé, 06]

Thank's