On Rerouting Connection Requests in Networks with Shared Bandwidth

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Warwick, March 24th 2009

Routing in WDM Networks

Physical Network, Links provide several wavelengths

multi-graph $G = (V, E)$

an edge $(u, v) \Leftrightarrow$ one wavelength on the link (u, v)

Routing of a set of requests/connections

set of requests $\mathcal{R} \subseteq 2^{V \times V}$

routing: for each request (u, v),

a path from u to v and 1 wavelength.

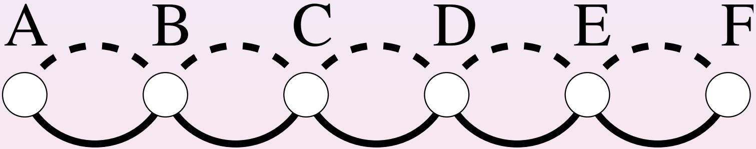
A wavelength on a link used by AT MOST 1 request

Problem: due to dynamicity of traffic, failures

how to maintain an efficient routing?

Basic Example

Network = Path with two wavelengths per link
(2 parallel edges).



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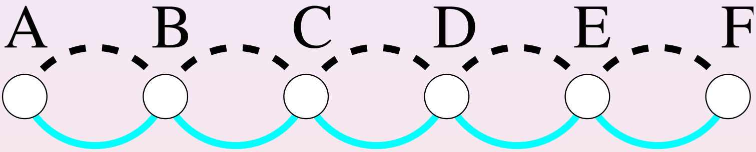

request for a A-F connection

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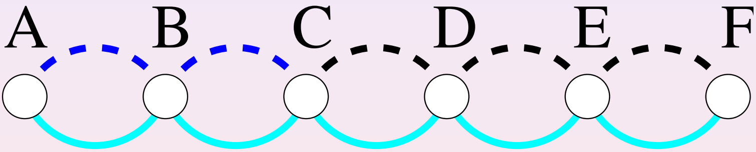

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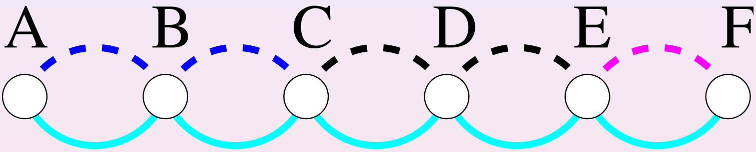

request for a E-F connection

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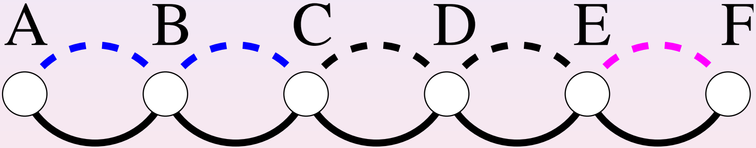

end of the A-F connection

Basic Example

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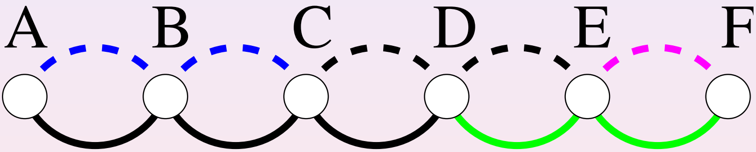

request for a D-F connection

Basic Example

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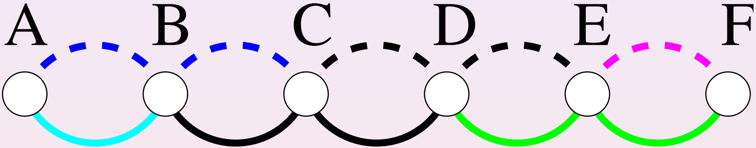

request for a A-B connection

Basic Example

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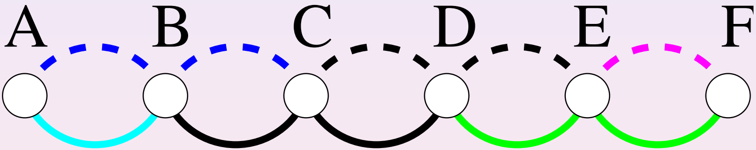

What if there is a request for a B-E connection?
(using ONE wavelength) Impossible with the current routing...

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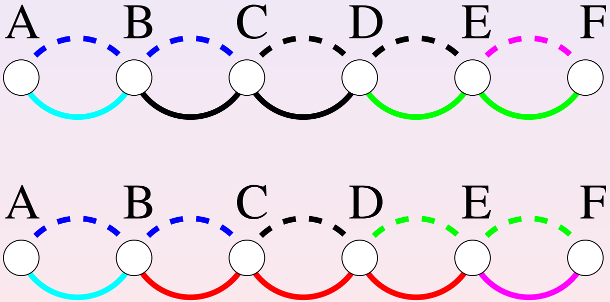

... While it is possible !!

What can we do ?

- Reject the new request $\rightarrow$ *blocking probabilities*
- Stop all requests and restart with new "optimal" routing
- Sequence of switching to converge to new routing
- Find the most suitable route for incoming request with eventual rerouting of pre-established connections

Our problem:

Inputs: Set of connection requests
+ current **and** new routing

Output: Scheduling for switching connection requests from current to new routes

Constraint: A connection is switched only once

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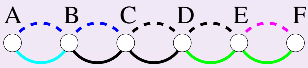
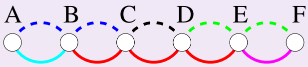
Tool: the Dependency Digraph (Jose & Somani, DRCN'03)

initial routing $\mathcal{I}$ + request BE



final routing $\mathcal{F}$ (pre-computed)

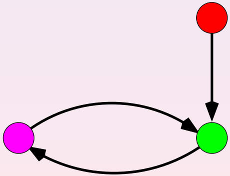


Dependancy Digraph

- one vertex per connection with different routes in I and F
- arc from u to v if ressources needed by u in $\mathcal{F}$ are used by v in $\mathcal{I}$

If cycles exist $\Rightarrow$ cyclic dependencies $\Rightarrow$

some requests must be interrupted



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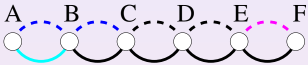

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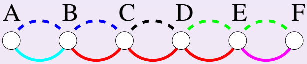

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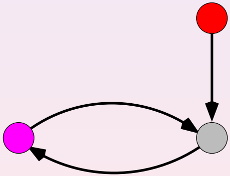

interrupt DF-connection

(Break-before-Make)

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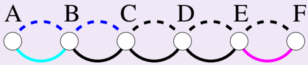

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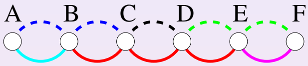

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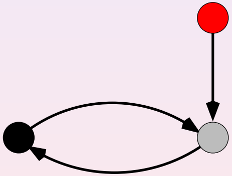

reroute EF-connection

(Make-before-Break)

Tool: the Dependency Digraph (Jose & Somani, DRCN'03)

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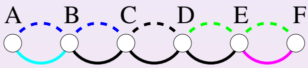

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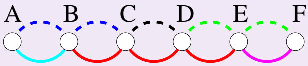

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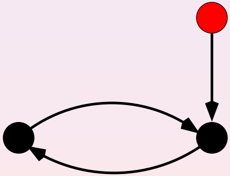

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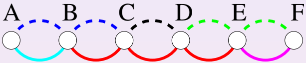

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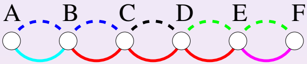

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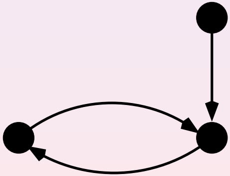
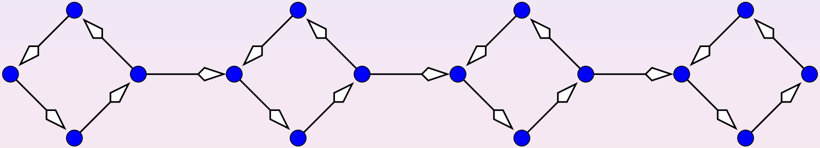


route BE-connection

Possible objectives

Minimize overall number of interrupted requests

Minimum Feedback Vertex Set (MFVS), here $N/4$



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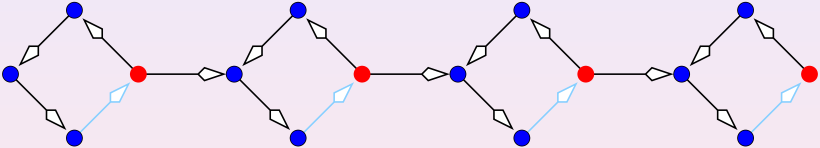

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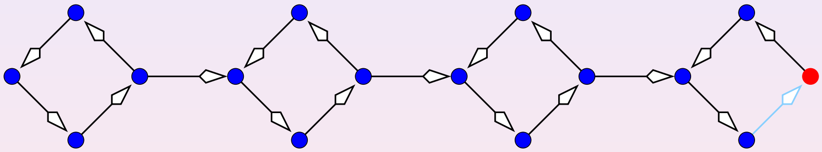

Minimize number of simultaneous interrupted requests

Process Number, pn = smallest number of requests that have to be **simultaneously** interrupted.
Here, $pn = 1 \Rightarrow$ Gap with MFVS up to $N/2$

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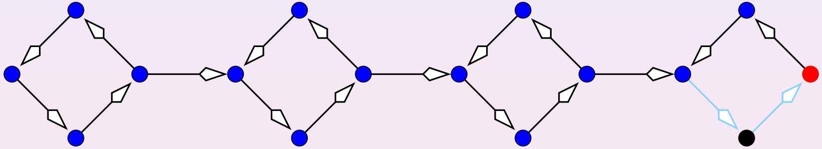

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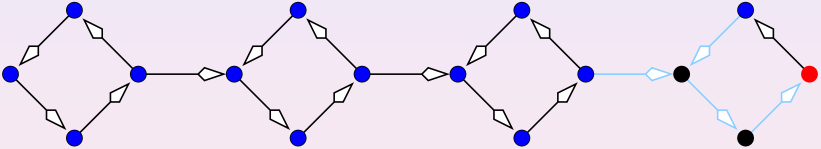

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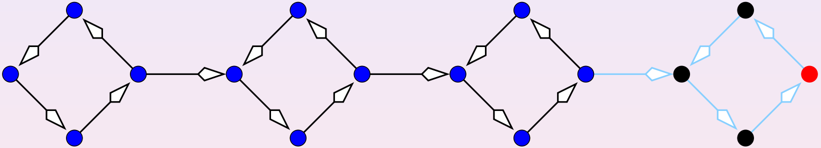

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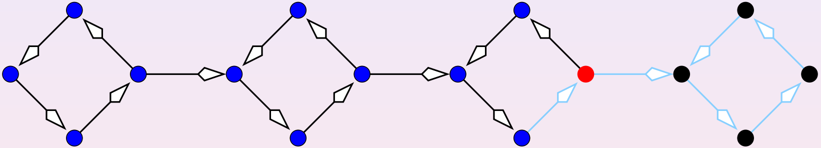

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Reconfiguration and Process number (Coudert, Sereni)

Game with Agents on the Dependency digraph D

Sequence of three basic operations,...

1. Place a agent at a node = **interrupt the request**;
2. Process a node if all its out-neighbors are either processed or occupied by an agent = **(Re)route a connection when final resources are available**;

   A processed node is removed from the dependency digraph.
3. Remove an agent from a node, after having processed it.

...that must result in processing all nodes

Process number $pn(D) = \min p \mid D$ can be processed with p agents

Remark: In undirected graphs or symmetric digraphs:

Graph Searching game when a fugitive is captured when surrounded

Example: DAG

Only one operation is used

1. Place a agent at a node = interrupt the request;
2. Process a node if all its out-neighbors are either processed or occupied by an agent = **(Re)route a connection when final resources are available**;
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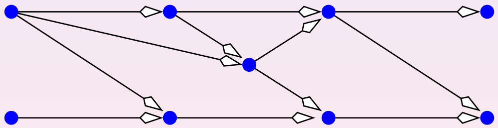
DAG



Theorem

$pn(D) = 0$ iff D is a DAG

Digraphs with process number 1

One agent is used

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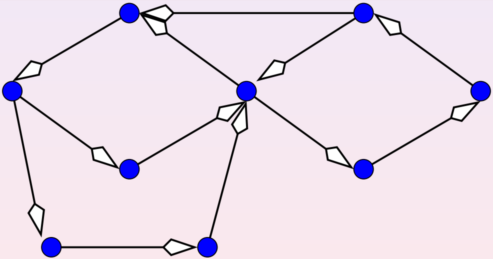

Theorem

$pn(D) = 1 \Leftrightarrow \forall SCC,\ \mathrm{MFVS}(SCC) = 1$ $\qquad O(N + M)$

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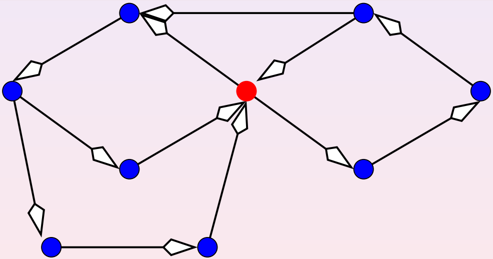

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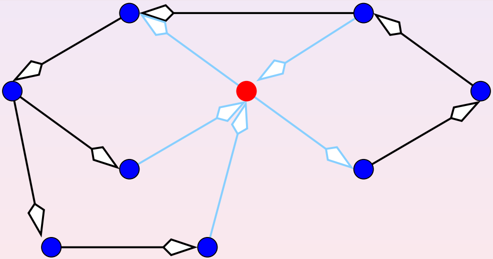

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Process number versus Other Parameters

a parameter of directed (and undirected) graphs

vs, vertex separation

in undirected graph or symetric digraph: vs = pathwidth

$vs(G) = pw(G)$ **Kinnersley [IPL 92]**

Theorem (Coudert & Sereni, 2007)

$vs(D) \leq pn(D) \leq vs(D) + 1$

Complexity: NP-Complete, Not APX

- Characterization of digraphs with process number 0, 1, 2 **(Coudert & Sereni, 2007)**

State of the Art

- distributed $O(n \log n)$-time exact algorithm in trees
  (Coudert, Huc, Mazauric [DISC 08])
- generalized Model handling priority connections

  connections that cannot be interrupted

  heuristic

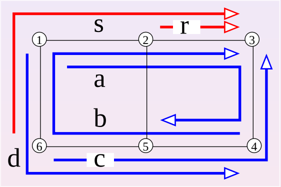
  (Coudert, Huc, Mazauric, Nisse, Sereni [ONDM 09])

When connections can share Bandwidth

Now: 1 wavelength on a link can be shared by several requests

More freedom, but Reconfiguration becomes more difficult

Example: Symmetric grid, one wavelength per link can be shared by 2 requests.



Routing 1,
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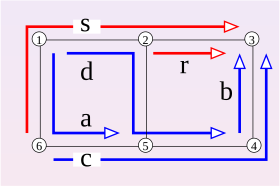


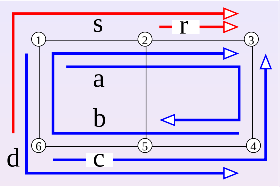
Routing 2

Theorem

NP-complete to decide whether the reconfiguration can be done without interruptions.
This is true even if capacities of wavelength are at most 3.

Recall that if capacities equal 1, this problem is equivalent to recognize a DAG

Ideas of Proof: generalized Dependency Digraph



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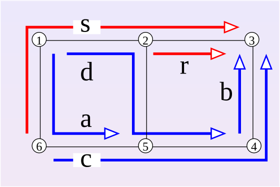

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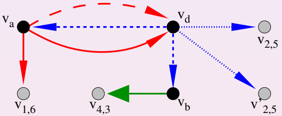
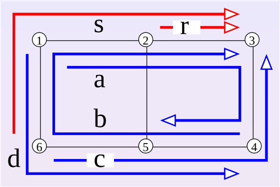


Dependency Digraph (a color ⇔ a network's link)

generalized Dependancy Digraph: multi digraph with labeled edges

- one vertex per request with different routes in I and F +
  ∀ network's link e, 1 *virtual* vertex per free unit of capacity (in Routing 1) on e
- for any network's link e, arc (u, v) labeled with e
  from request u to vertex v if e is used by u in $\mathcal{F}$ and
  either v is a request using e in $\mathcal{I}$, or v is a virtual vertex corresponding to e

Ideas of Proof: generalized Dependency Digraph



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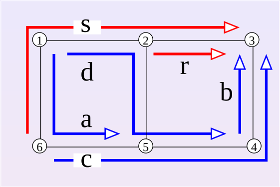

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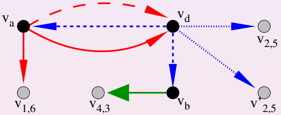

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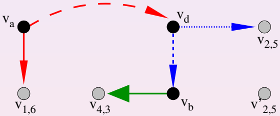


Possible reconfiguration

Remarks

- 1 color of the dependency digraph ⇒ directed complete bipartite graph
- possible reconfiguration ⇒
  - **1:** maximum matching for any color ⇒ "classical" Dependency Digraph
  - **2:** compute the process number of the obtain Dependency Digraph

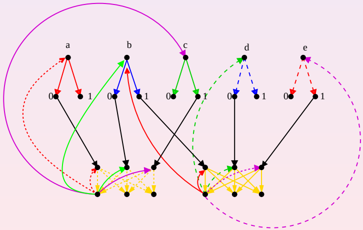
Ideas of Proof: NP-Hardness

Problem: Is there a possible reconfiguration without interrupting requests?

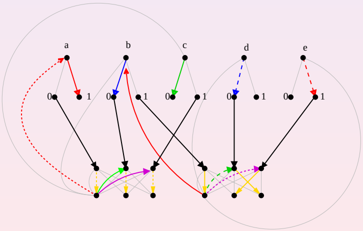
From previous remarks:

⇔ Find a set of maximal matchings (1 per color) s.t. the obtained digraph is a DAG

Reduction of 3-SAT



Reduction of Formula $(a \vee b \vee \neg c) \wedge (\neg b \vee d \vee \neg e)$



$\exists$ set of matchings inducing DAG ⇔ Formula satisfiable

Further work

Lot of questions remain:

- Heuristics
- Distributed algorithms
- Realistic scenarios
- Complexity when ≤ 2 requests can share a link?
- Other objectives, Tradeoffs:
  simultaneous interruptions / interruption time / overall time of reconfiguration
- ...