Tradeoffs for Routing Reconfiguration in WDM Networks

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Routing in WDM Networks

Physical Network, Links provide several wavelengths

**multi-graph** $G = (V, E)$

an edge $(u, v) \iff$ one wavelength on the link $(u, v)$

Routing of a set of requests/connections

set of requests $\mathcal{R} \subseteq 2^{V \times V}$

routing: for each request $(u, v)$, a path from $u$ to $v$ and 1 wavelength.

Problem: due to dynamicity of traffic, failures

how to maintain an efficient routing?
What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links
What happens in "real" world

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Example of a grid network with directed symmetric links

Request d: 1 → 3
What happens in "real" world

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Example of a grid network with directed symmetric links

Request d: 1 → 3
Request e: 6 → 5
What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

Request d : 1 → 3
Request e : 6 → 5
Request c : 2 → 3
What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

Request d: 1 → 3
Request e: 6 → 5
Request c: 2 → 3
Failure of link \{8, 9\}
What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

Request d: 1 → 3
Request e: 6 → 5
Request c: 2 → 3
Rerouting of request e
What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

Request d : 1 → 3
Request e : 6 → 5
Request c : 2 → 3
Request b : 1 → 5
New link \{8, 9\}
What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

Request d : 1 → 3
Request e : 6 → 5
Request c : 2 → 3
Request b : 1 → 5
Request a : 4 → 5
What happens in “real” world

Variation of traffic \( + \) dynamicity induced by failures
\Rightarrow \) Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

Leads to a poor usage of resources

Sometimes greedy routing is impossible even if several requests are allowed to be moved
What happens in "real" world

Variation of traffic + dynamicity induced by failures ⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

Leads to a poor usage of resources

Sometimes greedy routing is impossible even if several requests are allowed to be moved

If \{5, 8\} fails:
**Move-to-Vacant** impossible
What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links

2 questions arise:

1. Compute new routing
2. Switch from initial routing to final one

We focus on 2
Two ways of switching one request

Make-before-break:
Establish new path before switching the connection
⇒ Destination resources must be available

Break-before-make:
Break connection before establishing the new path
⇒ Traffic stopped = interruption
The Routing Reconfiguration Problem

How to go from the initial routing (left) to the final one (right)?

**Inputs:** Set of connection requests + current & new routing

**Output:** Scheduling for switching connection requests from current to new routes

**Constraint:** A connection is switched *only once*

**Objectives:** Number of Interruptions (detailed later)
Dependency digraph

\( u \rightarrow v \)
if \( u \) needs resources of \( v \)
if \( v \) must be rerouted/interrupted before \( u \)

\( b \) needs resources used by \( d \) and \( c \)
Dependency Digraph

- One vertex per connection with different routes in $I$ and $F$
- Arc from $u$ to $v$ if resources needed by $u$ in $F$ are used by $v$ in $I$
A game on dependency digraph

cyclic dependancies
⇒ Interruption required
A game on dependency digraph

put an agent on node $d$
break request $d$
A game on dependency digraph

process node c
reroute request c
A game on dependency digraph

A process node b
reroute request b
A game on dependency digraph

process node a
reroute request a
A game on dependency digraph

process node $d$
and remove agent
route request $d$
From now on: problem on digraphs

Any directed graph is a dependency digraph.
Two possible objectives

Minimize overall number of interrupted requests

Minimum Feedback Vertex Set (MFVS), here \( N/4 \)

**Remarks:** MFVS is NP-complete and non APX in digraphs
2-approx in undirected (directed symmetric) graphs
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Minimize number of simultaneous interrupted requests

Process Number, $pn =$ smallest number of requests that have to be simultaneously interrupted.
Here, $pn = 1 \Rightarrow$ Gap with MFVS up to $N/2$
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Minimize number of \textit{simultaneous} interrupted requests

\textbf{Process Number}, $pn = \text{smallest number of requests that have to be \textit{simultaneously} interrupted.}$

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Routing Reconfiguration, Process number

Game with Agents on the Dependency digraph $D$

Sequence of three basic operations,...

1. Place a searcher at a node = interrupt the request;
2. Process a node if all its out-neighbors are either processed or occupied by an agent = (Re)route a connection when final resources are available;
   A processed node is removed from the dependency digraph.
3. Remove an agent from a node, after having processed it.

...that must result in processing all nodes

Process number $pn(D) = \min p \mid D$ can be processed with $p$ agents

Remark: In undirected graphs or symmetric digraphs:

Graph Searching game when a fugitive is captured when surrounded
Example: DAG

Only one operation is used

1. Place a searcher at a node = interrupt the request;
2. Process a node if all its out-neighbors are either processed or occupied by an agent = (Re)route a connection when final resources are available;
3. Remove an agent from a node, after having processed it.

DAG

Theorem

\[ pn(D) = 0 \text{ iff } D \text{ is a DAG} \]
One agent is used

1. Place a searcher at a node = interrupt the request;
2. Process a node if all its out-neighbors are either processed or occupied by an agent = (Re)route a connection when final resources are available;
3. Remove an agent from a node, after having processed it.

Theorem

\[ pn(D) = 1 \iff \forall SCC, \text{MFVS}(SCC) = 1 \quad O(N + M) \]
Digraphs with process number 1

One agent is used

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\[ O(N + M) \]
Process number versus Other Parameters

A parameter of directed (and undirected) graphs vs, vertex separation

In undirected graph or symmetric digraph: vs = pathwidth

\( vs(G) = pw(G) \)

Kinnersley [IPL 92]

Theorem (Coudert & Sereni, 2007)

\( vs(D) \leq pn(D) \leq vs(D) + 1 \)

Complexity: NP-Complete, Not APX

- Characterization of digraphs with process number 0, 1, 2
  (Coudert & Sereni, 2007)
State of the Art

- distributed $O(n \log n)$-time exact algorithm in trees
  (Coudert, Huc, Mazauric [DISC 08])

- generalized Model handling priority connections
  connections that cannot be interrupted
  heuristic using random walk
  (Coudert, Huc, Mazauric, Nisse, Sereni [ONDM 09])
  heuristic using LP (Solano [Globecom 09])

- generalized Model allowing bandwidth sharing
  deciding whether reconfiguration may be done without interruption:
  NP-complete (Coudert, Mazauric, Nisse [AGT 09])
Tradeoff: total/ max simultaneous interruptions

#occupied vertices

mfvs_{pn} - pn

mfvs

pn - pn_{mfvs} - mfvs

#agents
Complexity

- Smallest number of agents such that the number of occupied vertices is minimum $= pn_{mfvs}(D)$
- $\mu = \frac{pn_{mfvs}(D)}{pn(D)}$
- Smallest total number of occupied vertices such that the number of agents is minimum $= mfvs_{pn}(D)$
- $\lambda = \frac{mfvs_{pn}(D)}{mfvs(D)}$

Theorem

The problems of determining $pn_{mfvs}(D)$, $mfvs_{pn}(D)$, $\mu$, and $\lambda$ are NP-Complete and not APX.
∃ digraphs with arbitrary large ratio: \( \mu = \frac{pn_{mfvs}(D)}{pn(D)} \).

\[ mfvs(D) = n \]
\[ pn(D) = 2 \]
\[ pn_{mfvs}(D) = n \]
∃ digraphs with arbitrary large ratio: \[ \mu = \frac{p_{n_{mfvs}}(D)}{p_n(D)}. \]

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\text{mfvs}(D) &= n \\
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\end{align*}
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∃ digraphs with arbitrary large ratio: $\lambda = \frac{mfvs_{pn}(D)}{mfvs(D)}$.

$mfvs(D) = 4$

$pn(D) = 3$

$mfvs_{pn}(D) = n + 4$
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Directed graphs with BOUNDED Process Number:
\[ \lambda = \text{occupied vertices} / \text{mfvs UNBOUNDED} \]

What if \( G \) is undirected??

Let \( G \) be a symmetric directed/undirected graph,
\[ \lambda = \frac{\text{mfvs}_{pn}(G)}{\text{mfvs}(G)} \leq \text{pn}(G) \]
Directed graphs with BOUNDED Process Number:
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Let \( G \) be a symmetric directed/undirected graph,
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\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} \leq pn(G)
\]
Consider a MFVS of $G$. $S$ using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS.
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\[
\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{Y+X}{Y+W}
\]
Consider a MFVS of $G$. $S$ using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS.

$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|}$$

$$|X| = |X \cap N(W)| + |R| \leq |W|.pn(G) + |R|$$
Consider a MFVS of $G$. $S$ using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS.

$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W| \cdot pn(G)+|R|}{|Y|+|W|}$$

$N(R) = \{v_1, \cdots, v_r\} \subseteq Y$ : ordering in which agents are removed.
Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS.

$$
\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W|.pn(G)+|R|}{|Y|+|W|}
$$

$$
|N(v_1)| \leq pn(G) - 1
$$
Consider a MFVS of $G$. $S$ using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS

\[ \lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y| + |X|}{|Y| + |W|} \leq \frac{|Y| + |W| \cdot pn(G) + |R|}{|Y| + |W|} \]

\[ |N(v_2) \setminus N(v_1)| \leq pn(G) - 1, \quad |N(v_i) \setminus \bigcup_{j<i} N(v_j)| \leq pn(G) - 1 \]
Consider a MFVS of $G$. $S$ using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS.

$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y| + |X|}{|Y| + |W|} \leq \frac{|Y| + |W| \cdot pn(G) + |R|}{|Y| + |W|}$$

so $|R| \leq |N(R)| (pn(G) - 1) \leq |Y| (pn(G) - 1)$
Consider a MFVS of $G$. $S$ using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS.

\[\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W|.pn(G)+|R|}{|Y|+|W|}\]

\[\lambda \leq \frac{|Y|+|W|.pn(G)+|Y|(pn(G)-1)}{|Y|+|W|} = pn(G)\]
∀ε, ∃ symmetric digraphs D: \[ \lambda = \frac{\text{mfvs}_{pn}(D)}{\text{mfvs}(D)} > 3 - \epsilon. \]
\( \forall \epsilon, \exists \) symmetric digraphs \( D \): 
\[ \lambda = \frac{mfvs_{pn}(D)}{mfvs(D)} > 3 - \epsilon. \]
\[ \forall \epsilon, \exists \text{ symmetric digraphs } D: \quad \lambda = \frac{mfv_{spn}(D)}{mfv(D)} > 3 - \epsilon. \]
\forall \epsilon, \exists \text{ symmetric digraphs } D: \lambda = \frac{mfvspn(D)}{mfvs(D)} > 3 - \epsilon.

mfvs(D) = n + 4

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machining

\begin{align*}
mfv(D) &= n + 4 \\
pn(D) &= n + 1 \\
mfv_{spn}(D) &= 3n + 2
\end{align*}
Some open questions

A lot of ”bad” news… No tradeoff?
Can we restrict the class of dependancy digraphs?
No… even if the physical network is a directed path…

Conjecture

Let $G$ be a symmetric directed/undirected graph,

$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} \leq 3$$

- Approximation and Heuristic algorithms for these parameters
- Link between random walks and separators of graphs?
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Merci