Tradeoffs in routing reconfiguration problems

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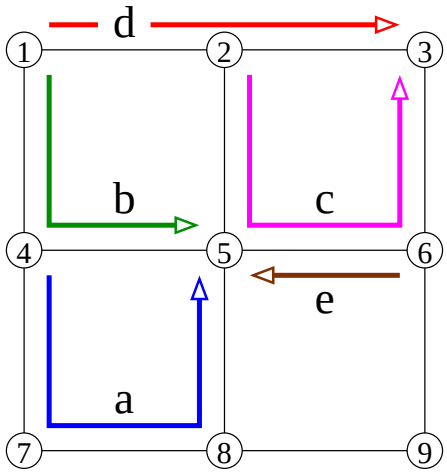
How to change the routing ?



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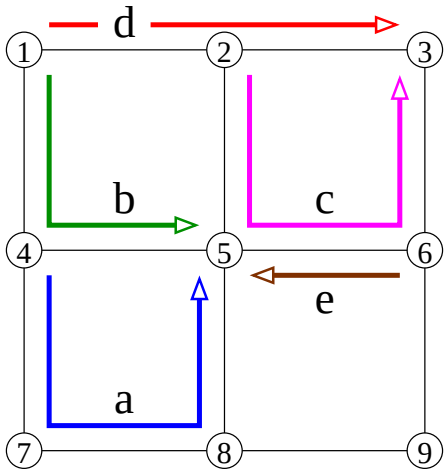
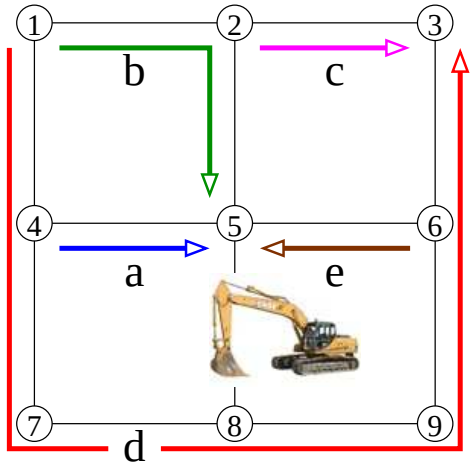




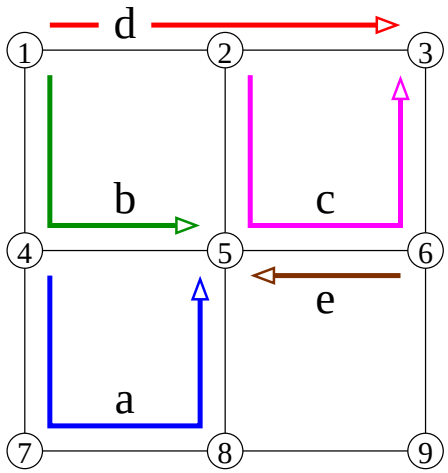
Motivation

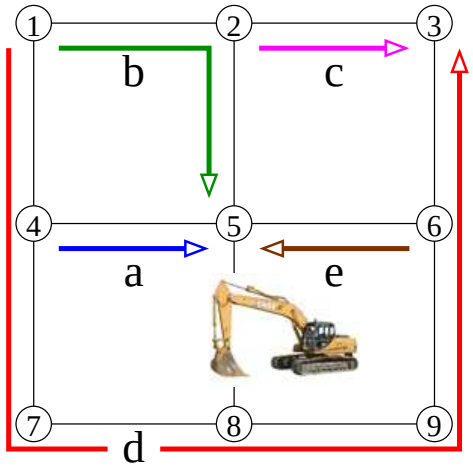
- Maintenance operations
- Optimize usage of resources (reduce blocking probability)
- Fault tolerance

Context

- Telephone: call repacking (70's)
- WDM, MPLS, ATM

Possible approaches





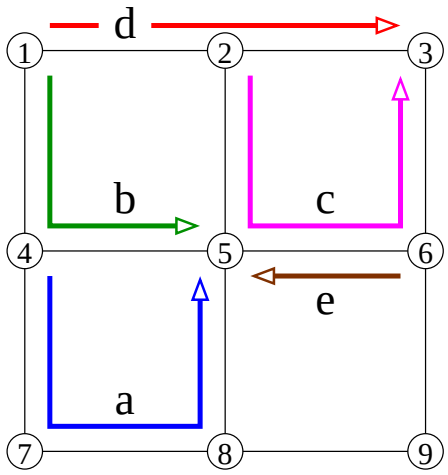
Move-To-Vacant

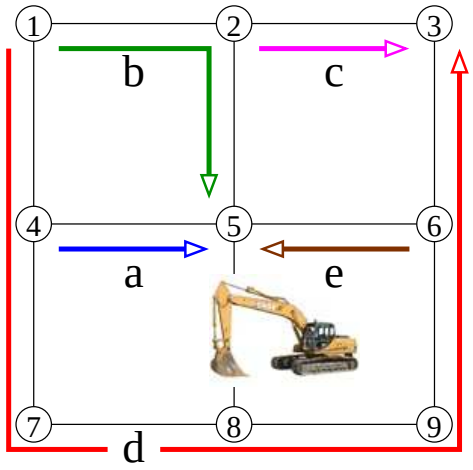
- Sequence of rerouting to converge to a valid routing
- ? Convergence time
- ? Existence

Compute the *closest* new *optimal* routing

- Minimizing number of changes (different routes)
- ? How to switch from current to new routing

The routing reconfiguration problem





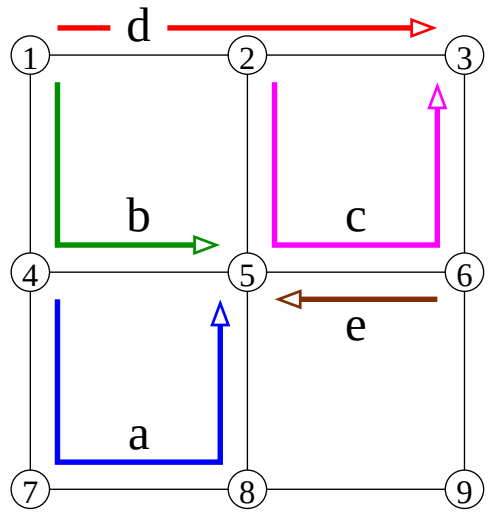
Inputs: Set of connection requests
+ current **and** new routing

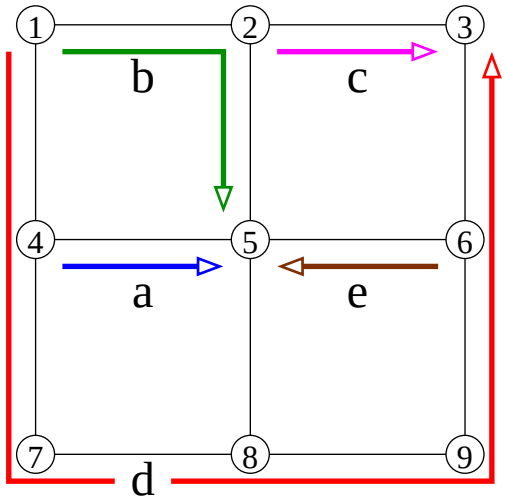
Output: Scheduling for rerouting connection requests from current to new routes

Constraint: A connection is switched only once

ObjectiveS: *detailed later*

Dependency digraph





Arc from u to v if resources needed by u in $\mathcal{F}$ are used by v in $\mathcal{I}$

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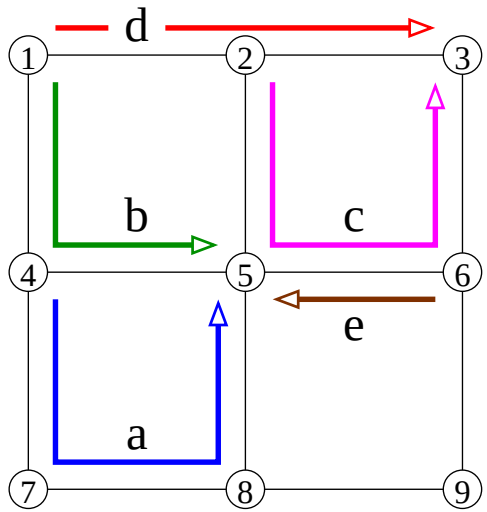

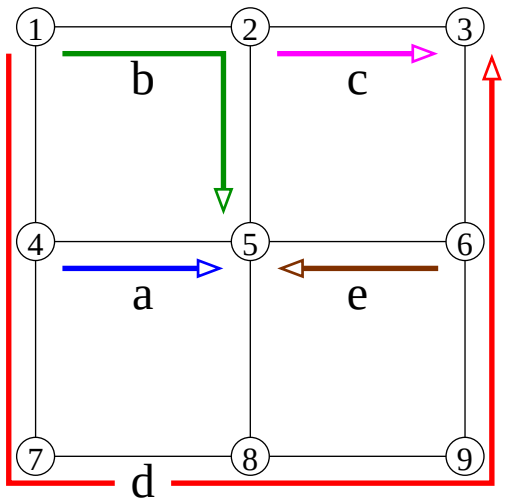

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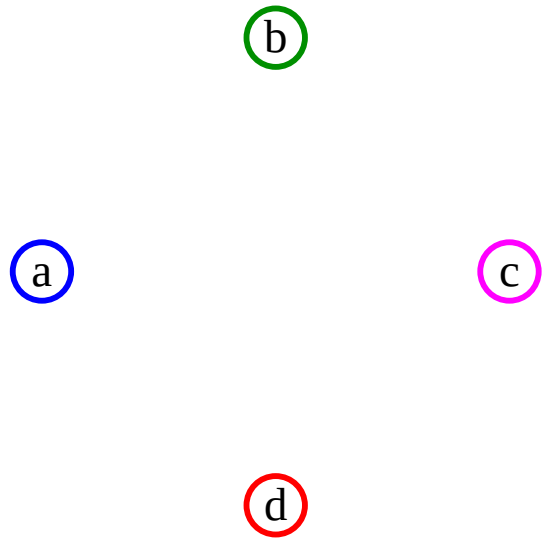

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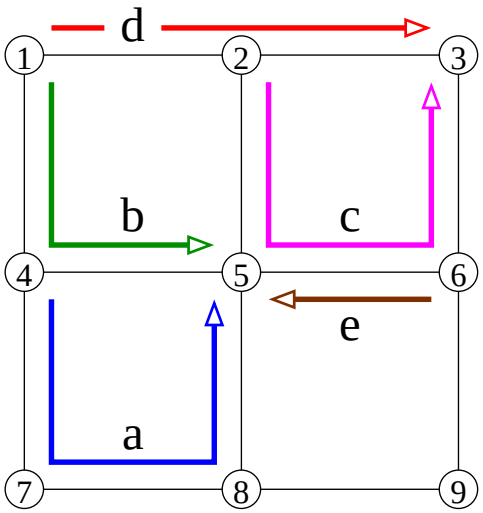

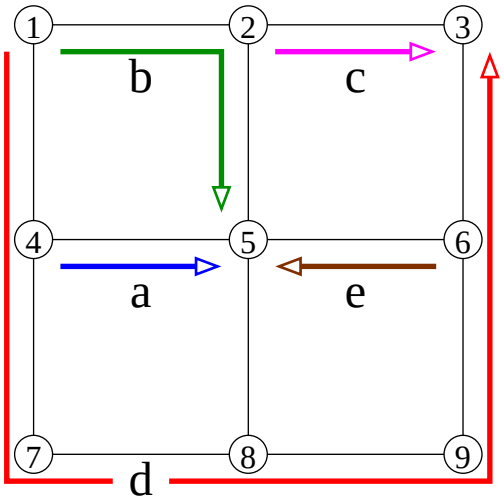

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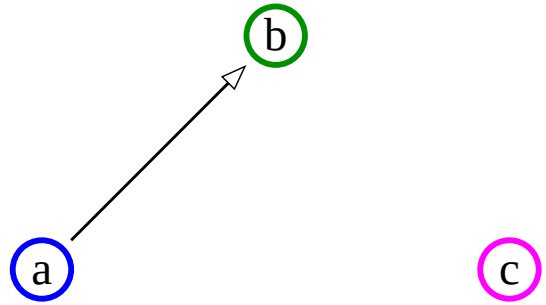

b before a

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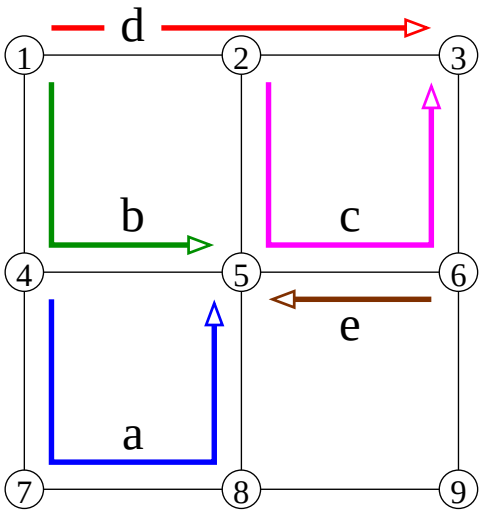

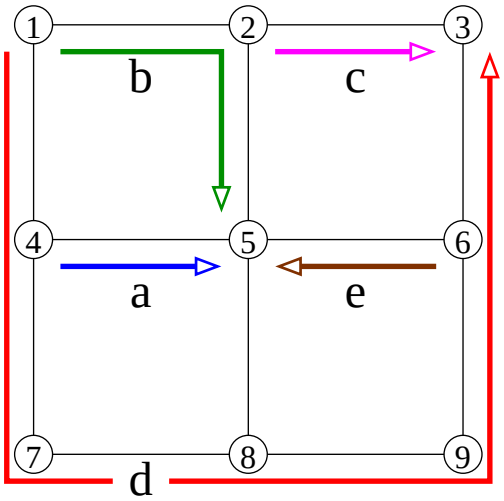

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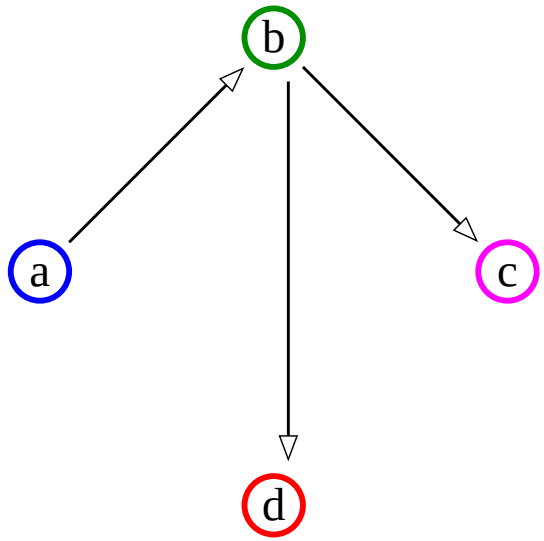

b before a
c and d before b

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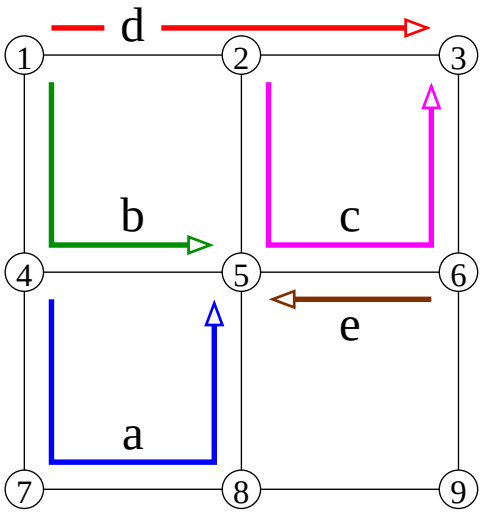

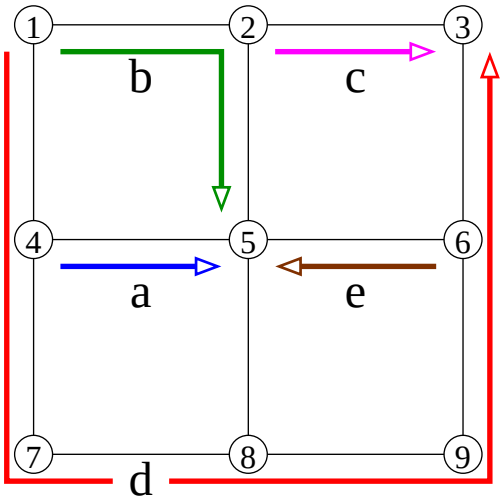

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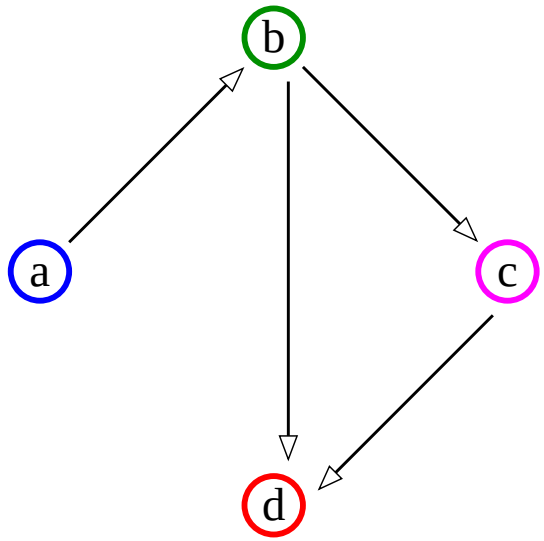
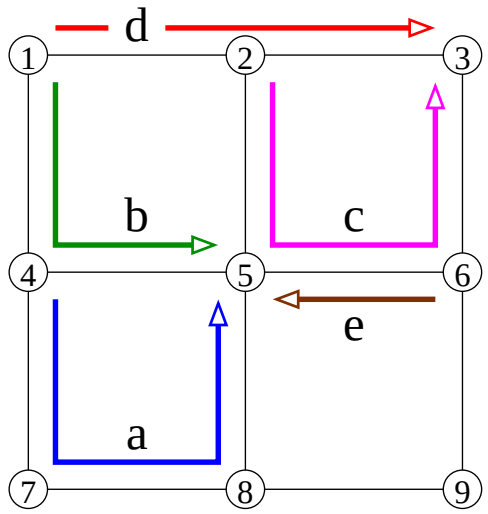
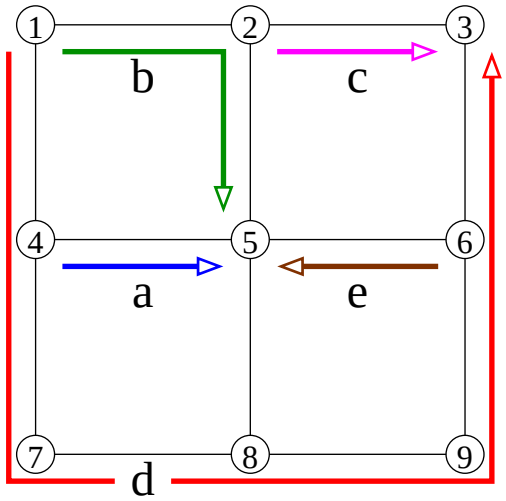


b before a
c and d before b
d before c

Dependency digraph





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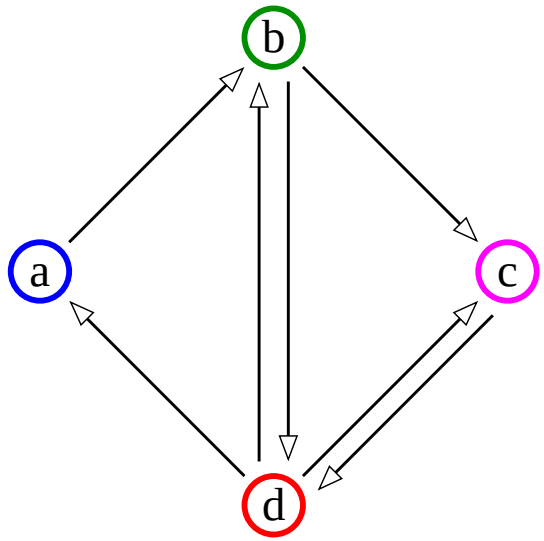
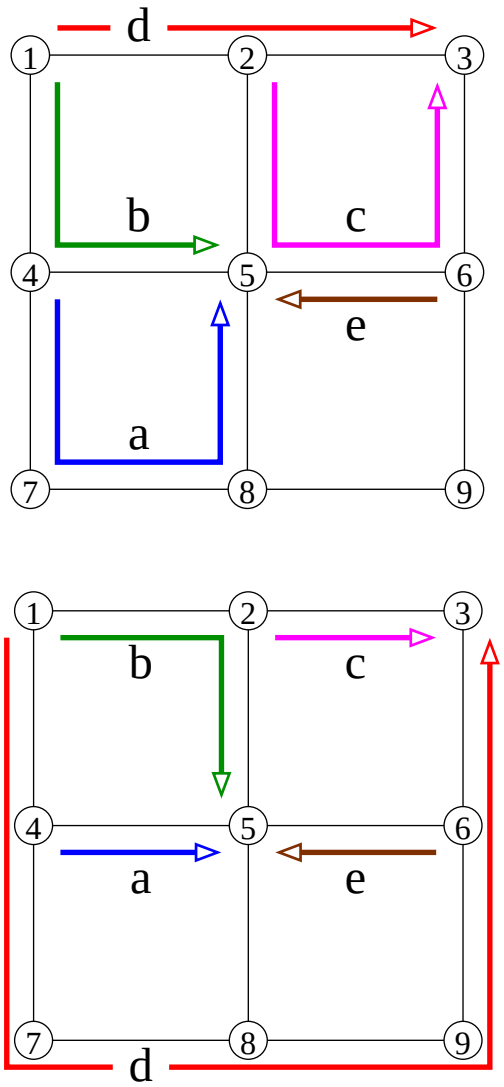


b before a
c and d before b
d before c
a, b, c before d

Dependency digraph



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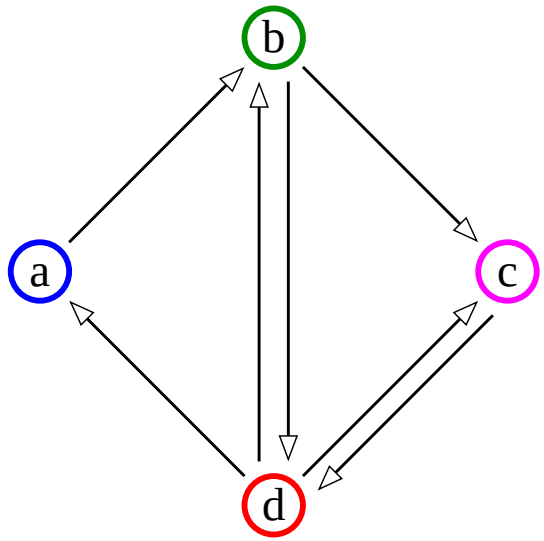


Cyclic dependencies
$\Rightarrow$ Interruptions required

Modeling, process strategy

A game with agents on the dependency digraph

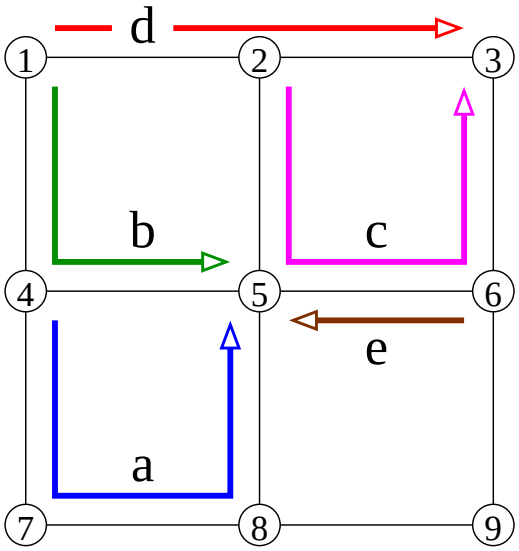
break-before-make

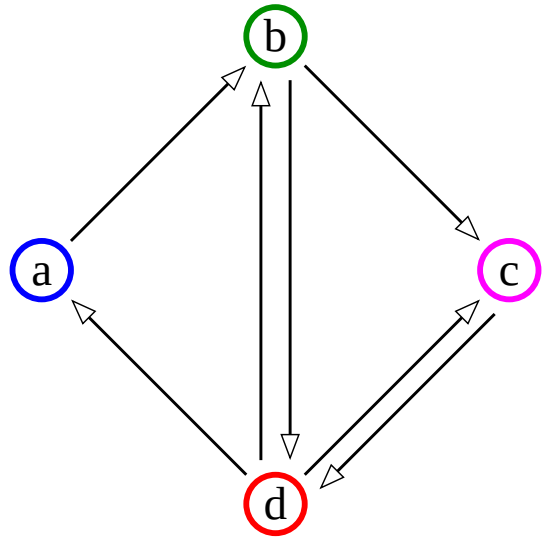
break a connection $\Longleftrightarrow$ place an agent on corresponding node
establish new path $\Longleftrightarrow$ remove agent from the node

Rules of the game

- R_1 Place an agent at a node = interrupt a connection
- R_2 Process a node if all its out-neighbors are either processed or occupied by an agent
  - = (Re)route a connection when final resources are available
- R_3 Remove an agent from a node, after having processed it

Example





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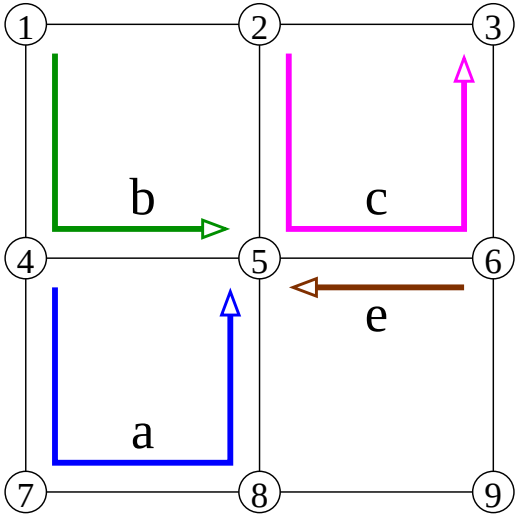
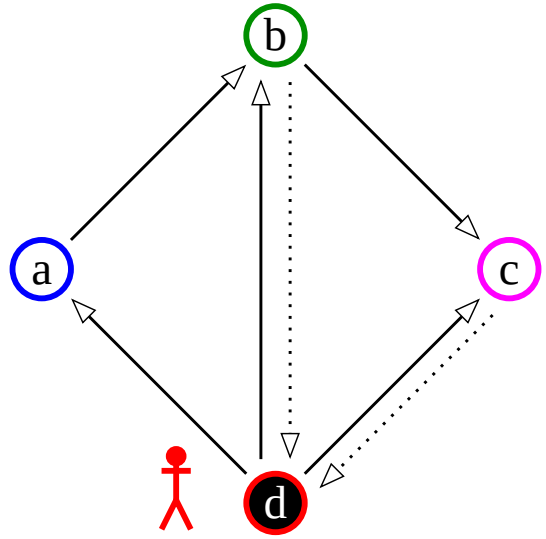


break request d



put an agent on node d

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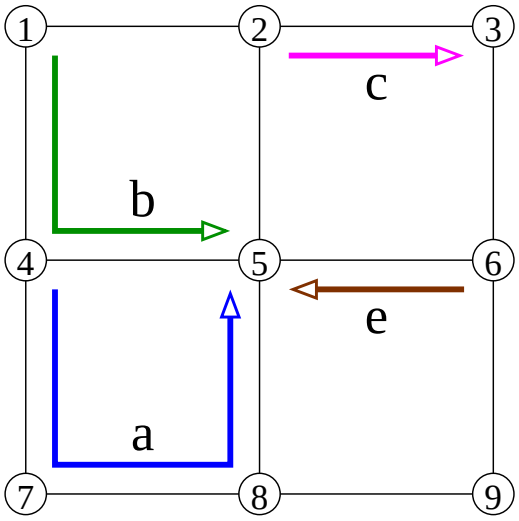
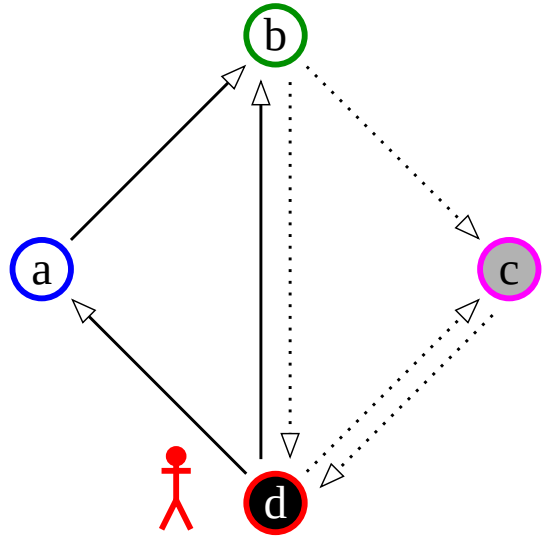


reroute request c



process node c

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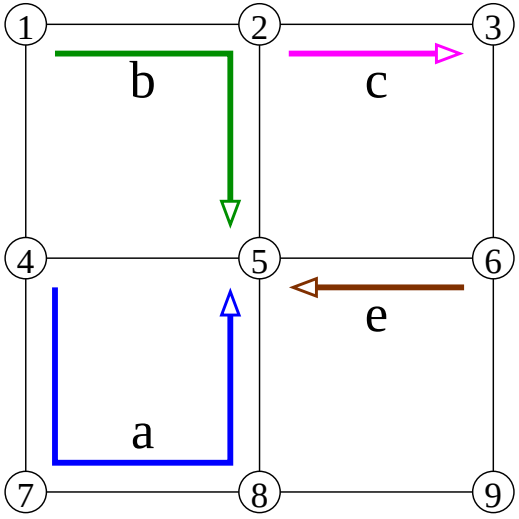
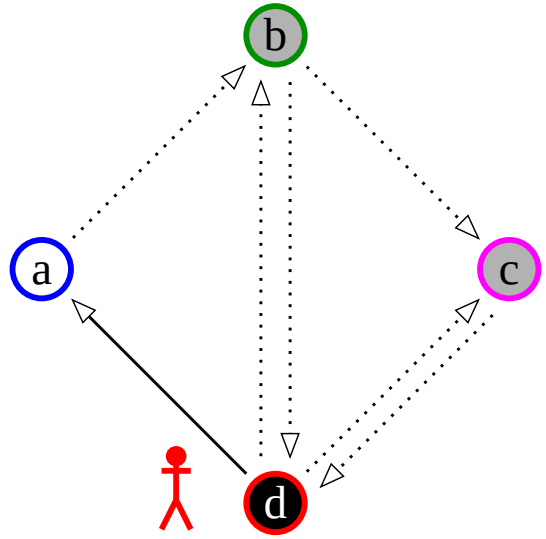


reroute request b



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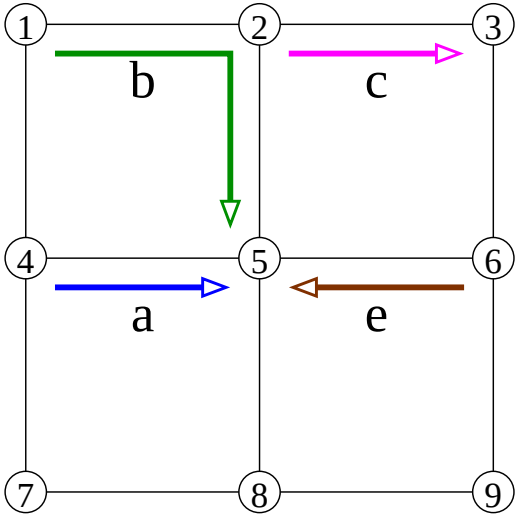
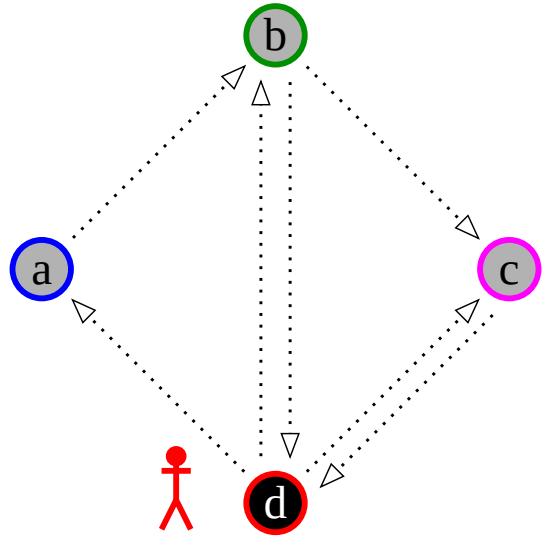


reroute request *a*



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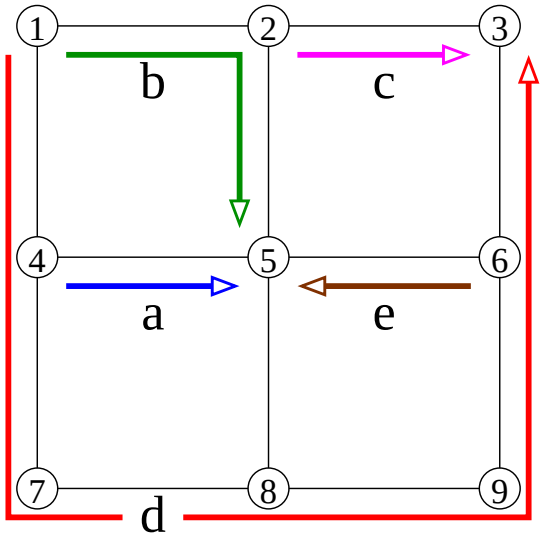
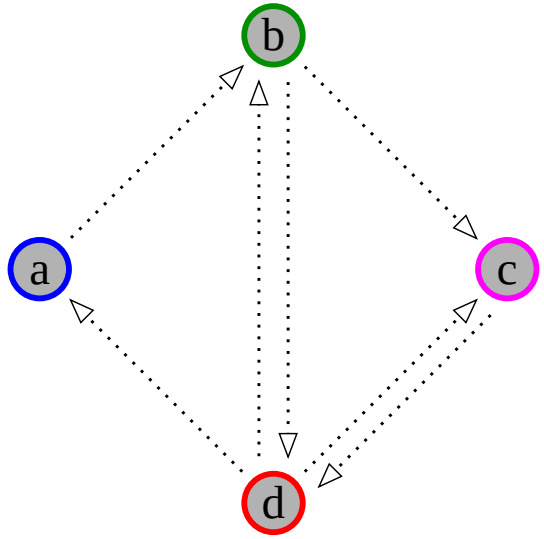


route request d



process node d
and remove agent

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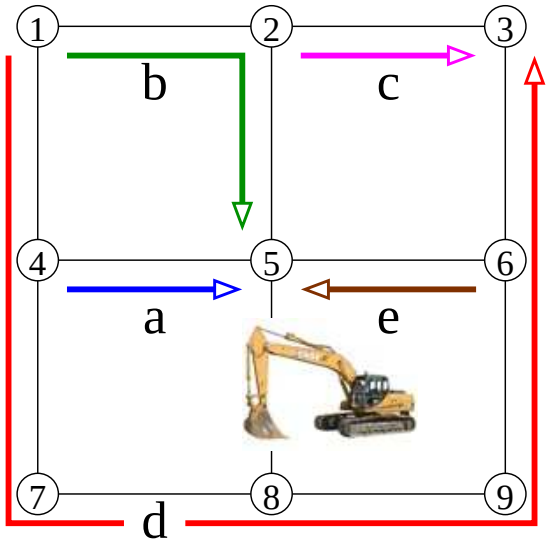
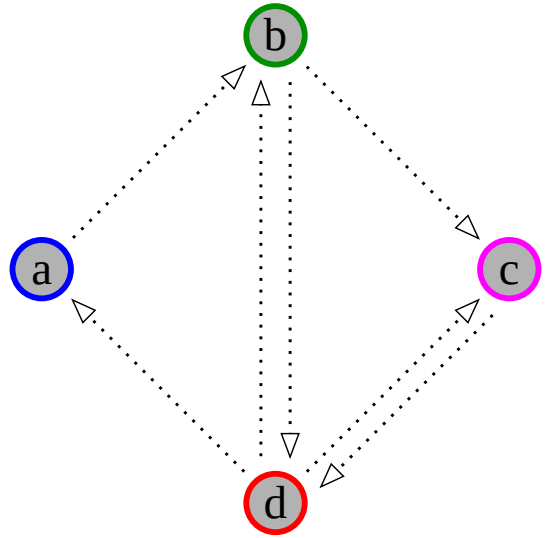


Ready for maintenance operation



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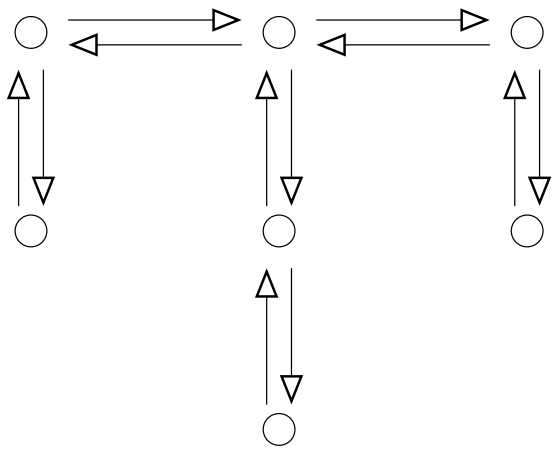

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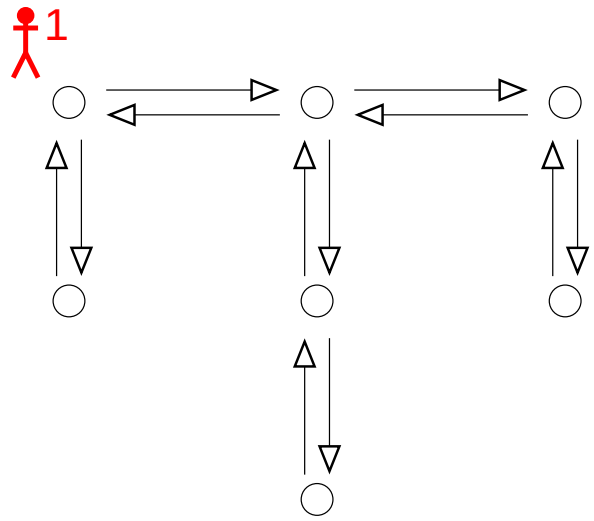

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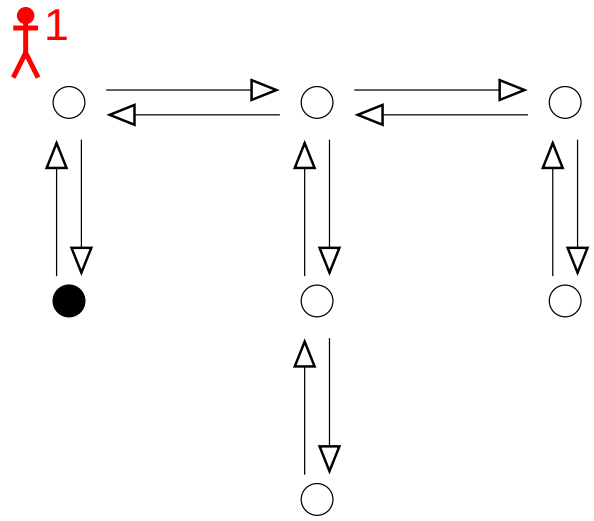

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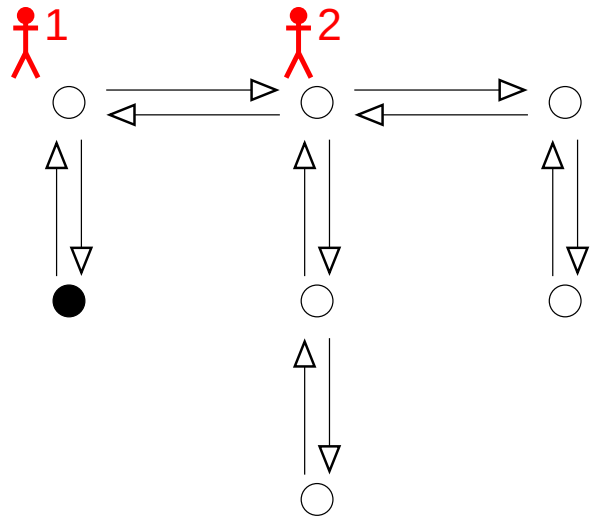

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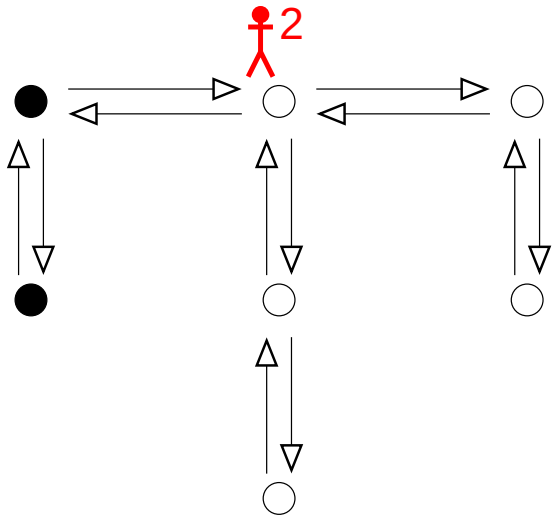

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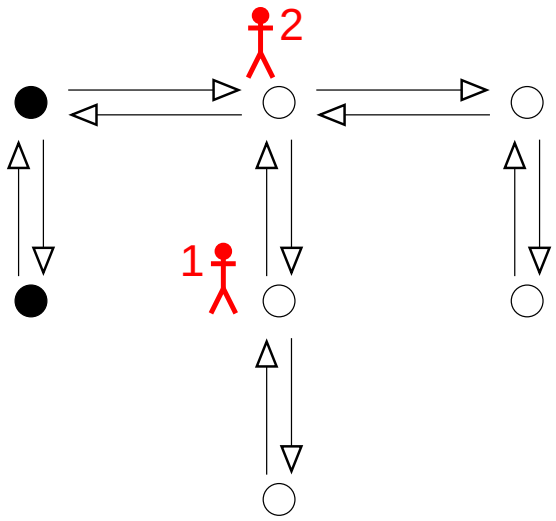

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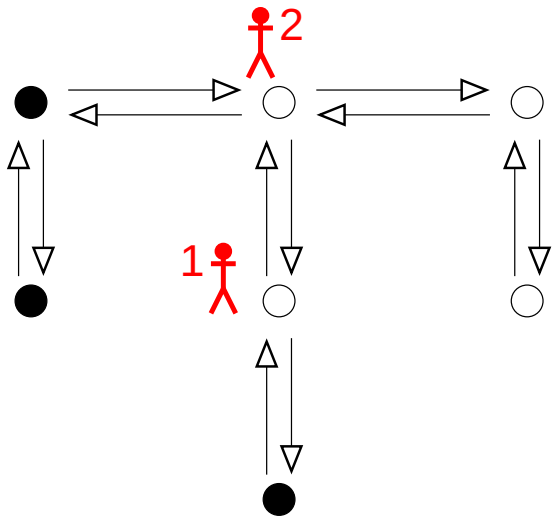

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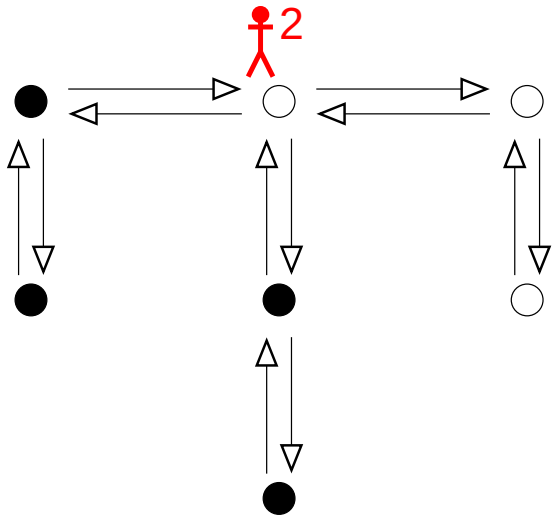

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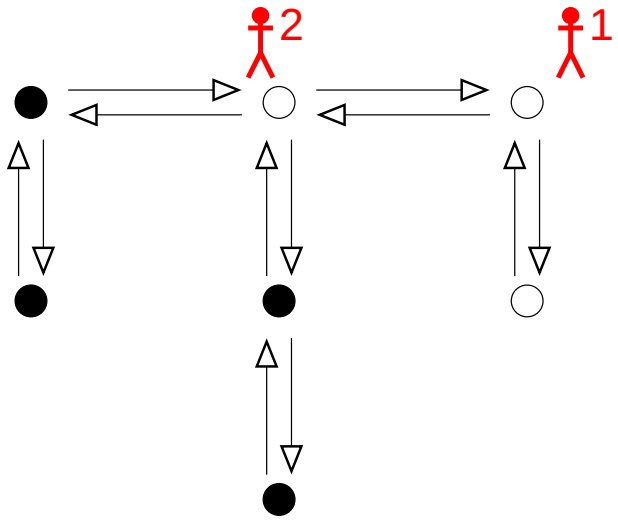
R_2 Process a node if all its out-neighbors are either processed or occupied by an agent

= (Re)route a connection when final resources are available

R_3 Remove an agent from a node, after having processed it

Example



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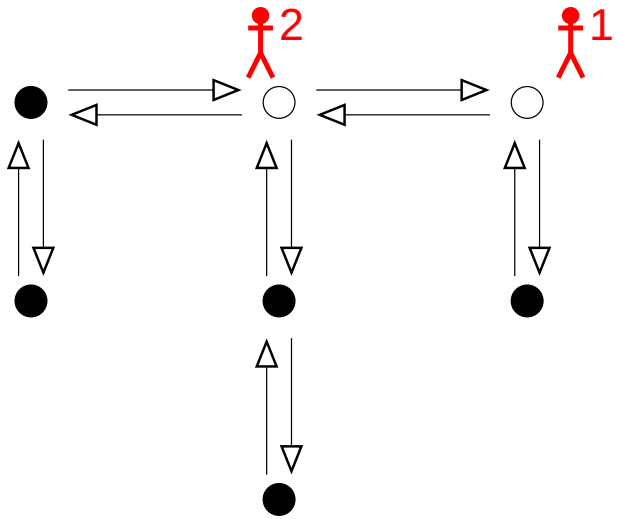

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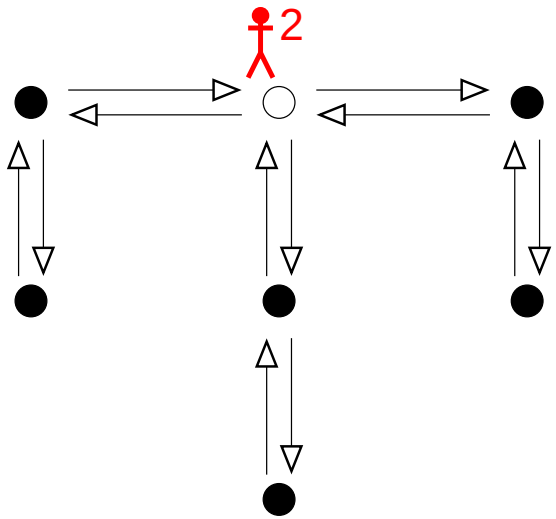

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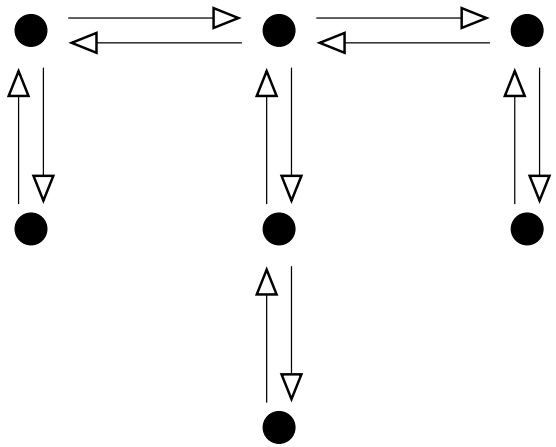

2 agents, 4 interruptions

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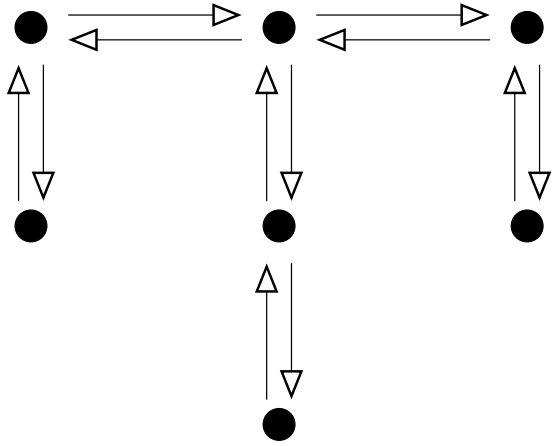

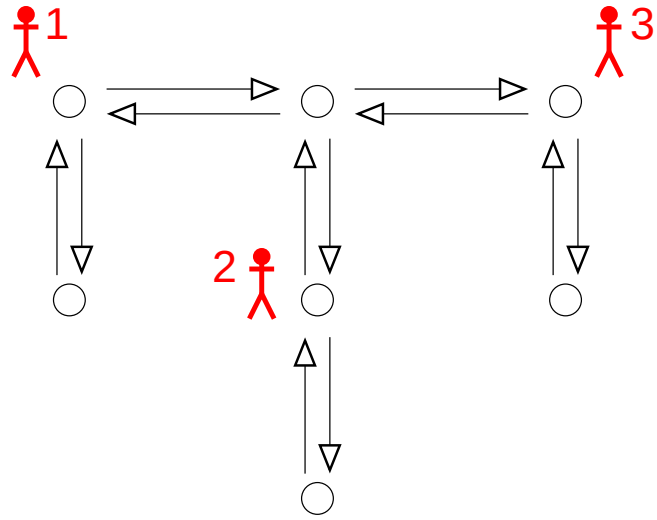

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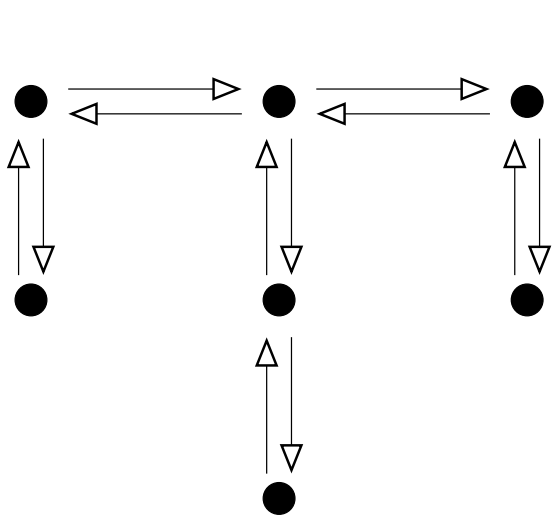

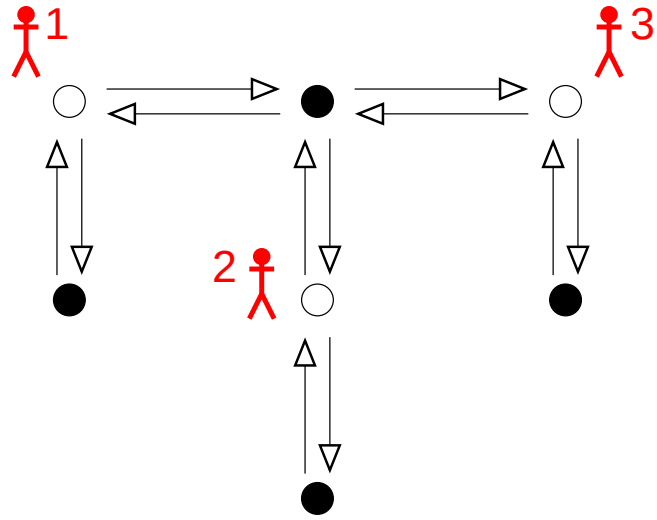

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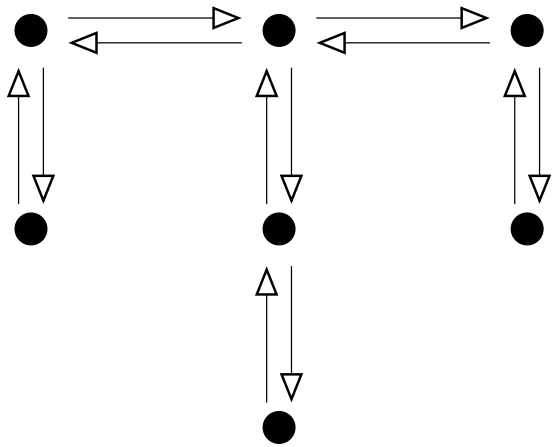

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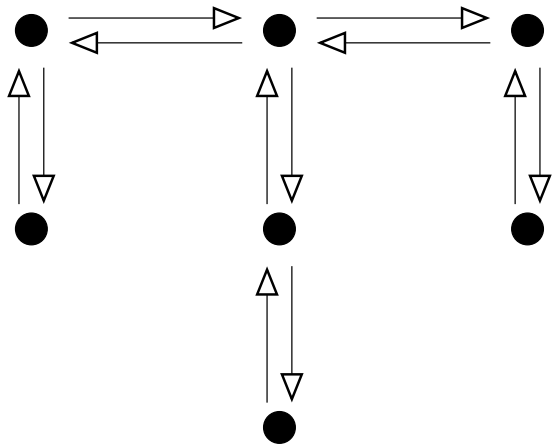

3 agents, 3 interruptions

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Objectives (on the dependency digraph)

Minimize...

- Total number of interruptions = minimum feedback vertex set
  $\Leftrightarrow$ # **of occupied vertices**
- Number of simultaneous interruptions = process number, pn
  $\Leftrightarrow$ # **of agents used**
  - Vertex separation, vs $\qquad vs(D) \leq pn(D) \leq vs(D)+1$
  - Pathwidth, pw
  - Node search number, ns $\qquad pw(G) = vs(G) = ns(G)-1$

cops-and-robber games

Complexity

- All problems are NP-hard and "computing pn" is not in APX

Tradeoffs

- # agents given # of occupied vertices
- # of occupied vertices given # agents

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- # agents given # of occupied vertices
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State of the art

Total number of interruptions,

- Dependancy digraph, heuristics [Jose, Somani'03]

Simultaneous interruptions, Process strategy

- Modeling as a cops-and-robber like game
- Complexity and hardness [Coudert, Perennes, Pham, Sereni'05]
- Characterization of (di)graphs with process number 0, 1, 2 [Coudert, Sereni'07]
- Heuristic algorithms [Coudert, Huc, Mazauric, Nisse, Sereni'09]
- Generalized model allowing bandwidth sharing [Coudert,Mazauric, Nisse'09]

Tradeoffs

- Linear Programs, heuristic [Solano'09]

Tradeoff Parameters

Objectives

Min. number of simulteanous interruptions **given** total number of interruptions

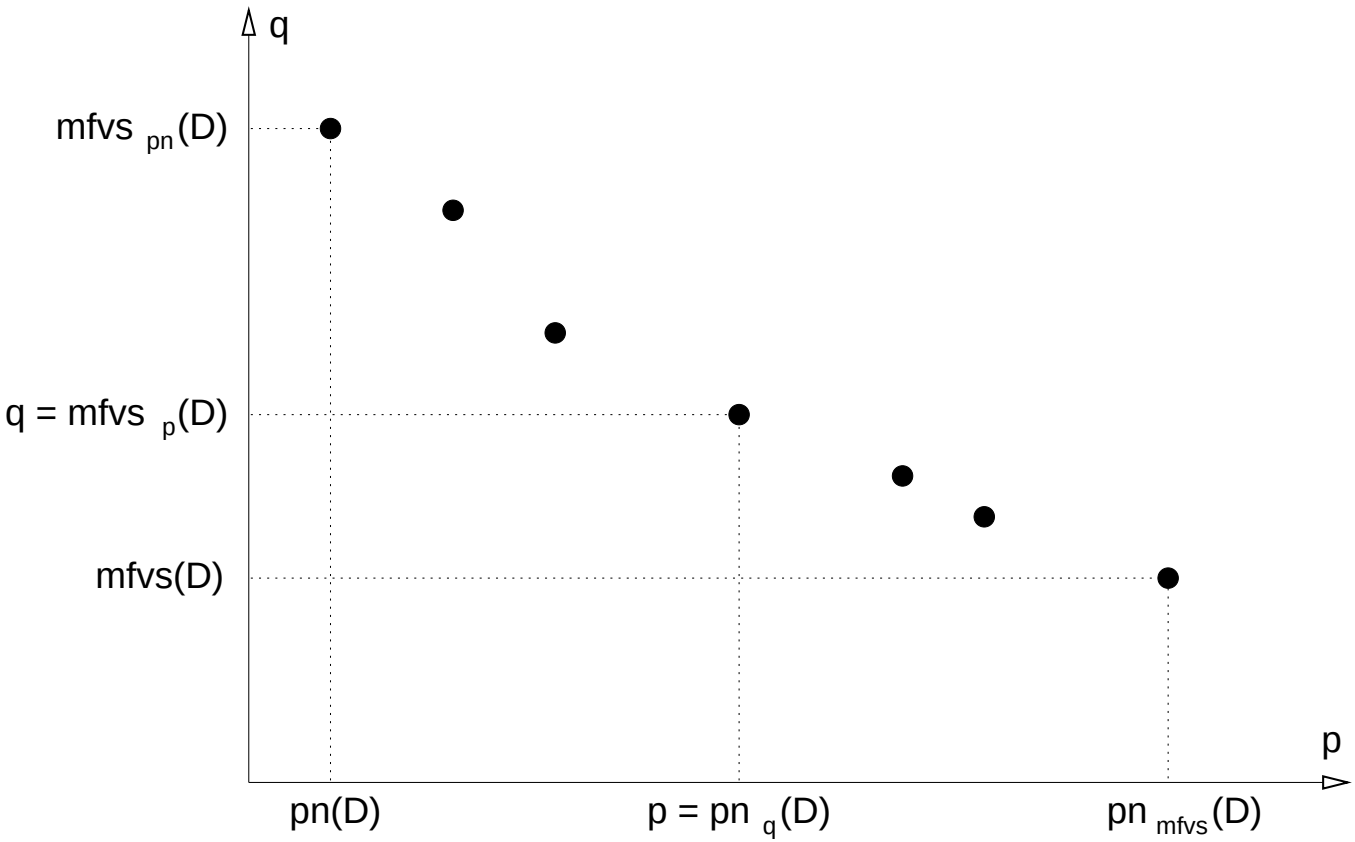
- $pn_m(D)$ = min. agents to occupy at most m nodes

Min. total number of interruptions **given** total maximum number of simulteanous interruptions

- $mfvs_p(D)$ = min. occupied nodes using p agents

Tradeoff Parameters

Representation of Minimal Values



Tradeoff Parameters

Complexity Results

Theorem

Let $\alpha \in [0,1]$. The problem that takes a digraph D as an input and aims at minimizing:

- $\alpha.pn_{mfvs}(D) + (1-\alpha)pn(D)$ *is not in APX,*
- $\alpha.mfvs_{pn}(D) + (1-\alpha)mfvs(D)$ *is APX-hard,*

Impossibility of Tradeoff

Unbounded Ratios

$\frac{pn_{mfvs+p}(D)}{pn(D)}$: how the number of agents increases if constrained to occupy "few" vertices? $(p \geq 0)$

$\frac{mfvs_{pn+p}(D)}{mfvs(D)}$: how the number of occupied vertices increases if constrained to occupy "few" agents? $(p \geq 0)$

Theorem

$\forall p \geq 0$, $\frac{pn_{mfvs+p}}{pn}$ *is not bounded* *(even in symmetric digraphs).*

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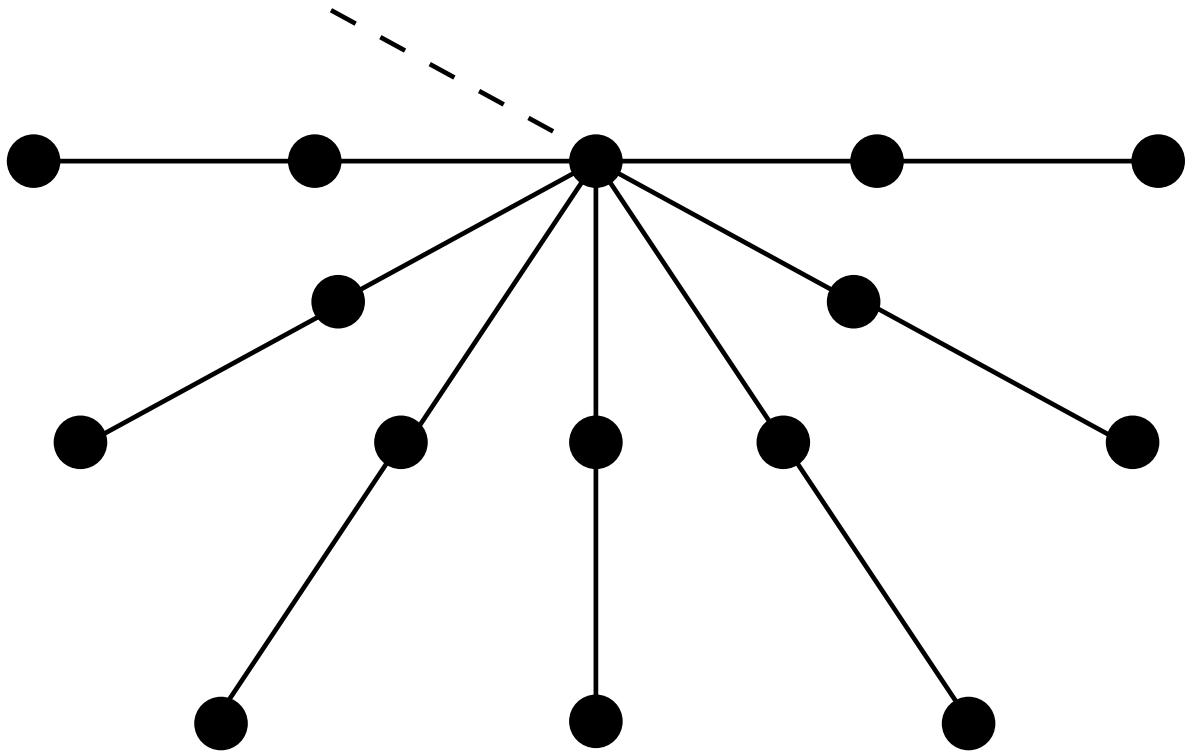
agents for minimizing # occupied vertices

$\exists$ digraphs with arbitrary large ratio: $\frac{pn_{mfvs}(D)}{pn(D)}$.



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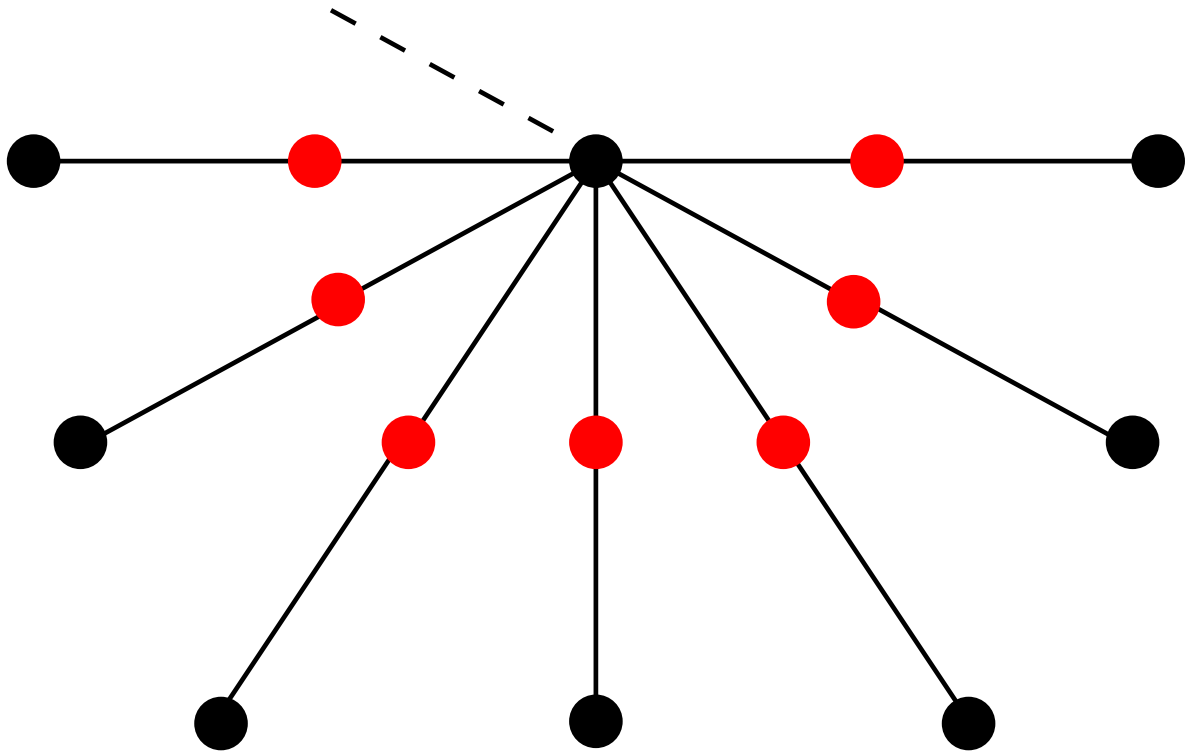

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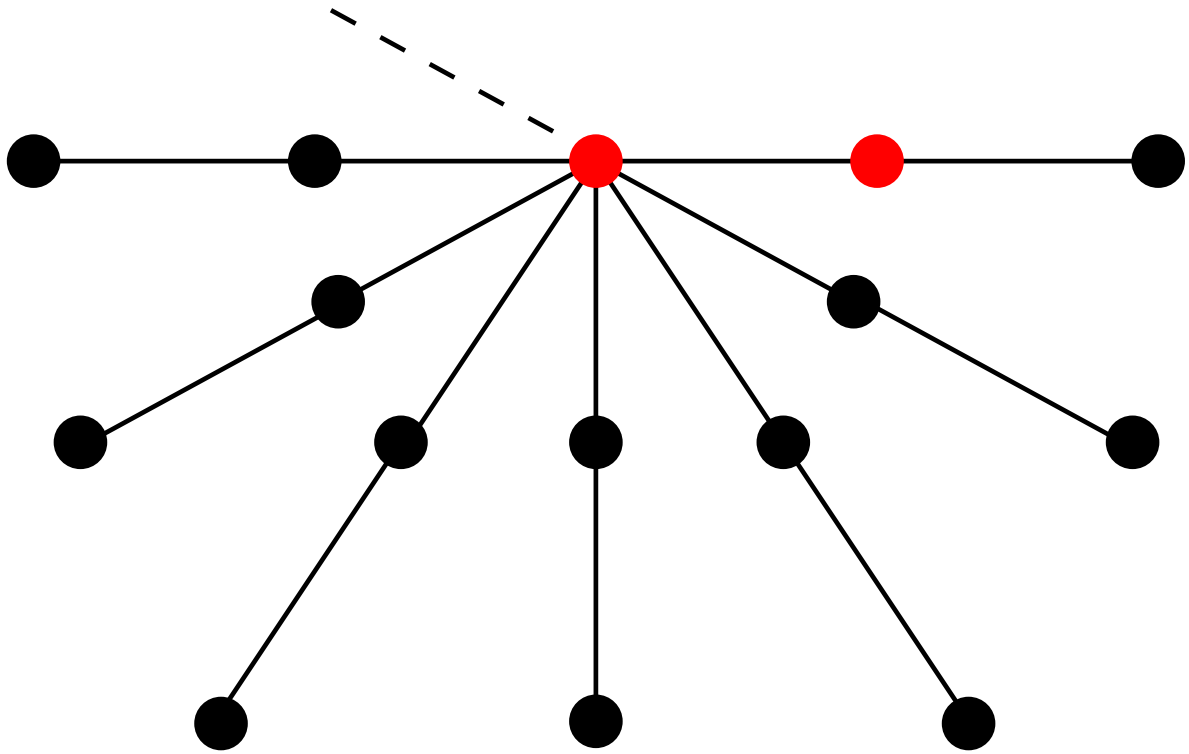

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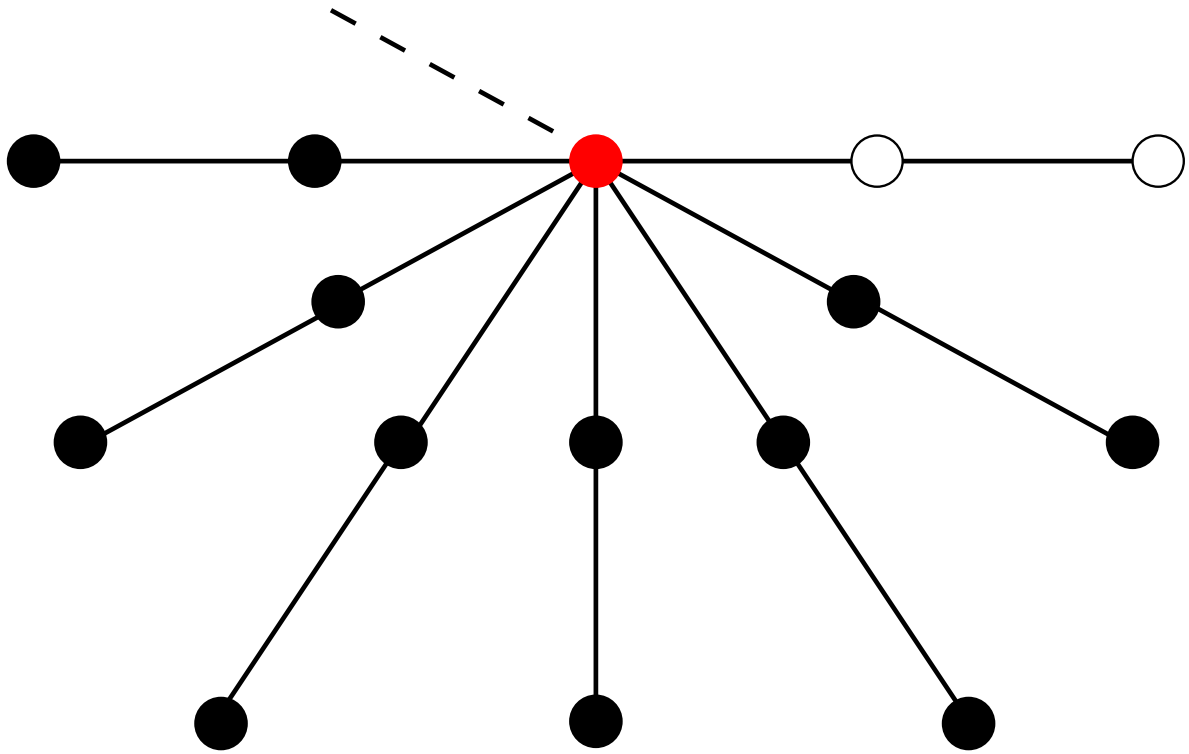

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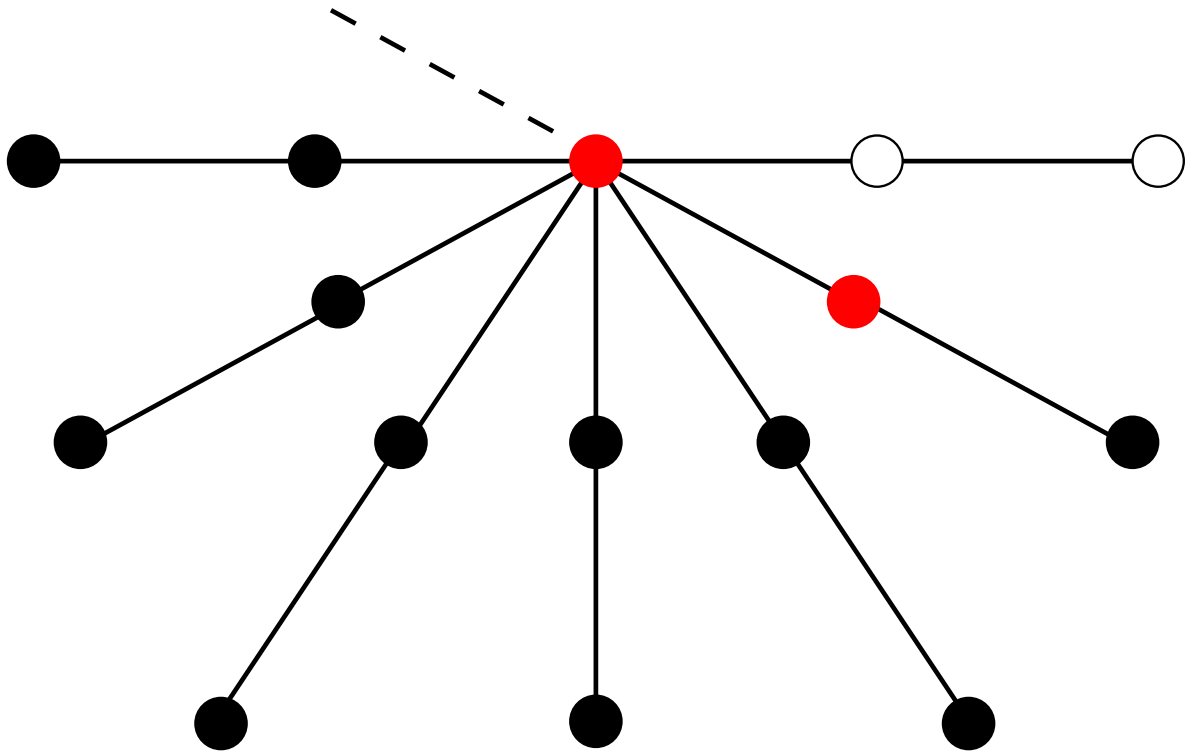

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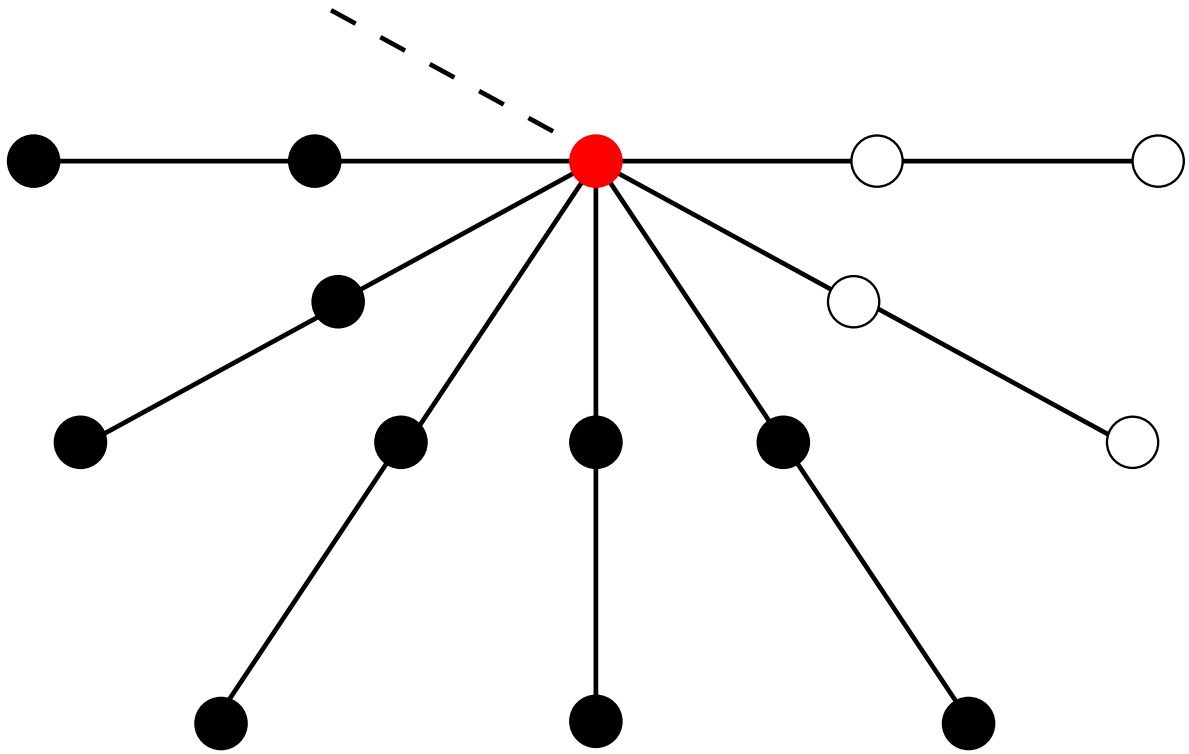

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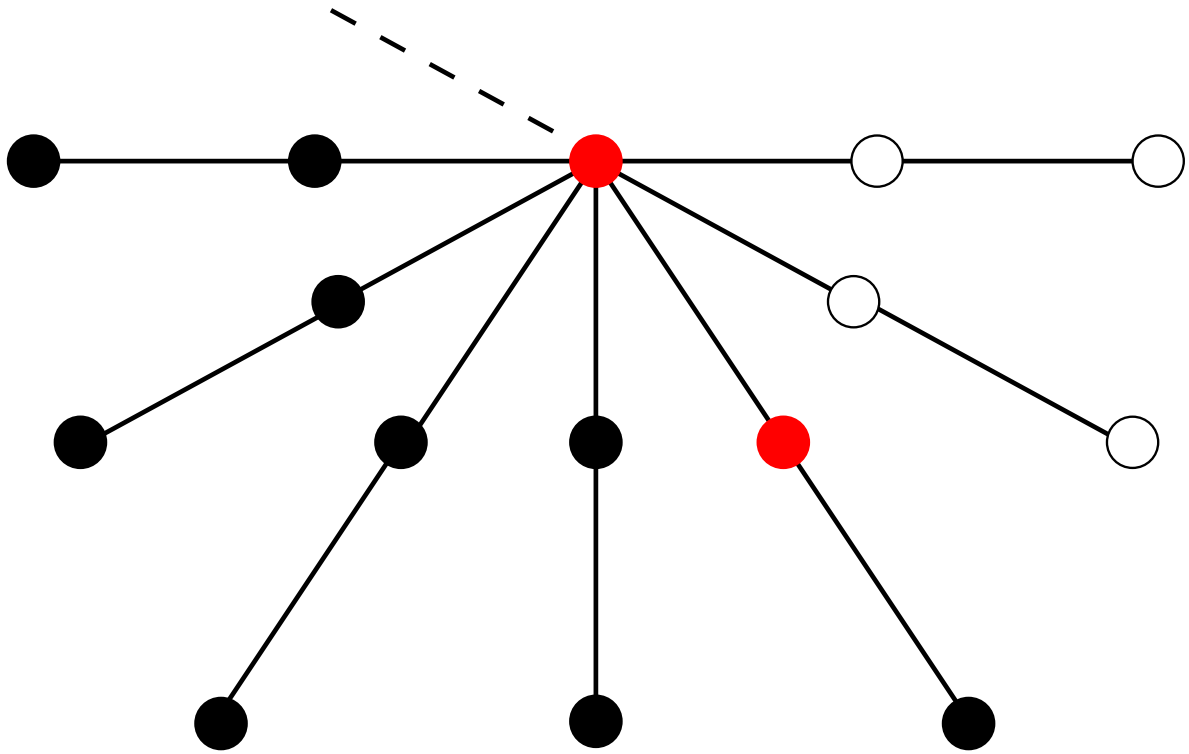

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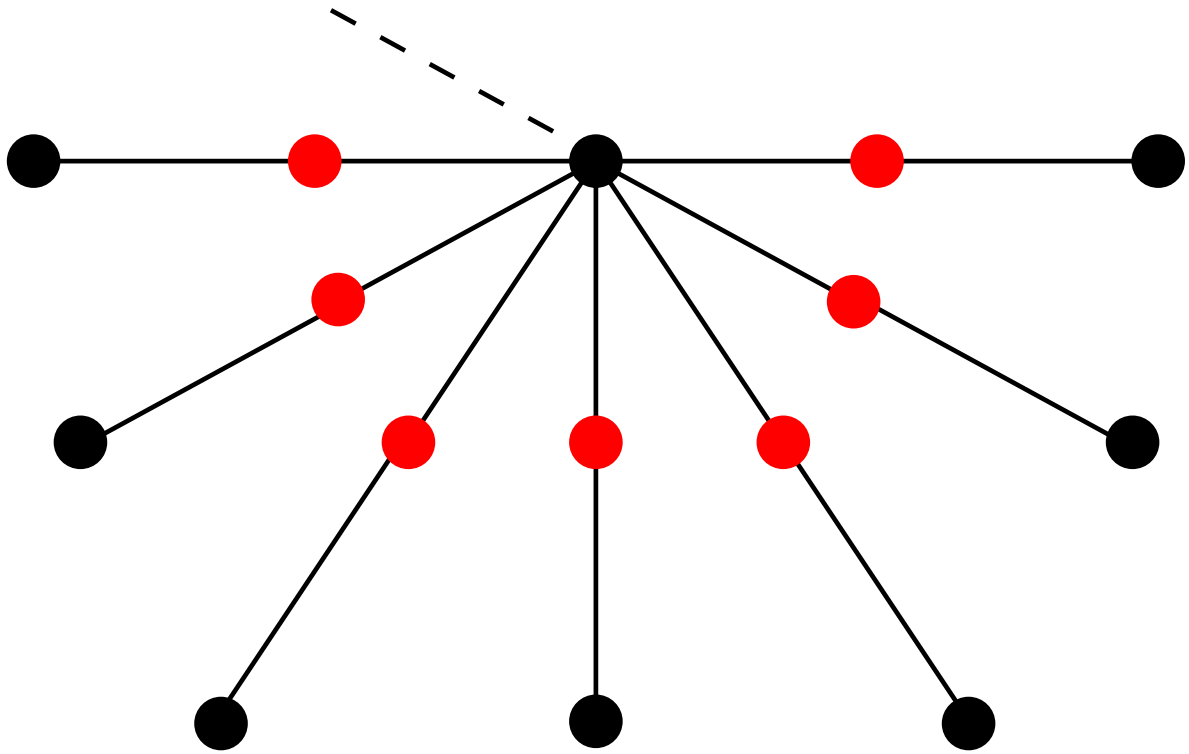

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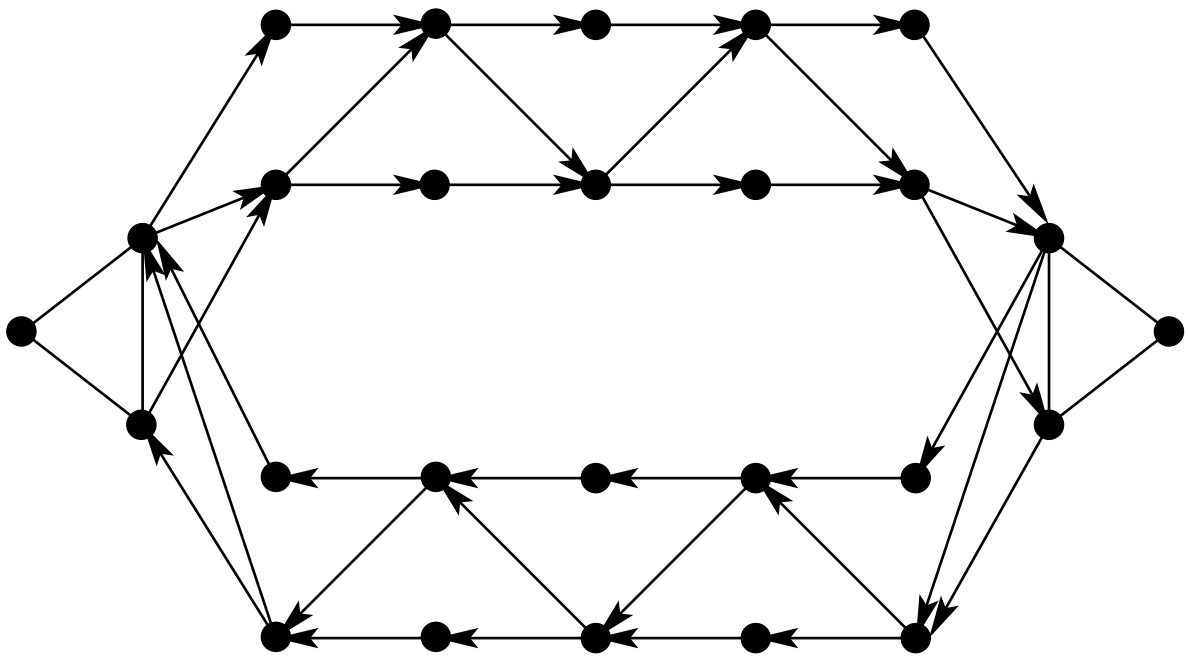

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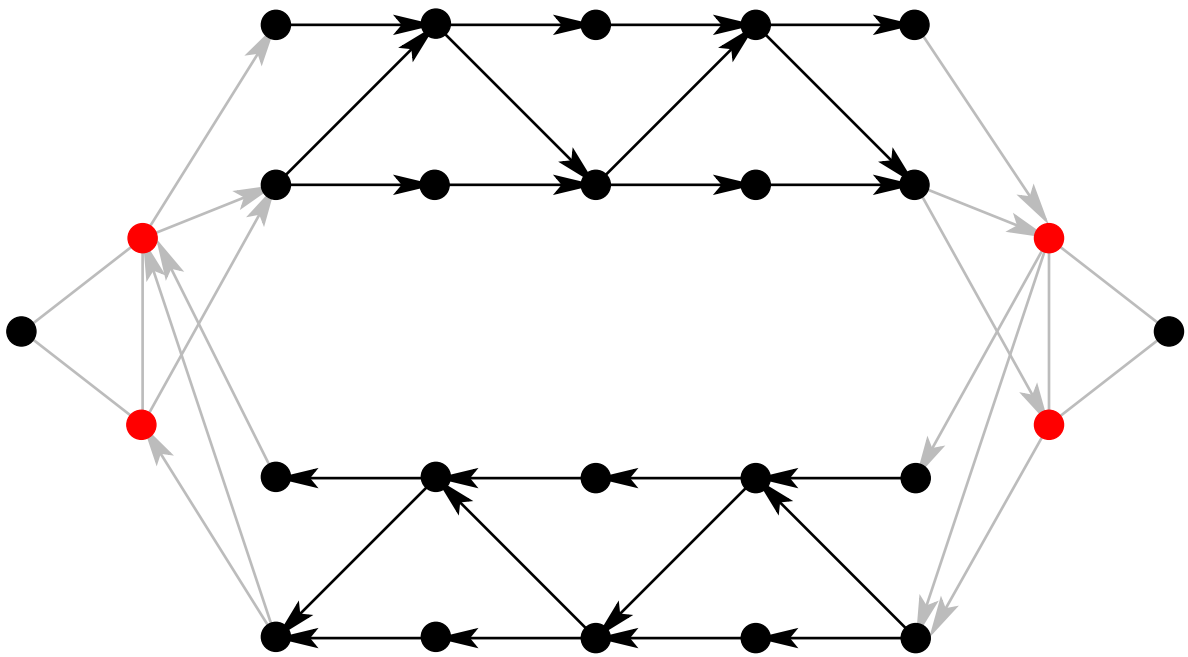

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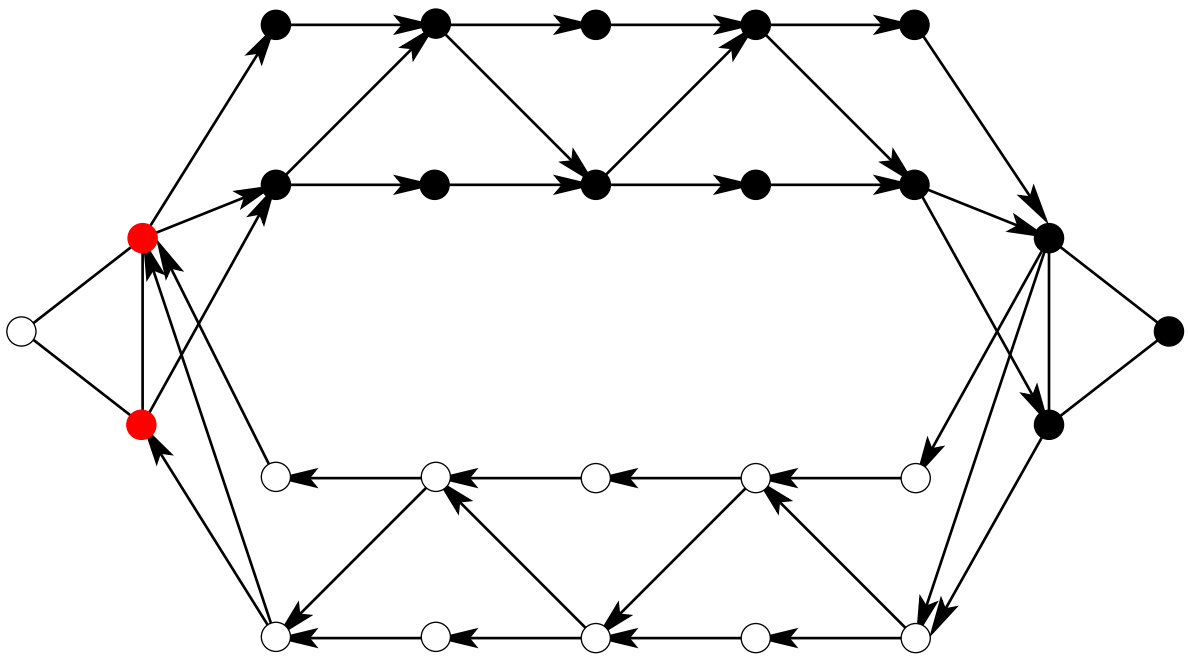

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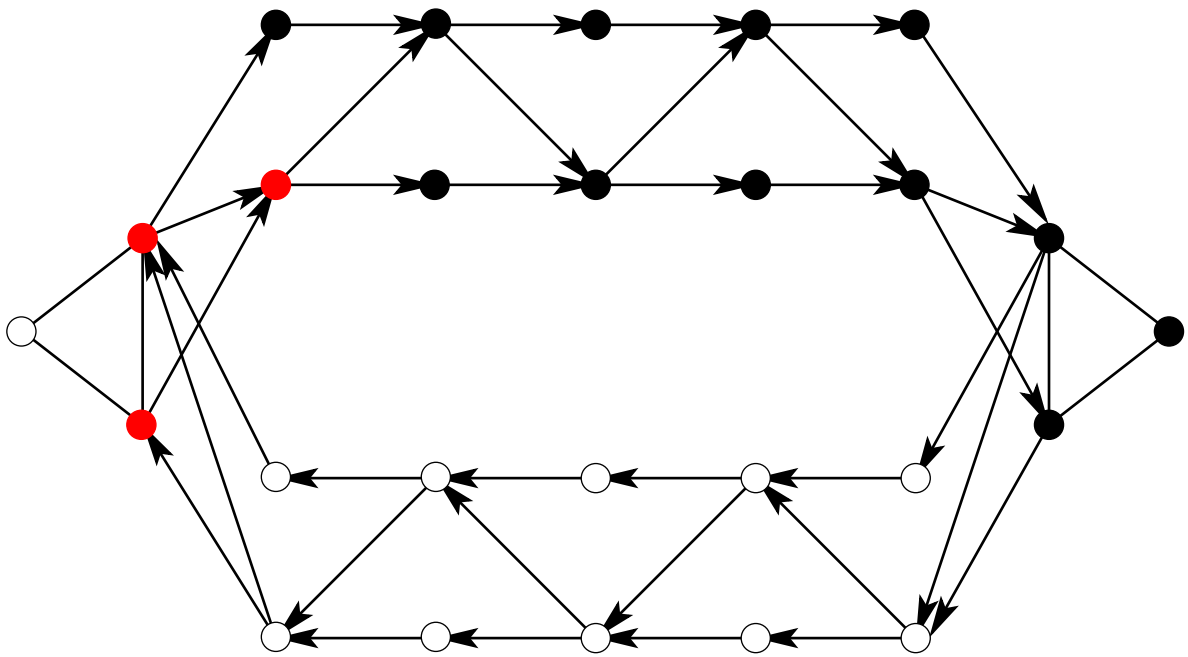

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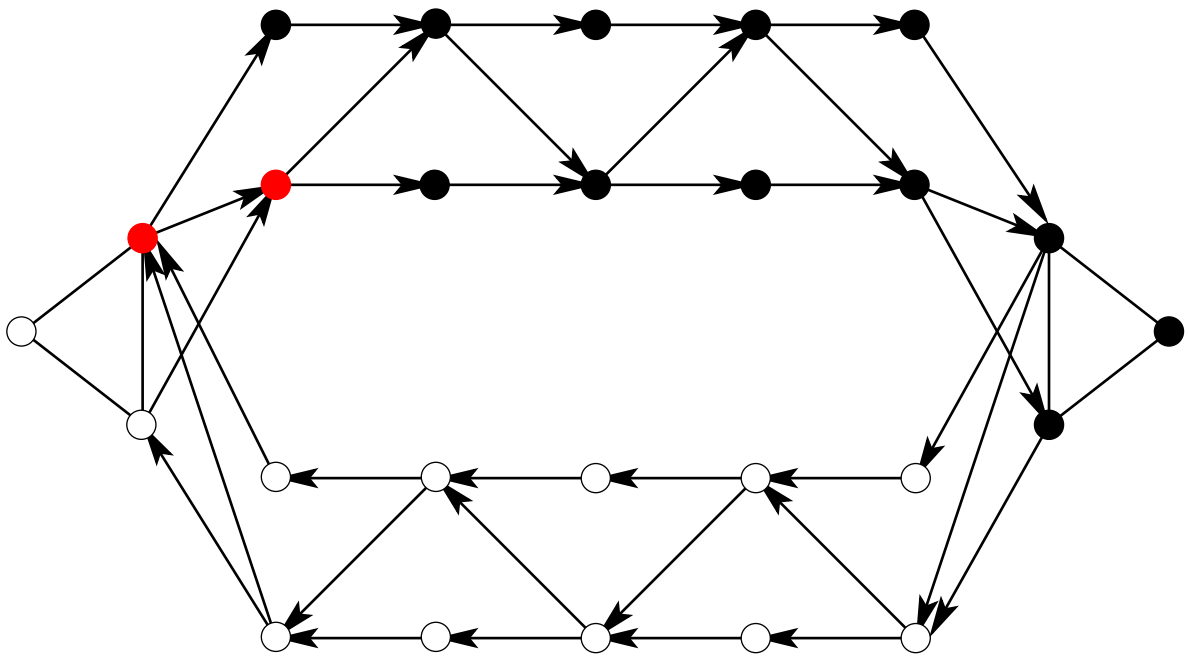

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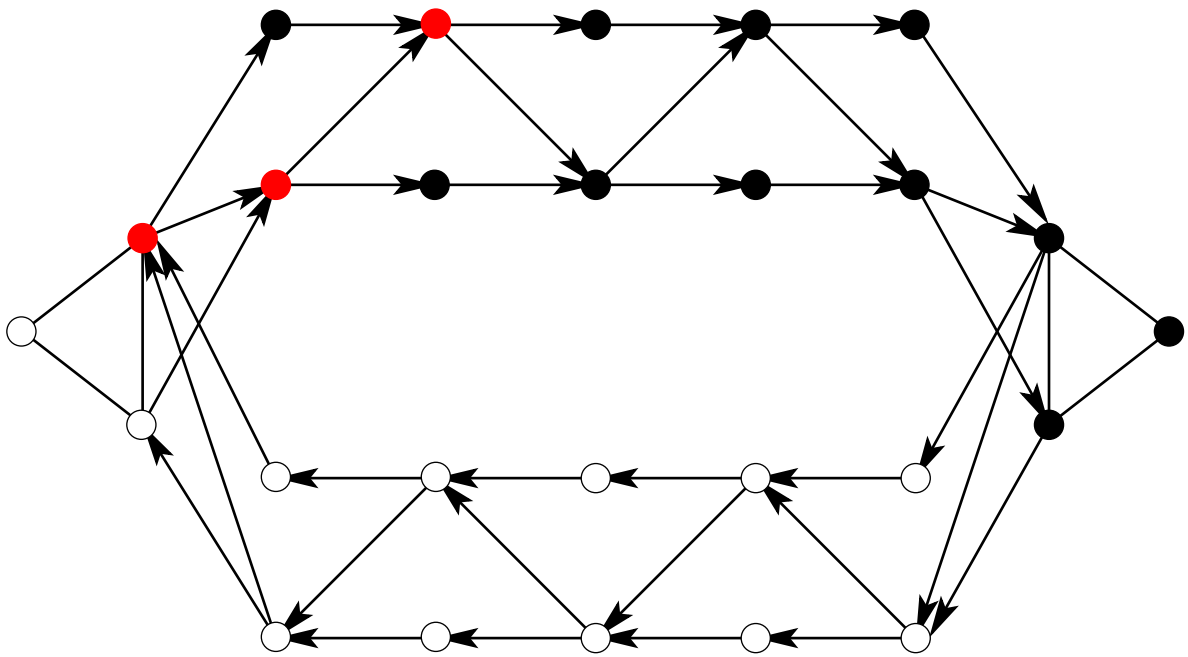

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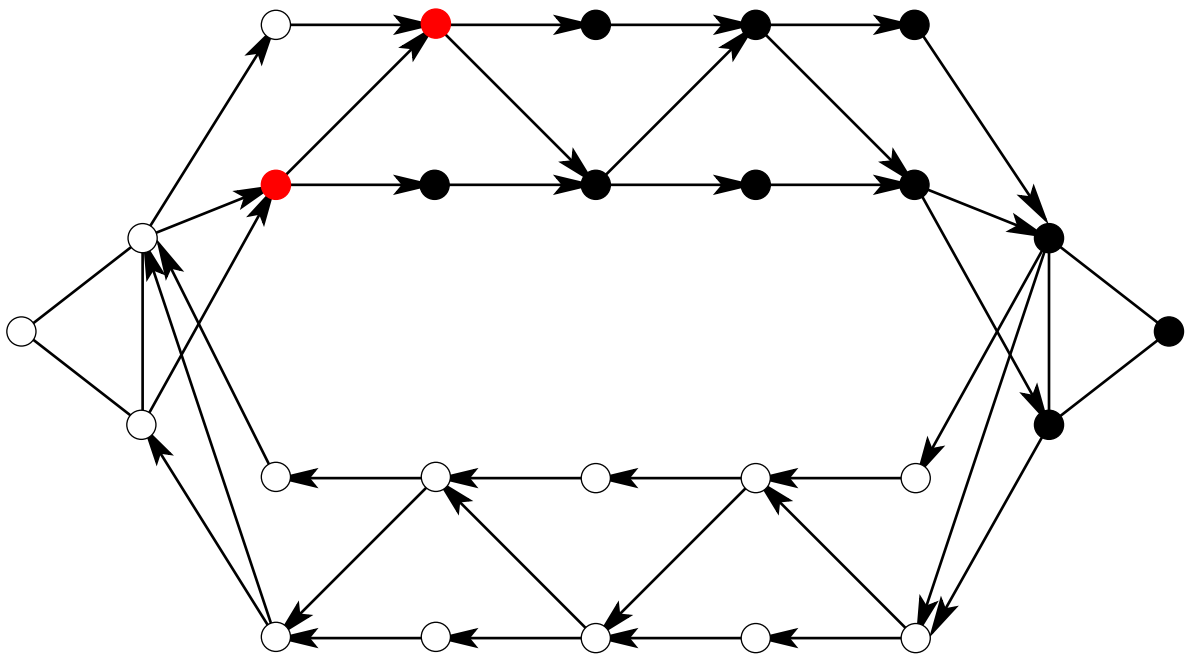

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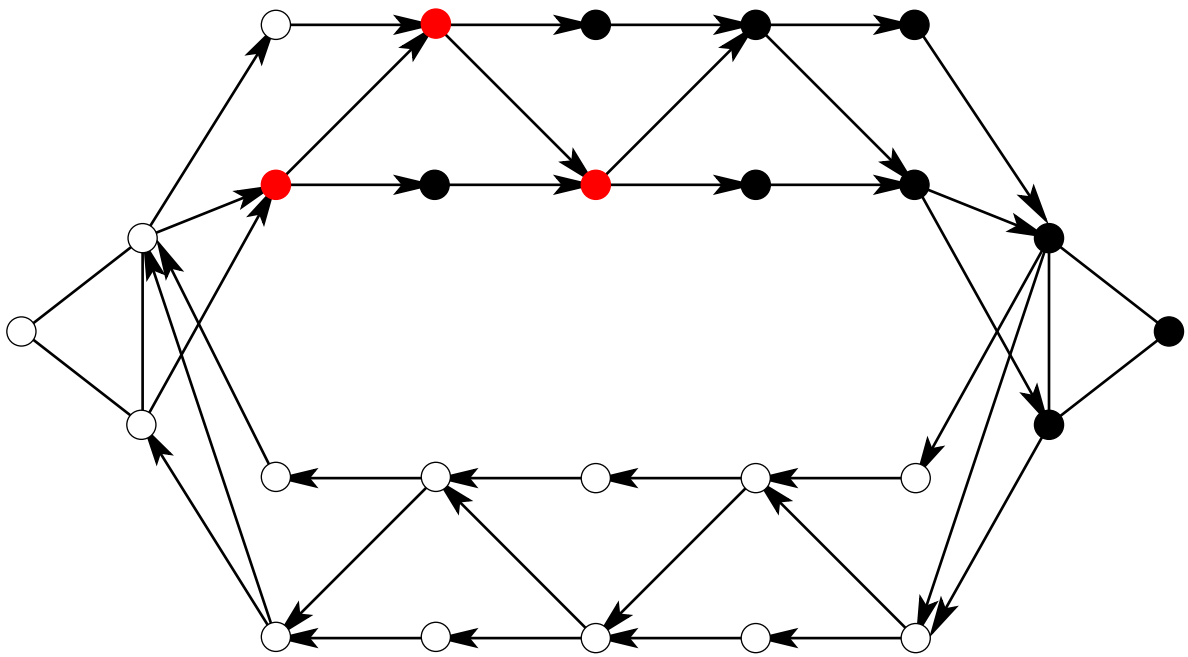

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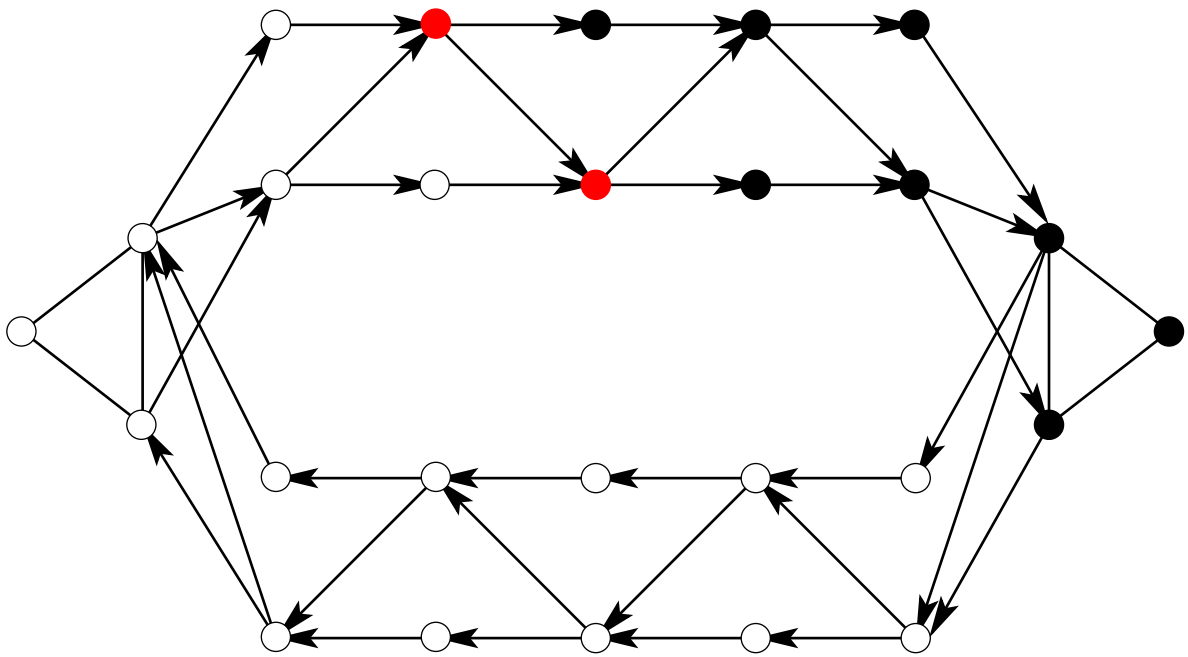

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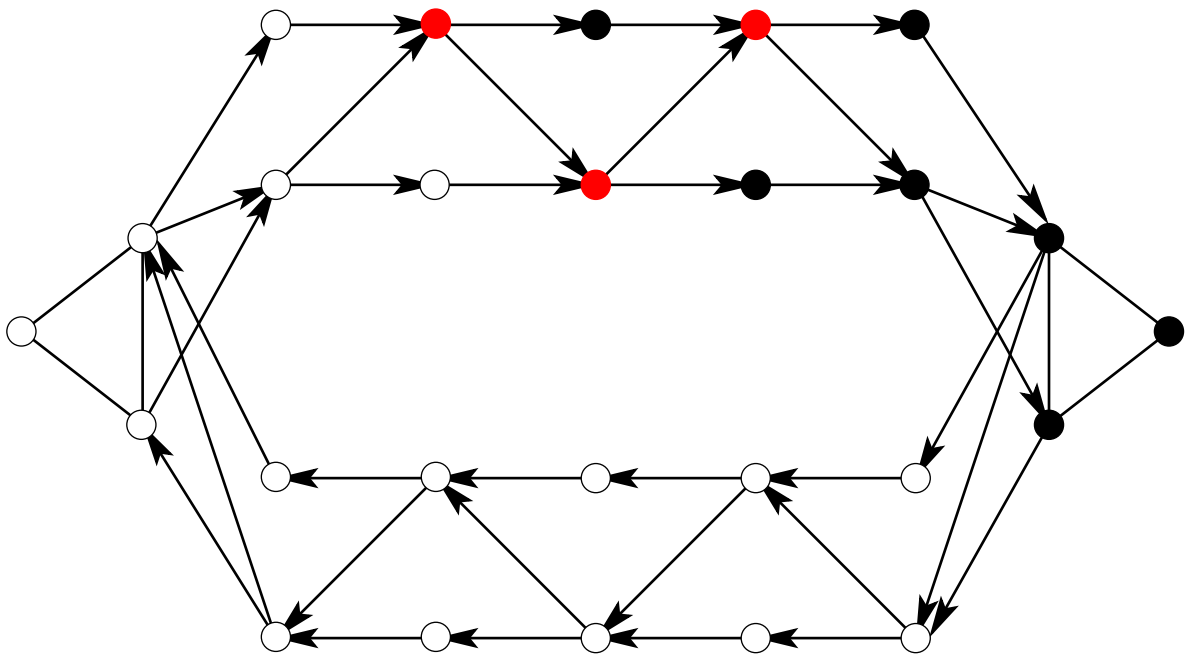

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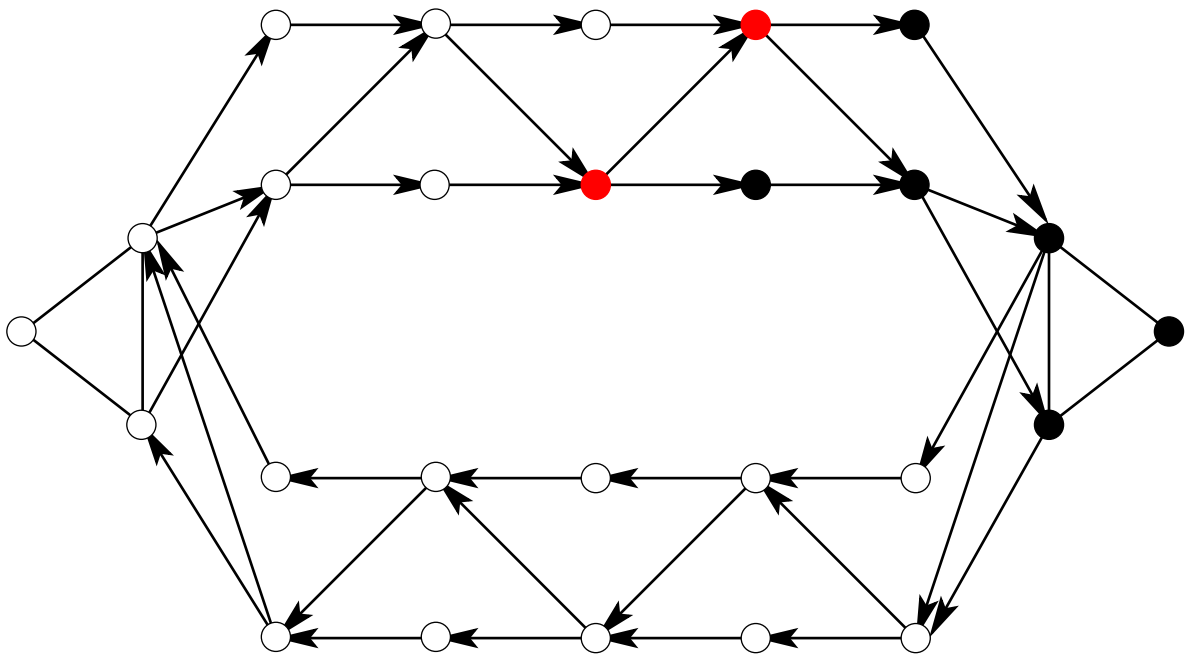

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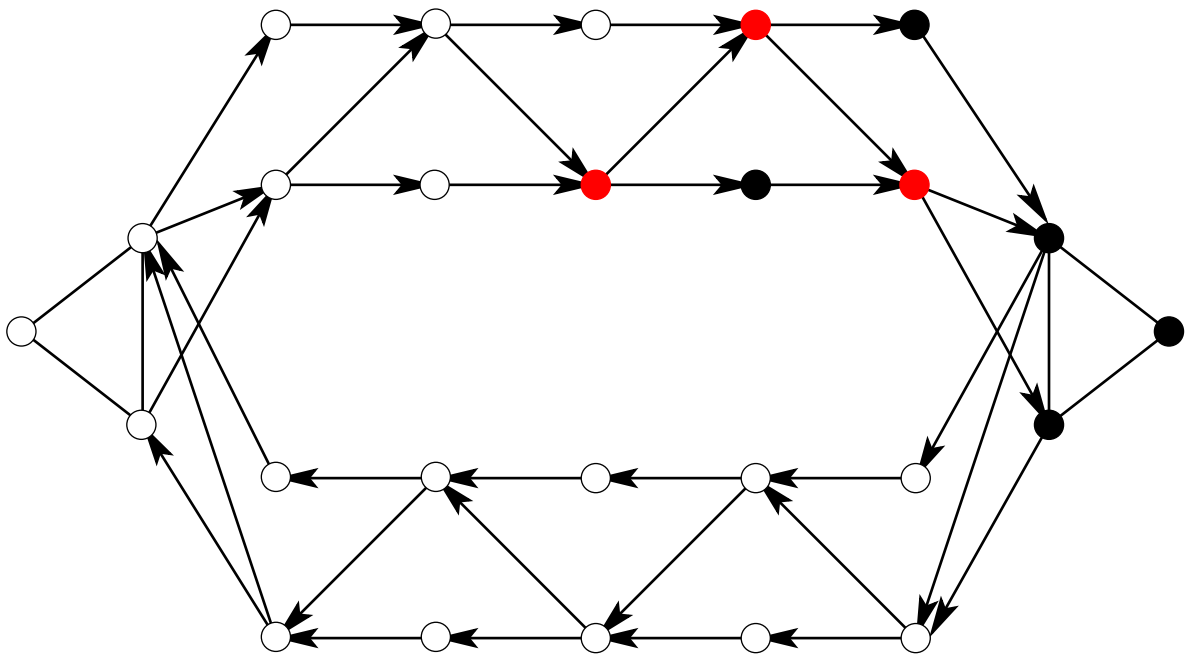

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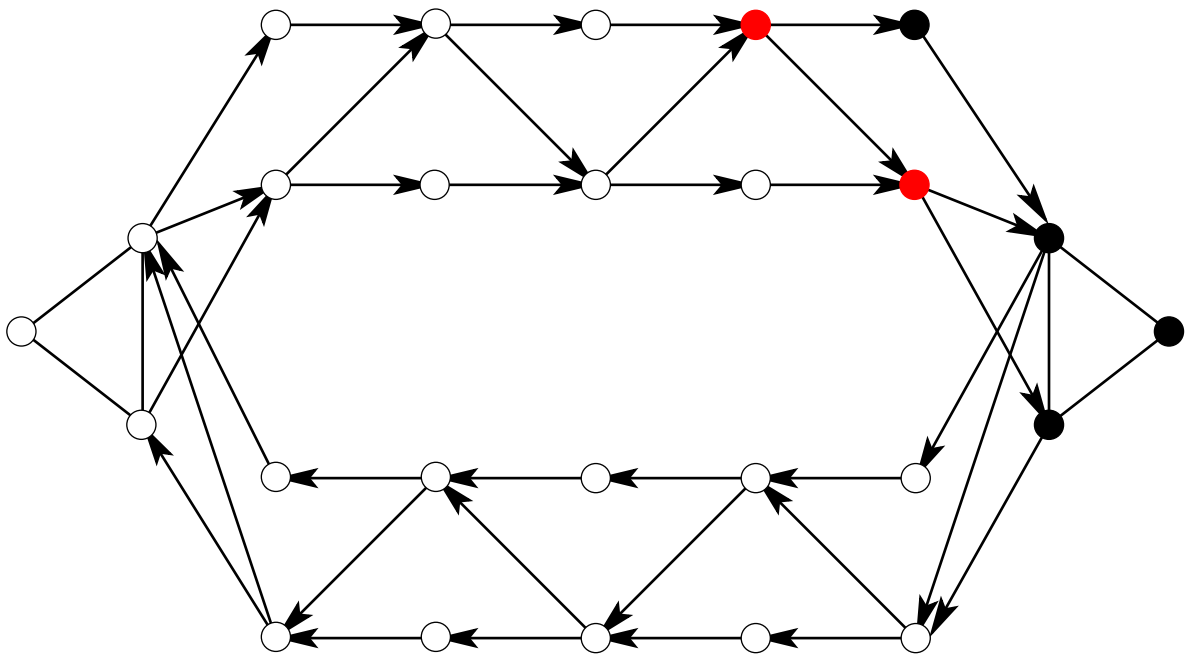

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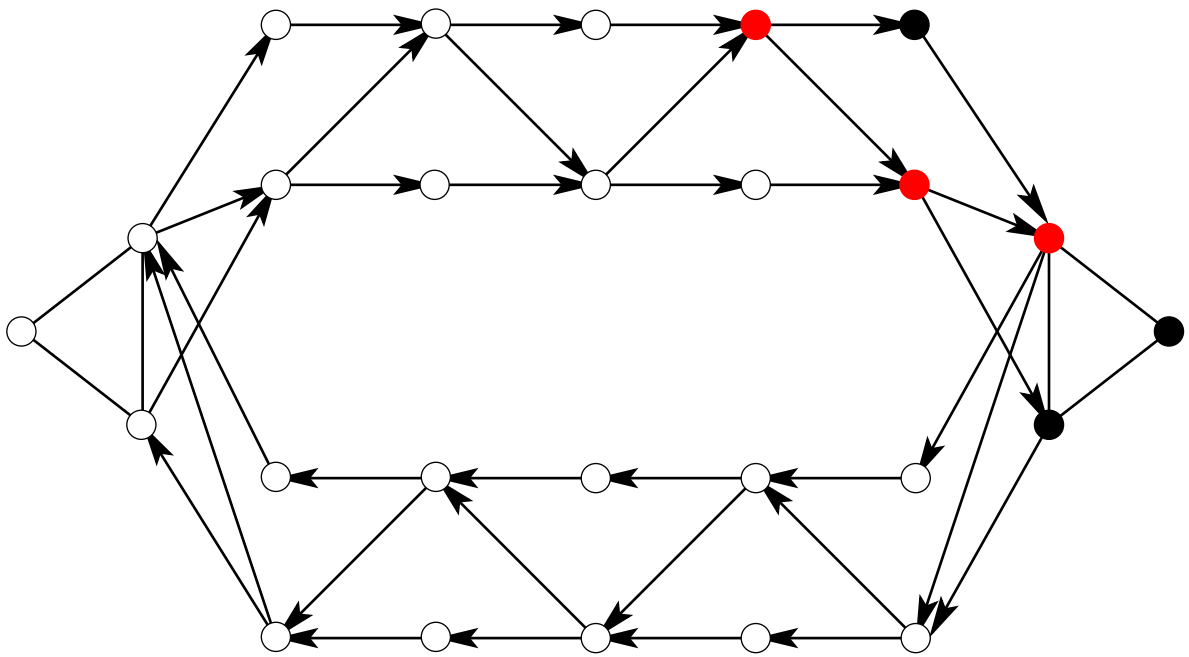

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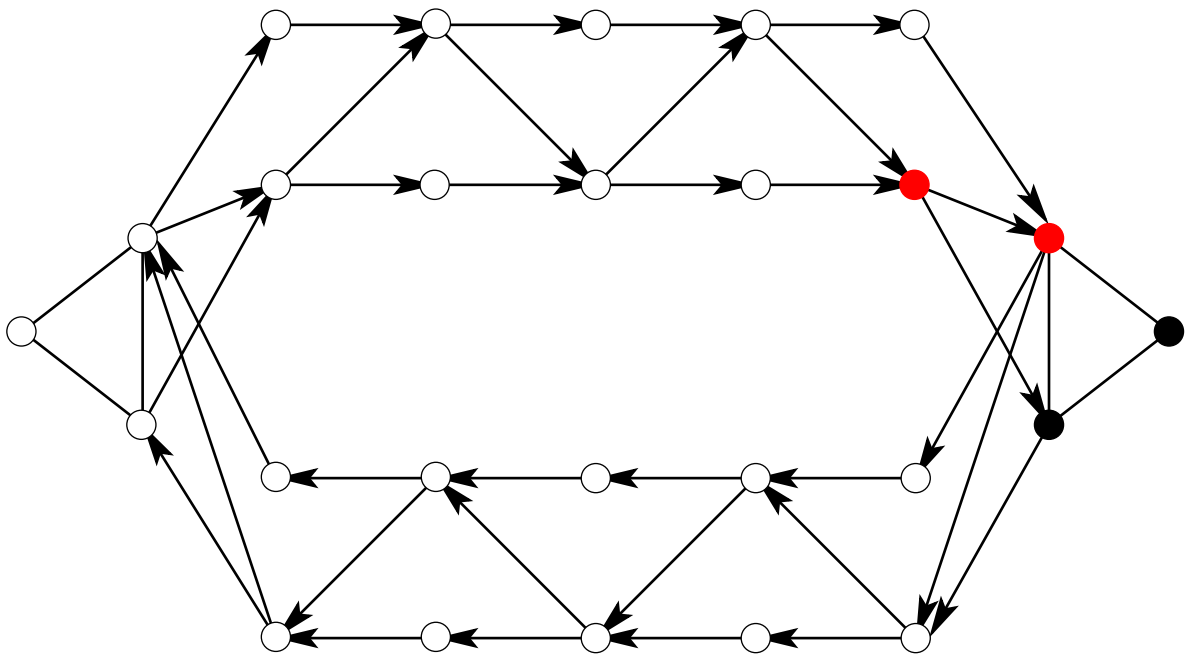

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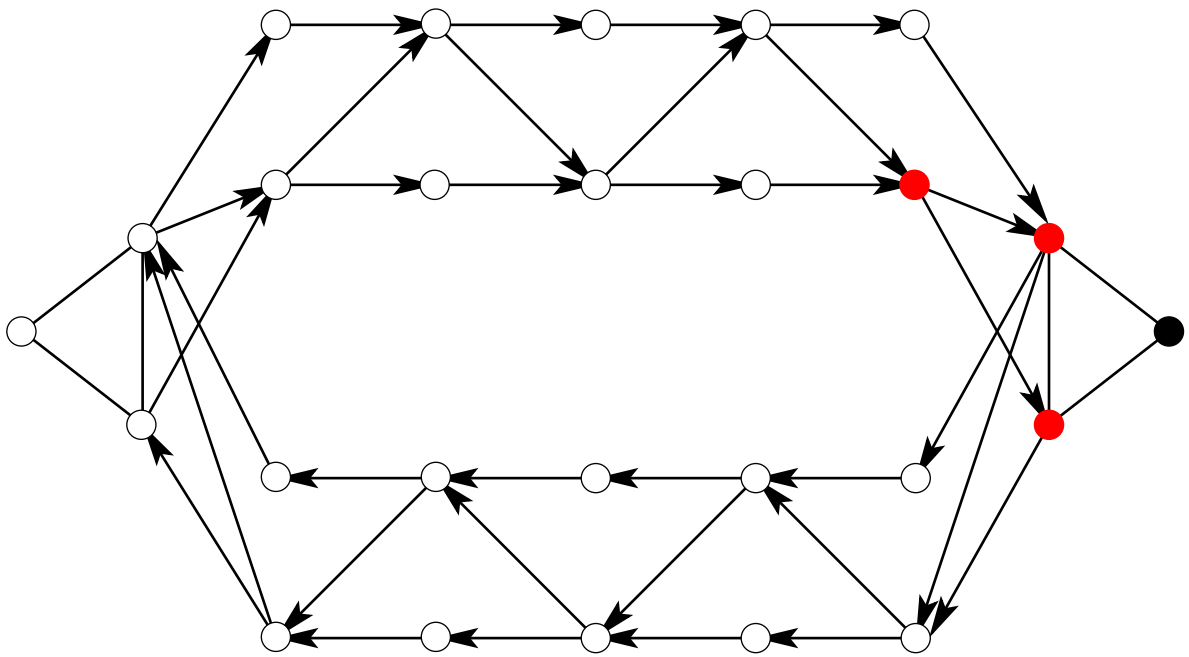

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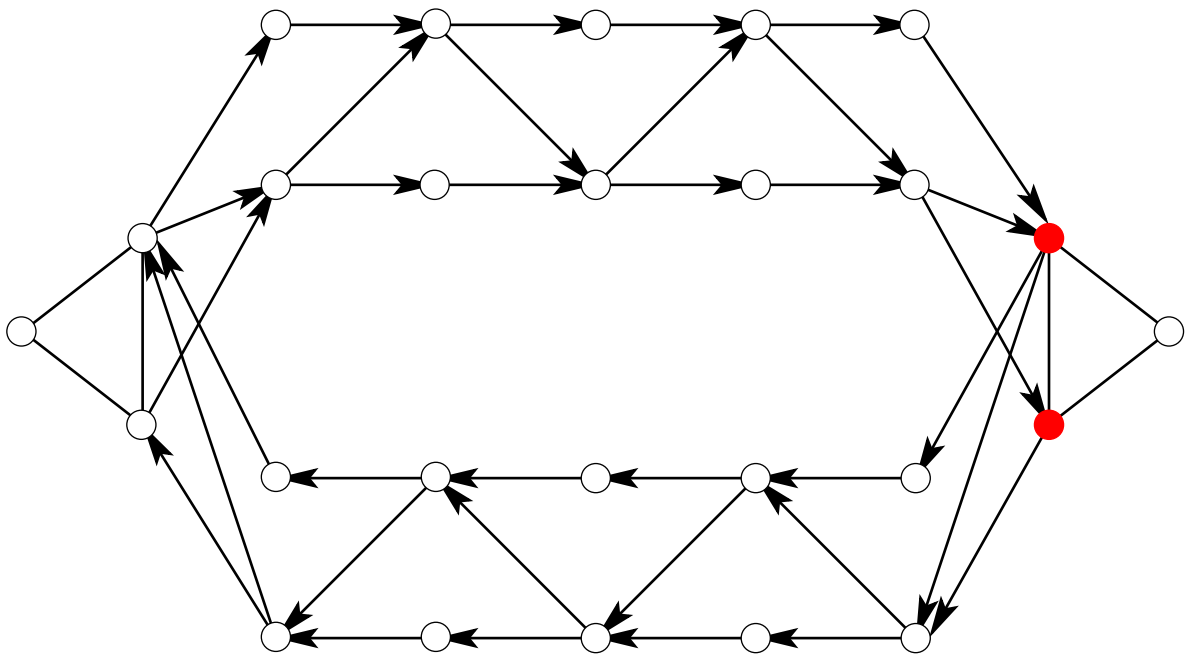
occupied vertices by the minimum # agents

$\exists$ digraphs with arbitrary large ratio: $\frac{mfvs_{pn}(D)}{mfvs(D)}$.



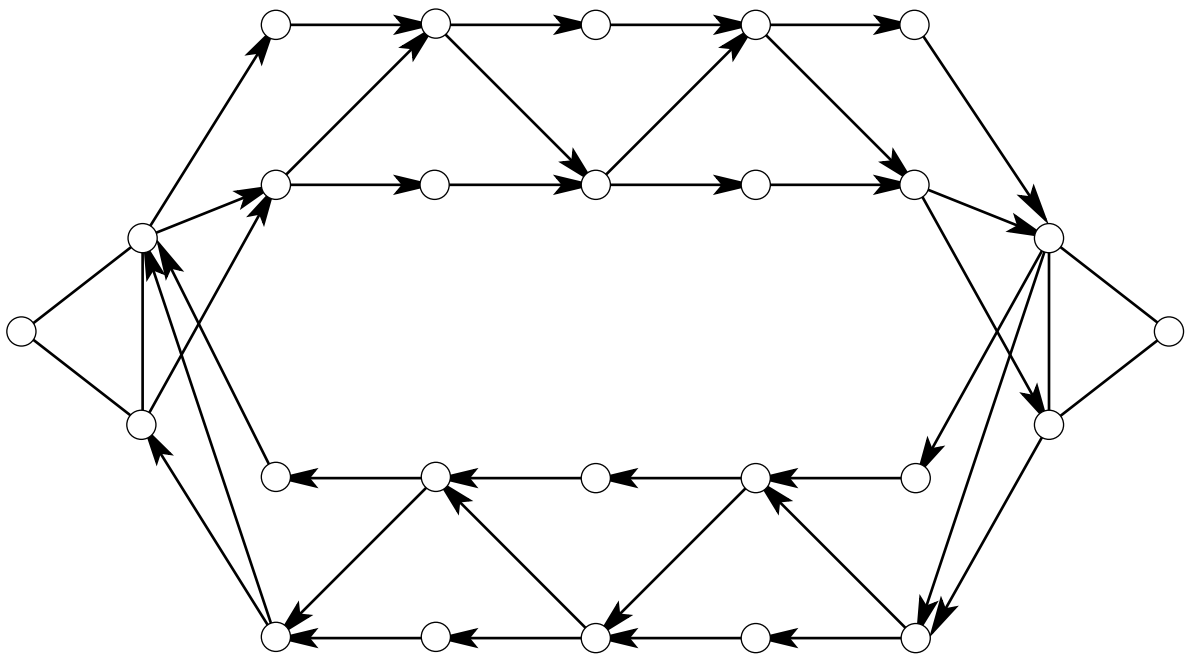
$mfvs(D) = 4$

$pn(D) = 3$

$mfvs_{pn}(D) = n + 4$

occupied vertices by the minimum # agents

$\exists$ digraphs with arbitrary large ratio: $\frac{mfvs_{pn}(D)}{mfvs(D)}$.



$mfvs(D) = 4$

$pn(D) = 3$

$mfvs_{pn}(D) = n + 4$

Open Question

Symmetric Digraphs

Maximum value of $\frac{mfvs_{pn}(D)}{mfvs(D)}$ for symmetric digraphs?

Lemma

There exists a symmetric digraph D with $\lambda > 3 - \varepsilon$ and $\forall D$ symmetric digraph, $\frac{mfvs_{pn}(D)}{mfvs(D)} \leq pn(D)$.

Conjecture: 3 is tight.

Perspectives

Among others...

- Determine routing + process strategy
- Mixte strategies using available resources
  - Tradeoff number of interruptions / number of switchings
- Inclusion of physical constraints (with Alcatel-Lucent)
  - Time to retune wavelengths after addition/removal
- Distributed strategies
- Sequence of maintenance operations
  - Mutualize route changes to reduce interruptions

Merch'i !!!! A l'année prochaine !!!!!