Tradeoffs for Routing Reconfiguration in WDM Networks

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Routing in WDM Networks

Physical Network, Links provide several wavelengths

multi-graph $G = (V, E)$

an edge $(u, v) \Leftrightarrow$ one wavelength on the link (u, v)

Routing of a set of requests/connections

set of requests $\mathcal{R} \subseteq 2^{V \times V}$

routing: for each request (u, v),

a path from u to v and 1 wavelength.

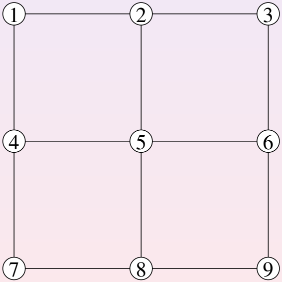
Problem: due to dynamicity of traffic, failures

how to maintain an efficient routing?

What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

Example of a grid network with directed symmetric links



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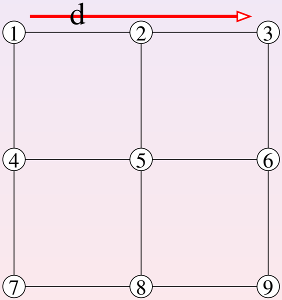

Request d : $1 \rightarrow 3$

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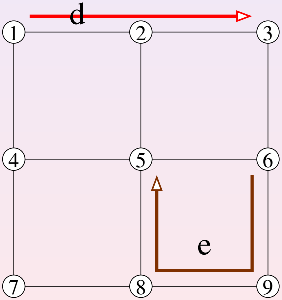

Request d : 1 → 3
Request e : 6 → 5

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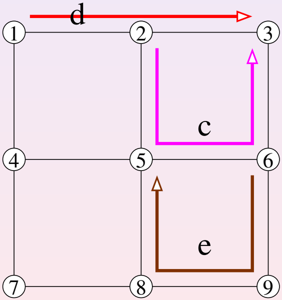

Request d : $1 \longrightarrow 3$
Request e : $6 \longrightarrow 5$
Request c : $2 \longrightarrow 3$

What happens in "real" world

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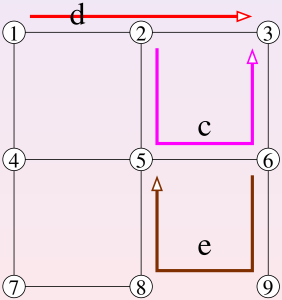

Request d : $1 \rightarrow 3$
Request e : $6 \rightarrow 5$
Request c : $2 \rightarrow 3$
Failure of link $\{8, 9\}$

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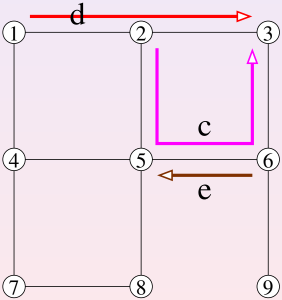

Request d : $1 \longrightarrow 3$
Request e : $6 \longrightarrow 5$
Request c : $2 \longrightarrow 3$
Rerouting of request e

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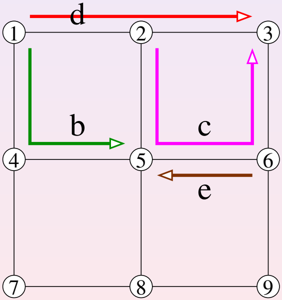

Request d : $1 \longrightarrow 3$
Request e : $6 \longrightarrow 5$
Request c : $2 \longrightarrow 3$
Request b : $1 \longrightarrow 5$
New link $\{8, 9\}$

What happens in "real" world

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$\Rightarrow$ Online processes to route all requests: e.g., greedy routing

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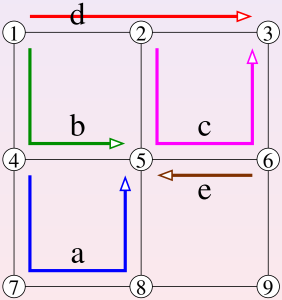

Request d : $1 \longrightarrow 3$
Request e : $6 \longrightarrow 5$
Request c : $2 \longrightarrow 3$
Request b : $1 \longrightarrow 5$
Request a : $4 \longrightarrow 5$

What happens in "real" world

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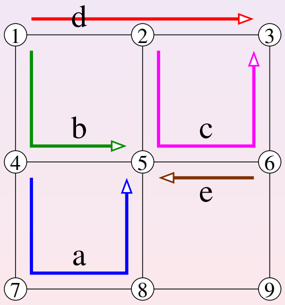
Example of a grid network with directed symmetric links



Leads to a poor usage of ressources

Sometimes greedy routing is impossible
even if several requests are allowed to be moved

What happens in "real" world

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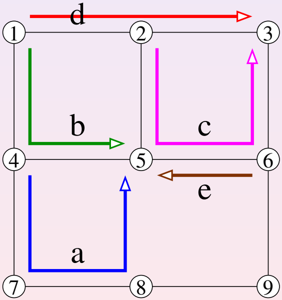

Leads to a poor usage of ressources

Sometimes greedy routing is impossible
even if several requests are allowed to be moved

If $\{5, 8\}$ fails:
Move-to-Vacant impossible

What happens in "real" world

Variation of traffic + dynamicity induced by failures
⇒ Online processes to route all requests: e.g., greedy routing

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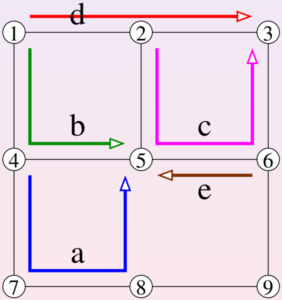


2 questions arise:

1. Compute new routing
2. Switch from initial routing to final one

We focus on 2

Two ways of switching one request

Make-before-break:

Establish new path before switching the connection

$\Longrightarrow$ Destination resources must be available

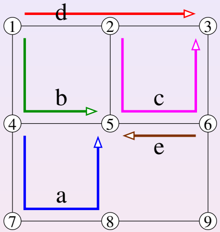
Break-before-make:

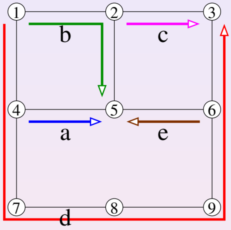
Break connection before establishing the new path

$\Longrightarrow$ Traffic stopped $=$ interruption

The Routing Reconfiguration Problem

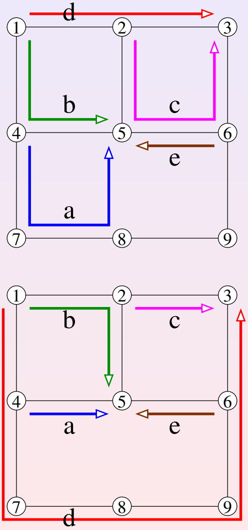
How to go from the initial routing (left) to the final one (right)?

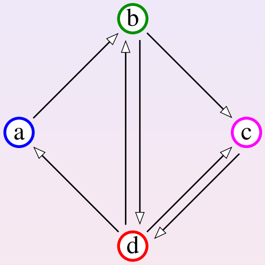




Inputs:	Set of connection requests + current & new routing
Output:	Scheduling for switching connection requests from current to new routes
Constraint:	A connection is switched **only once**
ObjectiveS	Number of Interruptions (detailled later)

Dependency digraph

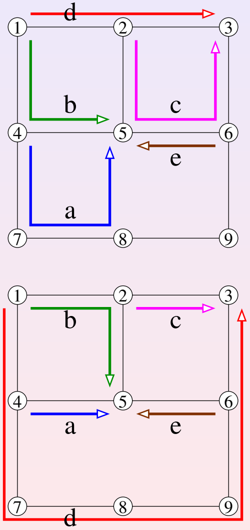


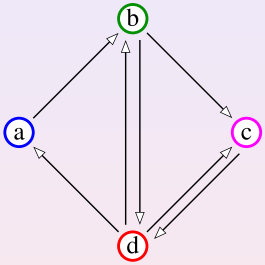


$u \longrightarrow v$
if u needs ressources of v
if v must be rerouted/interrupted before u

b needs ressources used by d and c

Dependency digraph

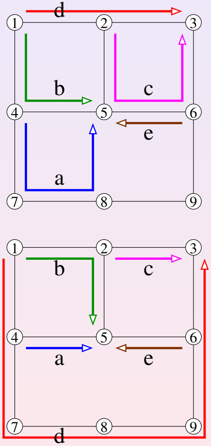


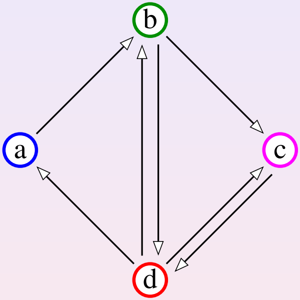


Dependancy Digraph

- one vertex per connection with different routes in $\mathcal{I}$ and $\mathcal{F}$
- arc from u to v if ressources needed by u in $\mathcal{F}$ are used by v in $\mathcal{I}$

A game on dependency digraph

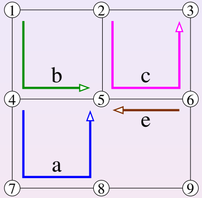


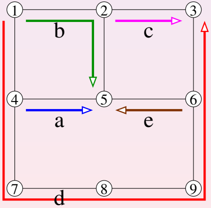


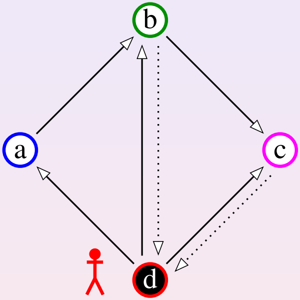
cyclic dependancies

$\Rightarrow$ Interruption required

A game on dependency digraph

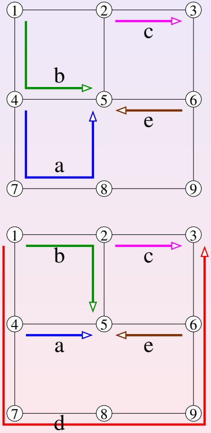


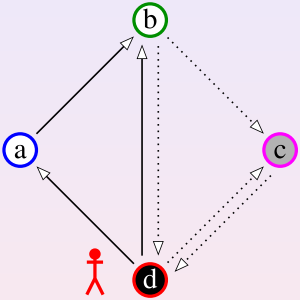




put an agent on node *d*
break request *d*

A game on dependency digraph

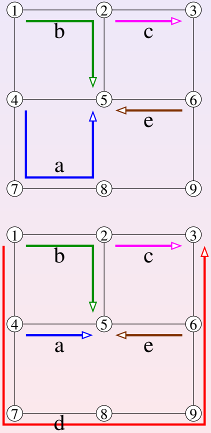


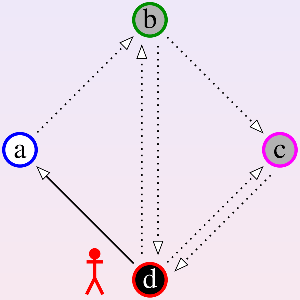


process node c

reroute request c

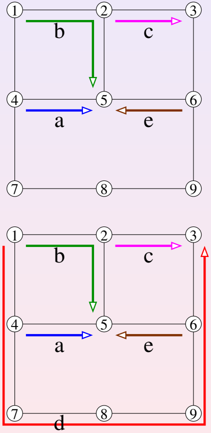
A game on dependency digraph

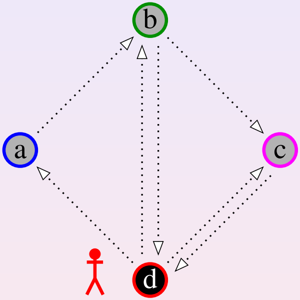




process node b
reroute request b

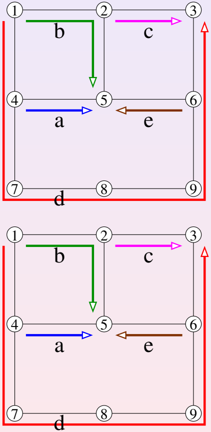
A game on dependency digraph

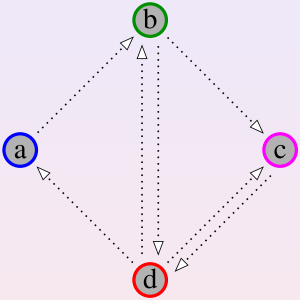




process node *a*
reroute request *a*

A game on dependency digraph

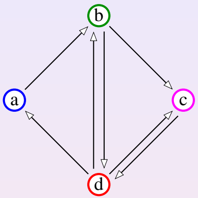


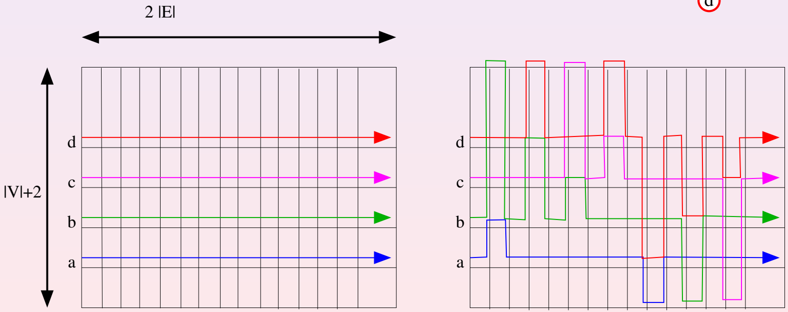


process node d
and remove agent
route request d

From now on: problem on digraphs

Any directed graph is a dependency digraph

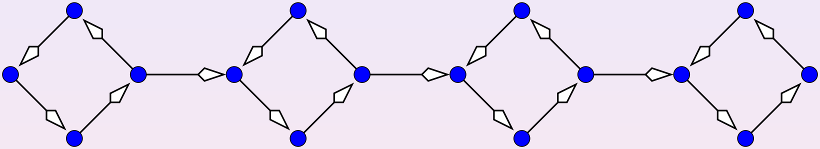




Two possible objectives

Minimize overall number of interrupted requests

Minimum Feedback Vertex Set (MFVS), here $N/4$



Remarks: MFVS is NP-complete and non APX in digraphs
2-approx in undirected (directed symmetric) graphs

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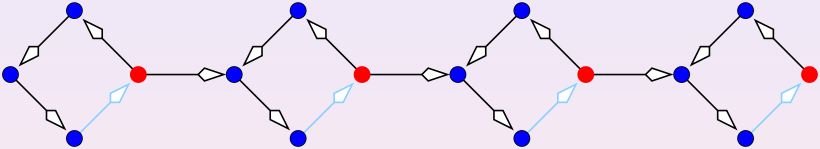

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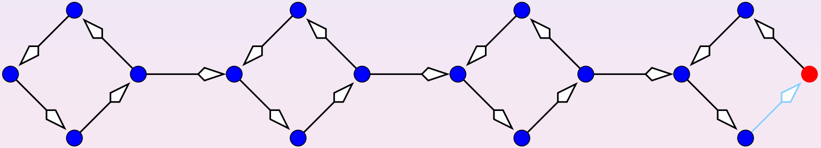

Minimize number of simultaneous interrupted requests

Process Number, $pn =$ smallest number of requests that have to be **simultaneously** interrupted.

Here, $pn = 1 \Rightarrow$ Gap with MFVS up to $N/2$

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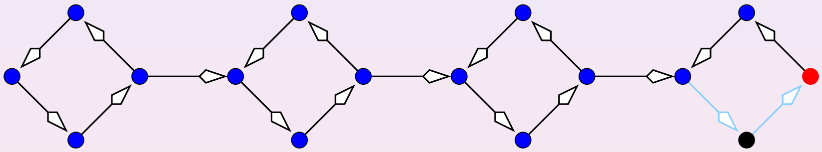

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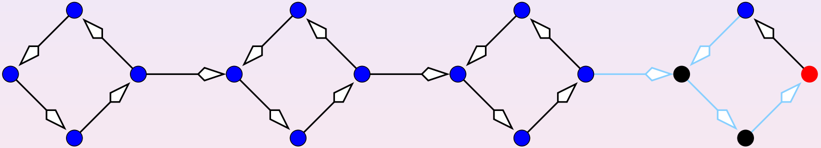

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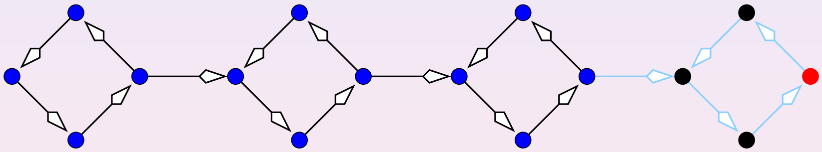

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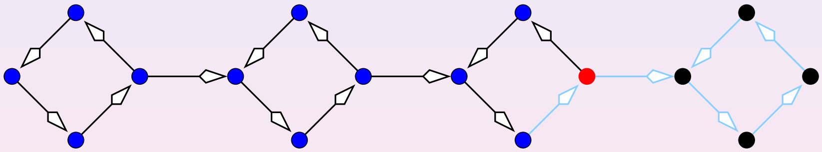

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Routing Reconfiguration, Process number

Game with Agents on the Dependency digraph D

Sequence of three basic operations,...

1. Place a searcher at a node = **interrupt the request**;
2. Process a node if all its out-neighbors are either processed or occupied by an agent = **(Re)route a connection when final resources are available**;

   A processed node is removed from the dependency digraph.
3. Remove an agent from a node, after having processed it.

...that must result in processing all nodes

Process number $pn(D) = \min p \mid D$ can be processed with p agents

Remark: In undirected graphs or symmetric digraphs:

Graph Searching game when a fugitive is captured when surrounded

Example: DAG

Only one operation is used

1. Place a searcher at a node = interrupt the request;
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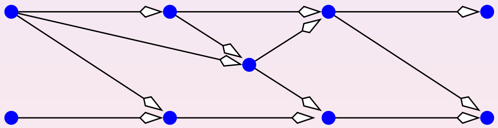
DAG



Theorem

$pn(D) = 0$ iff D is a DAG

Digraphs with process number 1

One agent is used

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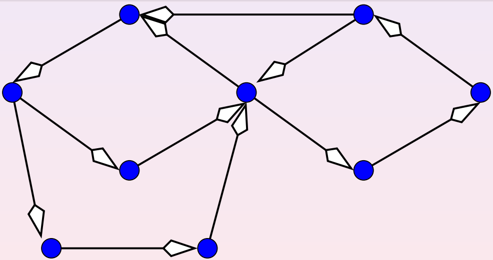

Theorem

$pn(D) = 1 \Leftrightarrow \forall SCC$, MFVS$(SCC) = 1$ $\qquad O(N + M)$

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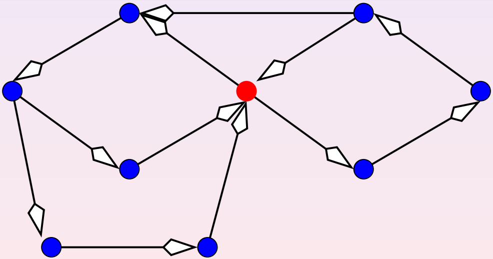

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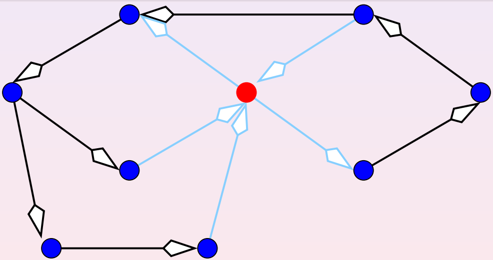


Theorem

$pn(D) = 1 \Leftrightarrow \forall SCC,\ \mathrm{MFVS}(SCC) = 1$ $\quad O(N + M)$

Process number versus Other Parameters

a parameter of directed (and undirected) graphs

vs, vertex separation

in undirected graph or symetric digraph: vs = pathwidth

$vs(G) = pw(G)$ **Kinnersley [IPL 92]**

Theorem **(Coudert & Sereni, 2007)**

$vs(D) \leq pn(D) \leq vs(D) + 1$

Complexity: NP-Complete, Not APX

- Characterization of digraphs with process number 0, 1, 2
  (Coudert & Sereni, 2007)

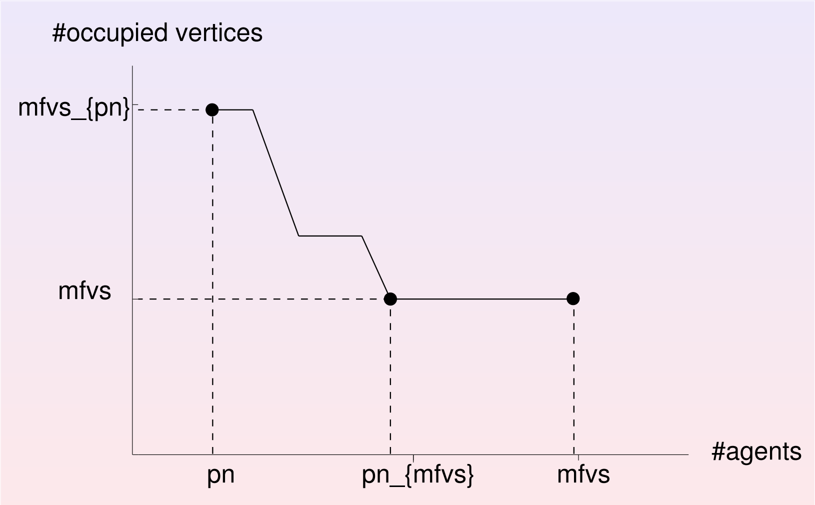
State of the Art

- distributed $O(n \log n)$-time exact algorithm in trees
  (Coudert, Huc, Mazauric [DISC 08])

- generalized Model handling priority connections
  connections that cannot be interrupted
  heuristic using random walk
  (Coudert, Huc, Mazauric, Nisse, Sereni [ONDM 09])
  heuristic using LP **(Solano [Globecom 09])**

- generalized Model allowing bandwidth sharing
  deciding whether reconfiguration may be done without interruption:
  NP-complete **(Coudert, Mazauric, Nisse [AGT 09])**

Tradeoff: total/ max simultaneous interruptions



Complexity

- Smallest number of agents such that the number of occupied vertices is minimum $= pn_{mfvs}(D)$
- $\mu = \frac{pn_{mfvs}(D)}{pn(D)}$

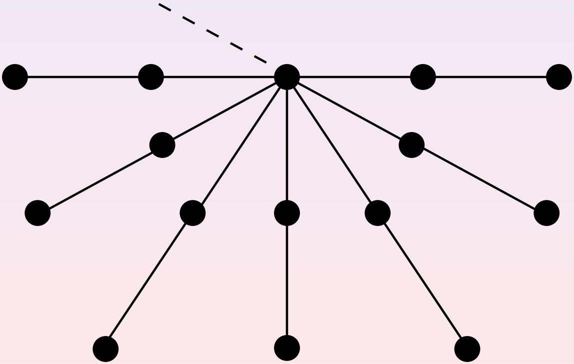
- Smallest total number of occupied vertices such that the number of agents is minimum $= mfvs_{pn}(D)$
- $\lambda = \frac{mfvs_{pn}(D)}{mfvs(D)}$

Theorem

The problems of determining $pn_{mfvs}(D)$, $mfvs_{pn}(D)$, μ, and λ are NP-Complete and not APX.

$\#$ agents for minimizing $\#$ occupied vertices

$\exists$ digraphs with arbitrary large ratio: $\mu = \frac{pn_{mfvs}(D)}{pn(D)}$.



$mfvs(D) = n$

$pn(D) = 2$

$pn_{mfvs}(D) = n$

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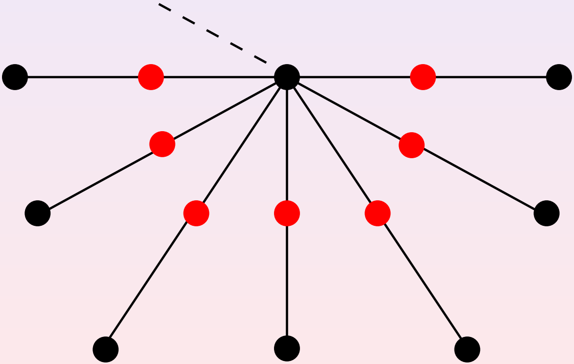

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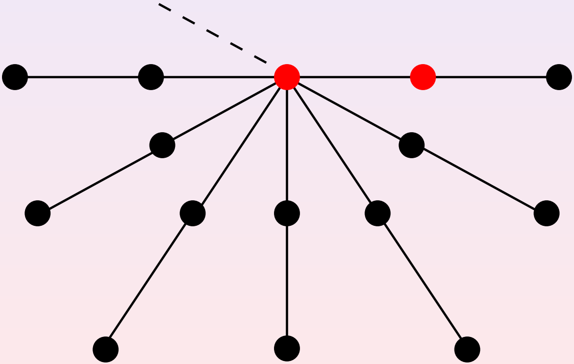
$$pn_{mfvs}(D) = n$$

$\#$ agents for minimizing $\#$ occupied vertices

$\exists$ digraphs with arbitrary large ratio: $\mu = \frac{pn_{mfvs}(D)}{pn(D)}$.



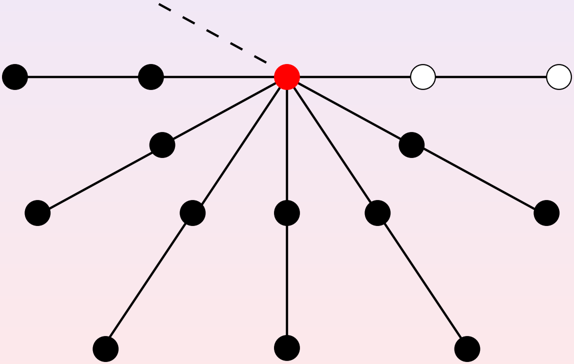
$$mfvs(D) = n$$

$$pn(D) = 2$$

$$pn_{mfvs}(D) = n$$

agents for minimizing # occupied vertices

$\exists$ digraphs with arbitrary large ratio: $\mu = \frac{pn_{mfvs}(D)}{pn(D)}$.



$mfvs(D) = n$

$pn(D) = 2$

$pn_{mfvs}(D) = n$

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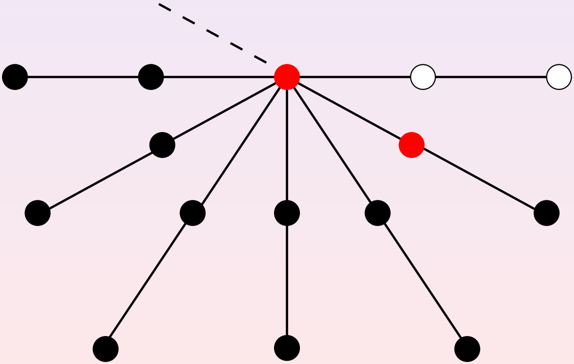

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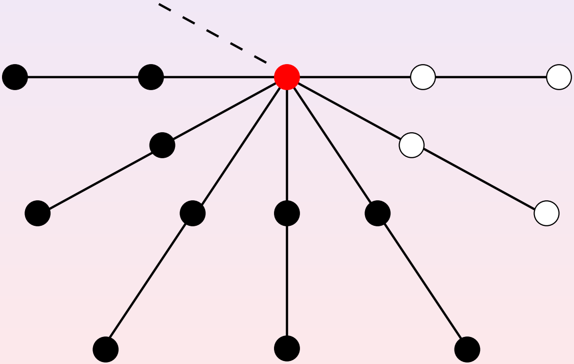

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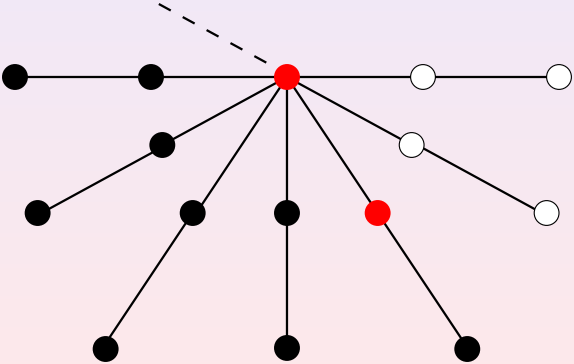

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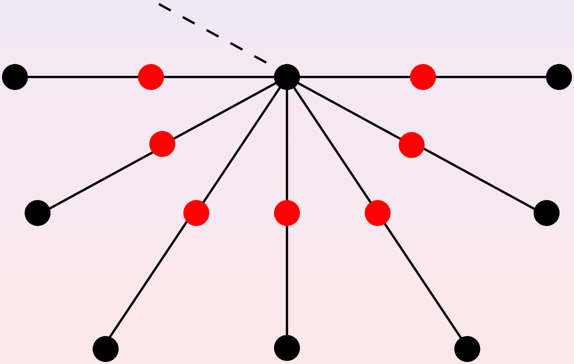


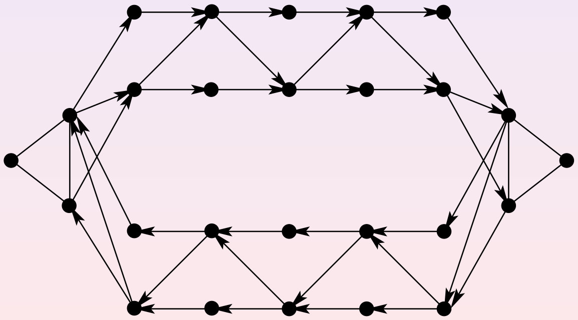
$mfvs(D) = n$

$pn(D) = 2$

$pn_{mfvs}(D) = n$

occupied vertices by the minimum # agents

$\exists$ digraphs with arbitrary large ratio: $\lambda = \frac{mfvs_{pn}(D)}{mfvs(D)}$.



$mfvs(D) = 4$

$pn(D) = 3$

$mfvs_{pn}(D) = n + 4$

occupied vertices by the minimum # agents

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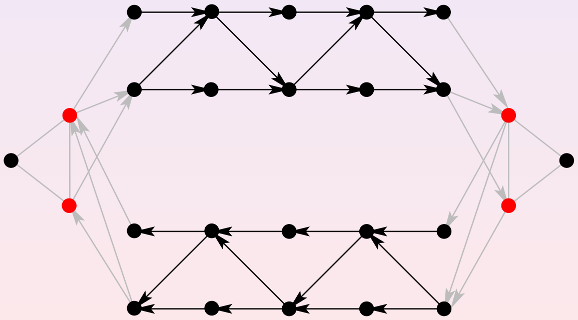

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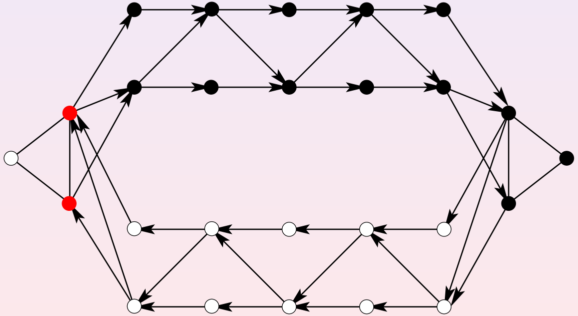

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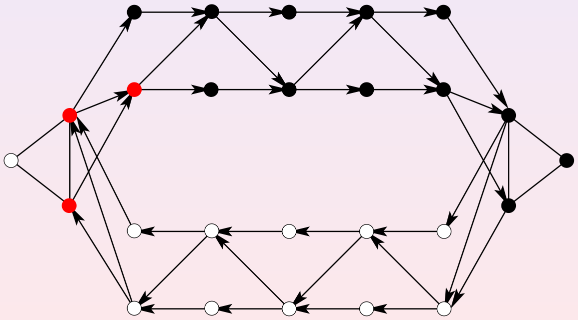

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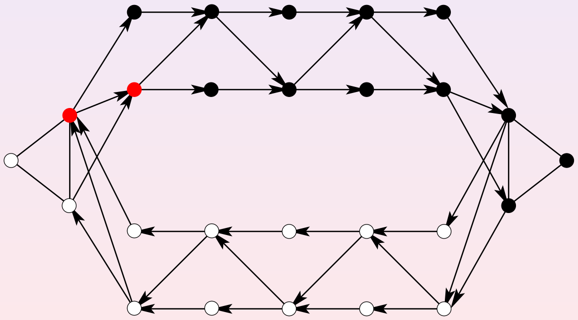

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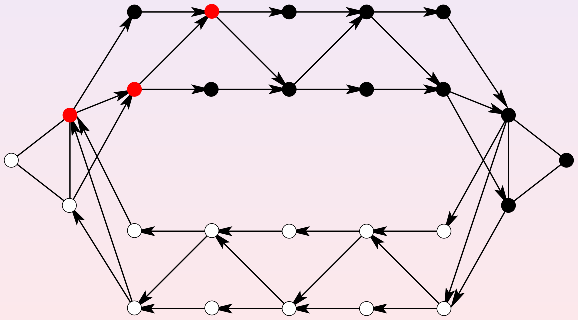

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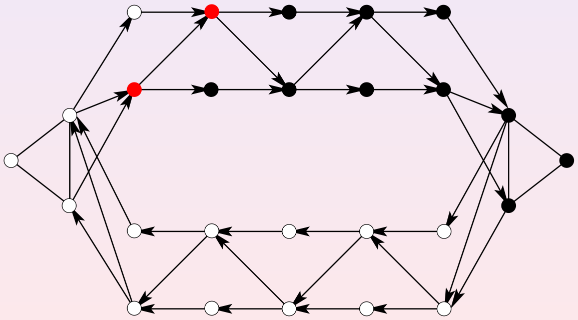

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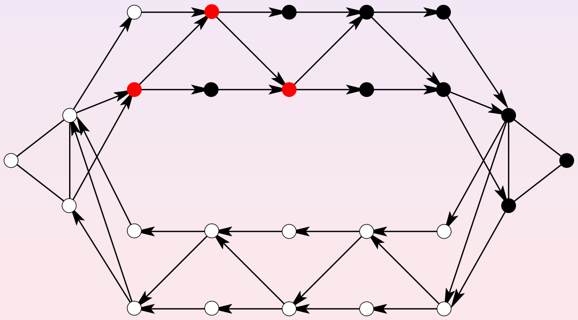

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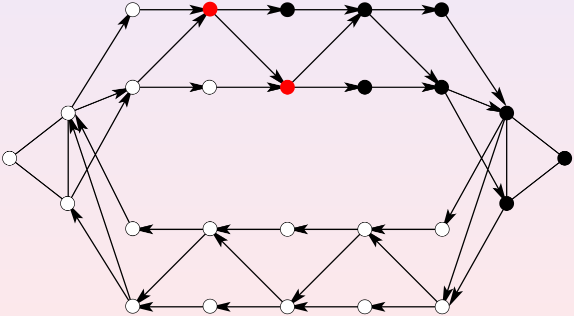
occupied vertices by the minimum # agents

∃ digraphs with arbitrary large ratio: $\lambda = \frac{mfvs_{pn}(D)}{mfvs(D)}$.



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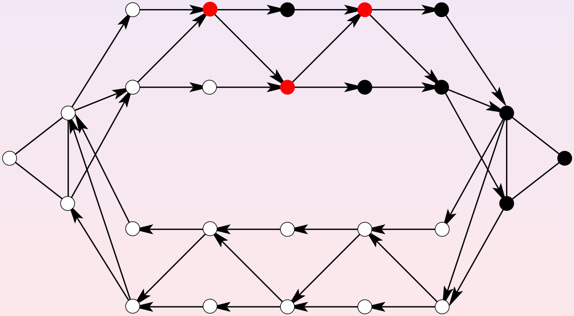

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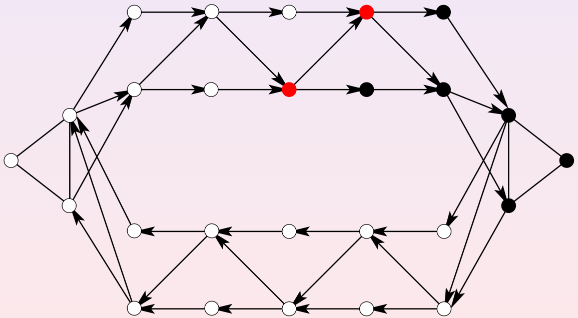

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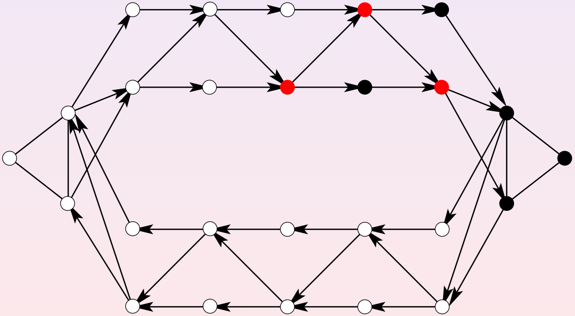

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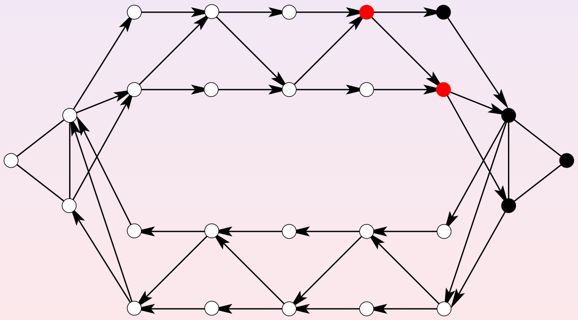

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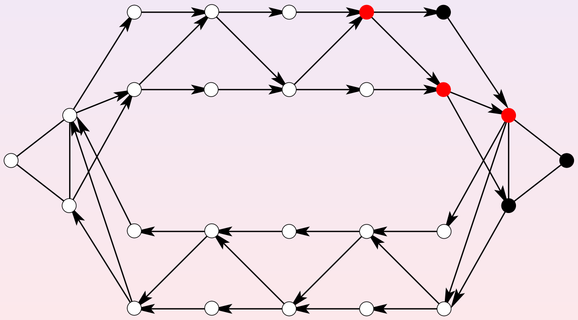

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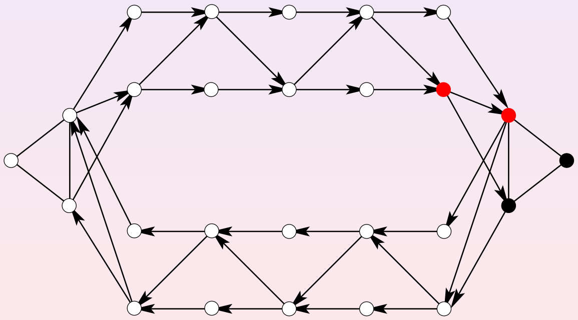

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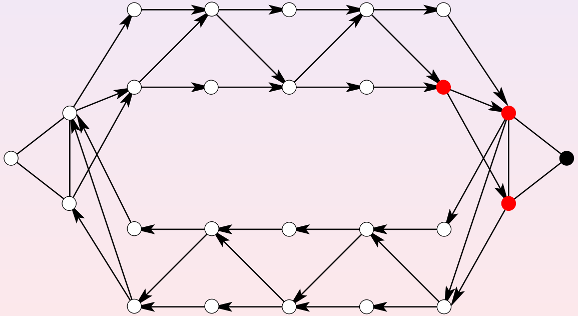

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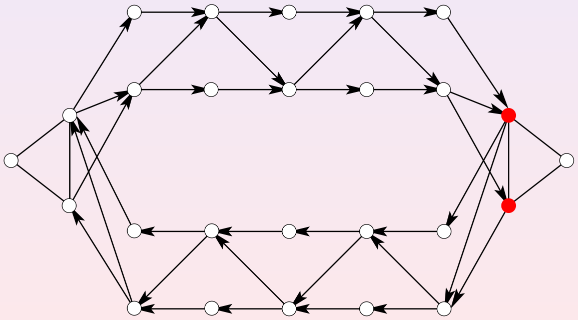

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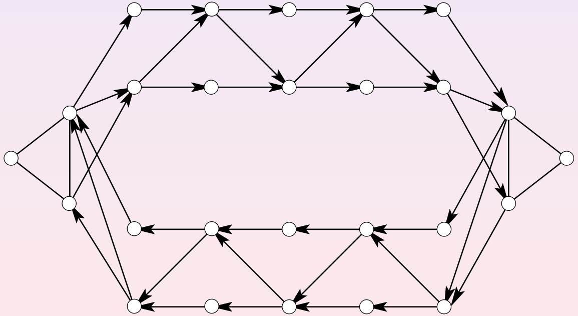


$mfvs(D) = 4$

$pn(D) = 3$

$mfvs_{pn}(D) = n + 4$

$\#$ occupied vertices by the minimum $\#$ agents

Directed graphs with BOUNDED Process Number:
$\lambda =$ occupied vertices / mfvs UNBOUNDED

What if G is undirected ??

Let G be a symmetric directed/undirected graph,
$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} \leq pn(G)$

$\#$ occupied vertices by the minimum $\#$ agents

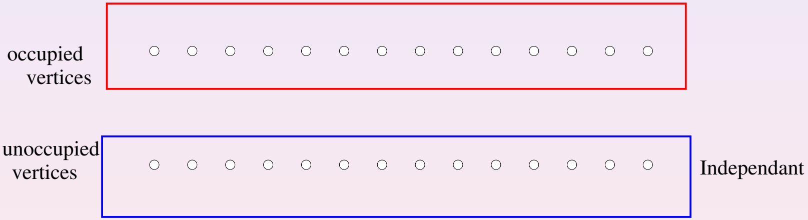
Directed graphs with BOUNDED Process Number:
$\lambda =$ occupied vertices / mfvs UNBOUNDED

What if G is undirected ??

Let G be a symmetric directed/undirected graph,
$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} \leq pn(G)$

occupied vertices by the minimum # agents

Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS



$\#$ occupied vertices by the minimum $\#$ agents

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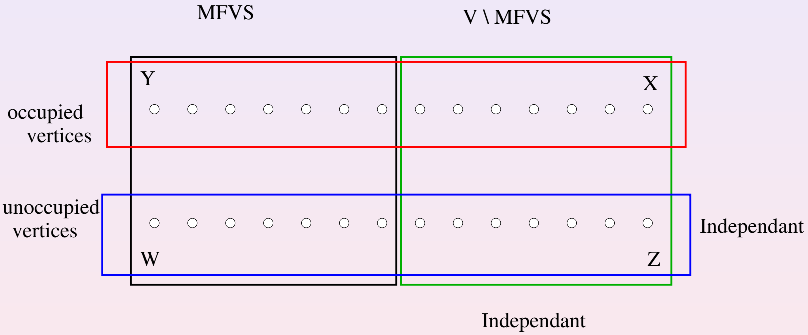
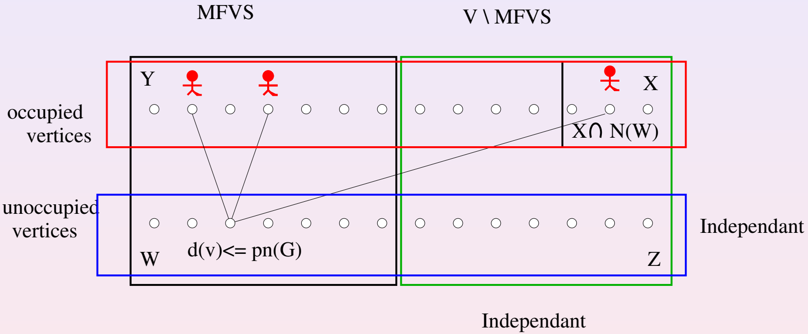


$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{Y+X}{Y+W}$$

occupied vertices by the minimum # agents

Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS

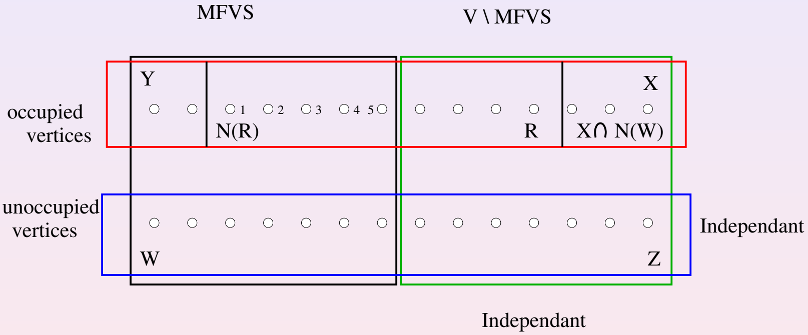


$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|}$$

$$|X| = |X \cap N(W)| + |R| \leq |W|.pn(G) + |R|$$

$\#$ occupied vertices by the minimum $\#$ agents

Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS

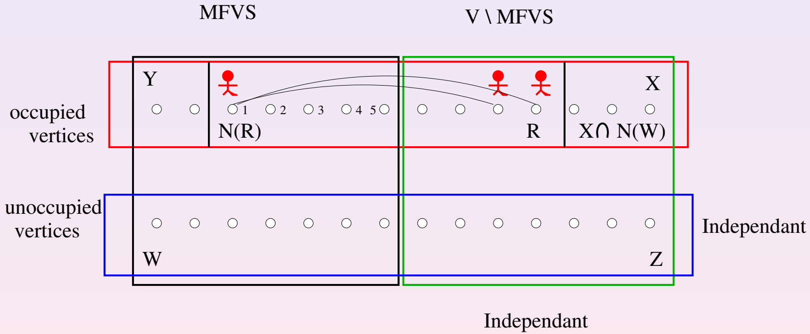


$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W|.pn(G)+|R|}{|Y|+|W|}$$

$N(R) = \{v_1, \cdots, v_r\} \subseteq Y$: ordering in which agents are removed

occupied vertices by the minimum # agents

Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS

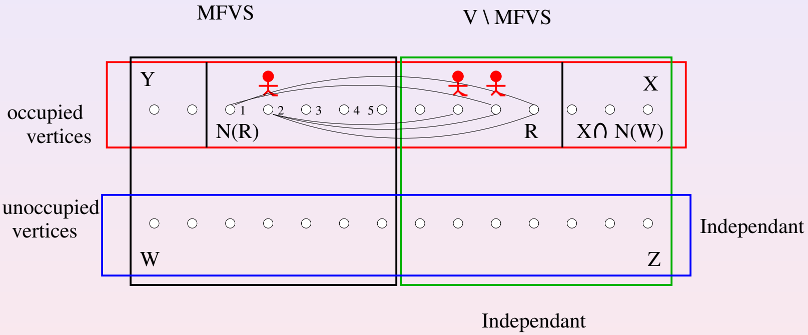


$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W|.pn(G)+|R|}{|Y|+|W|}$$

$$|N(v_1)| \leq pn(G) - 1$$

occupied vertices by the minimum # agents

Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS

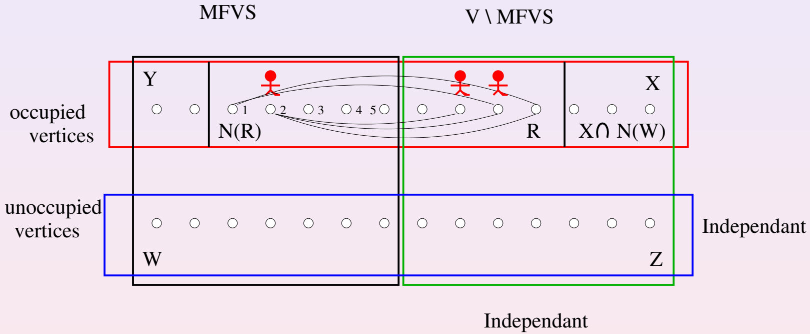


$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W|.pn(G)+|R|}{|Y|+|W|}$$

$|N(v_2) \setminus N(v_1)| \leq pn(G) - 1$, $|N(v_i) \setminus \bigcup_{j<i} N(v_j)| \leq pn(G) - 1$

occupied vertices by the minimum # agents

Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS

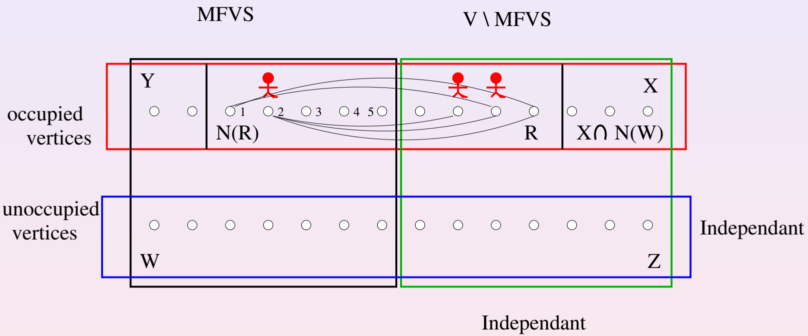


$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W|.pn(G)+|R|}{|Y|+|W|}$$

so $|R| \leq |N(R)|(pn(G)-1) \leq |Y|(pn(G)-1)$

occupied vertices by the minimum # agents

Consider a MFVS of G. S using $pn(G)$ agents and occupying $mfvs_{pn}(G)$ vertices, such that occupies the minimum number of vertices in MFVS

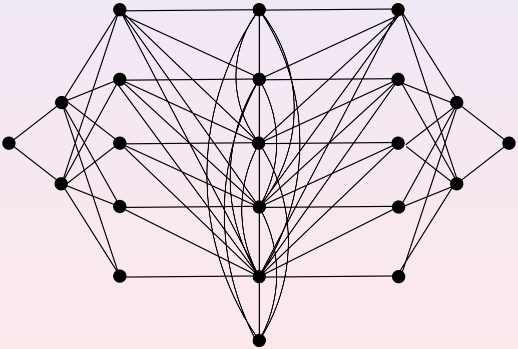


$$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} = \frac{|Y|+|X|}{|Y|+|W|} \leq \frac{|Y|+|W|.pn(G)+|R|}{|Y|+|W|}$$

$$\lambda \leq \frac{|Y|+|W|.pn(G)+|Y|(pn(G)-1)}{|Y|+|W|} = pn(G)$$

occupied vertices by the minimum # agents

$\forall \epsilon$, $\exists$ symmetric digraphs D: $\lambda = \frac{mfvs_{pn}(D)}{mfvs(D)} > 3 - \epsilon$.



$\#$ occupied vertices by the minimum $\#$ agents

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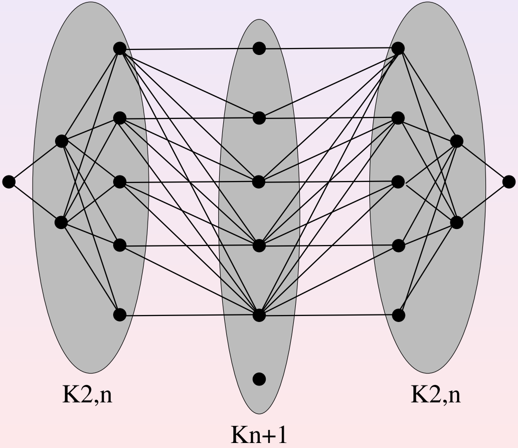

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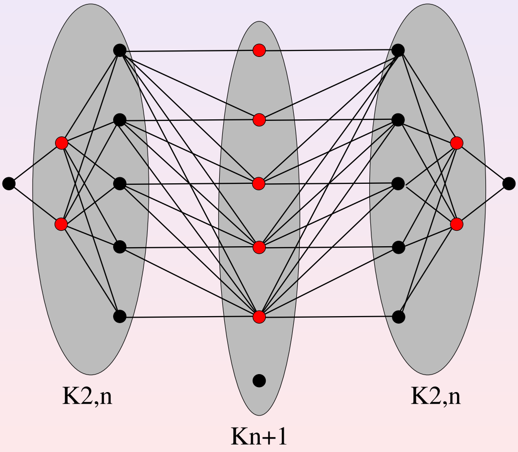

$mfvs(D) = n + 4$

$\#$ occupied vertices by the minimum $\#$ agents

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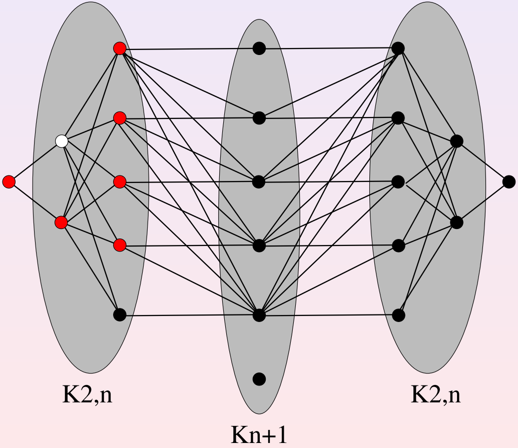

$mfvs(D) = n + 4$

$pn(D) = n + 1$

occupied vertices by the minimum # agents

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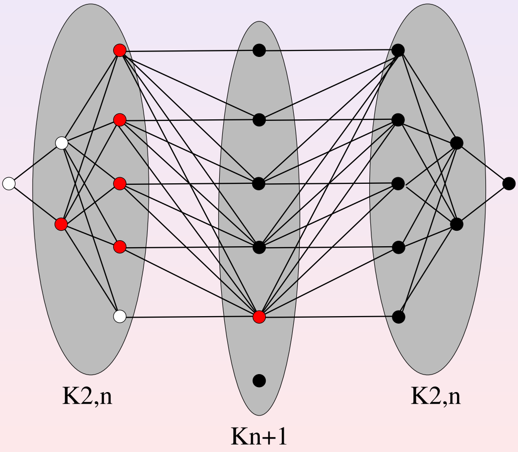

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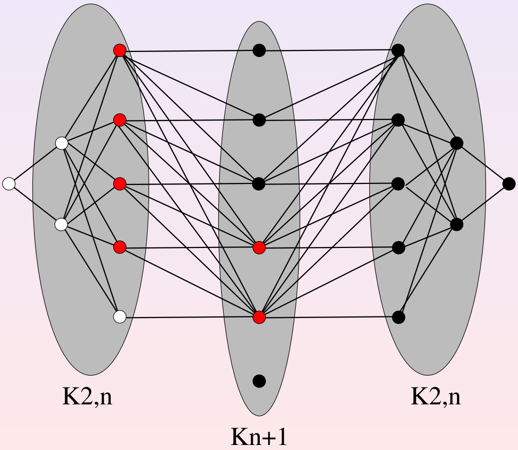

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occupied vertices by the minimum # agents

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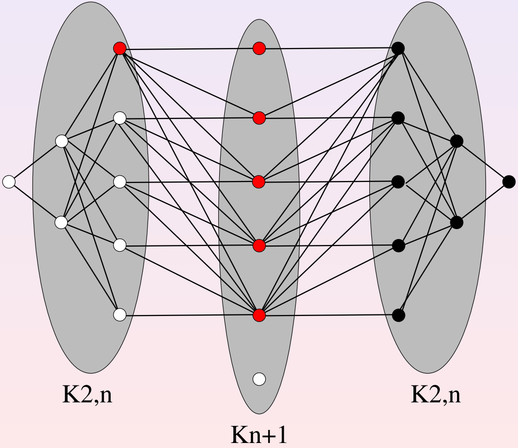

$mfvs(D) = n + 4$

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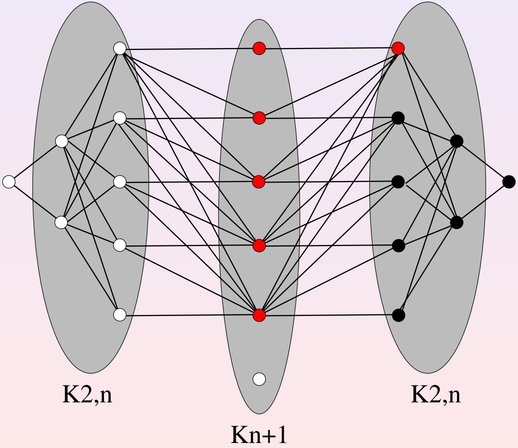
$\#$ occupied vertices by the minimum $\#$ agents

$$\forall \epsilon,\ \exists \text{ symmetric digraphs } D\text{: } \lambda = \frac{mfvs_{pn}(D)}{mfvs(D)} > 3 - \epsilon.$$



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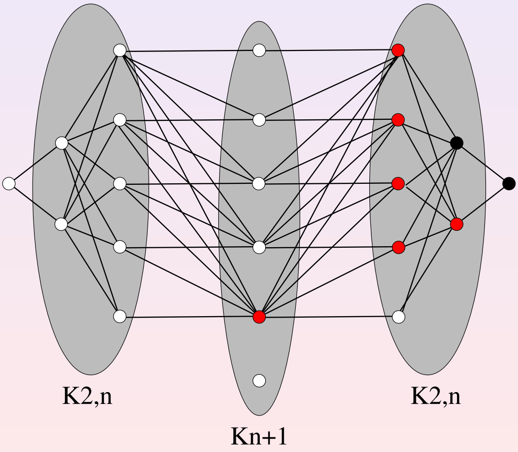
occupied vertices by the minimum # agents

$\forall \epsilon$, $\exists$ symmetric digraphs D: $\lambda = \frac{mfvs_{pn}(D)}{mfvs(D)} > 3 - \epsilon$.



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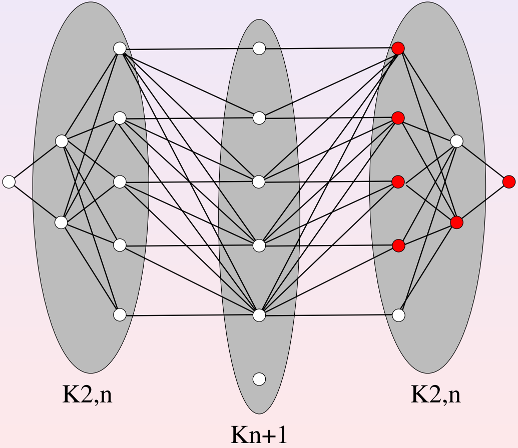

$mfvs(D) = n + 4$

$pn(D) = n + 1$

$mfvs_{pn}(D) = 3n + 2$

Some open questions

A lot of "bad" news... No tradeoff ?

Can we restrict the class of dependancy digraphs ?

No... even if the physical network is a directed path...

Conjecture

Let G be a symmetric directed/undirected graph,
$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} \leq 3$

- Approximation and Heuristic algorithms for these parameters

Link between random walks and separators of graphs ?

Some open questions

A lot of "bad" news... No tradeoff ?

Can we restrict the class of dependancy digraphs ?

No... even if the physical network is a directed path...

Conjecture

Let G be a symmetric directed/undirected graph,
$\lambda = \frac{mfvs_{pn}(G)}{mfvs(G)} \leq 3$

- Approximation and Heuristic algorithms for these parameters

Link between random walks and separators of graphs ?

Merci