Fast Data Gathering in Radio Grid Networks

J-C. Bermond and N. Nisse and P. Reyes and H. Rivano

Projet MASCOTTE, INRIA-I3S(CNRS/UNSA), *Sophia-Antipolis, France.*[†]

Nous présentons des algorithmes efficaces pour la collecte d'informations par une station de base au sein d'un réseau sans-fil multi sauts en présence d'interférences. Nous nous focalisons sur les réseaux en grille car ils sont un bon modèle des réseaux d'accès comme des réseaux aléatoires de capteurs. Le temps est divisé en étapes élémentaires. Au cours d'une étape, un nœud peut transmettre au plus un message à l'un de ces voisins. Chaque appareil est équipé d'un interface half duplex et ne peut donc émettre et recevoir à la même étape. Ainsi, au cours d'une étape, l'ensemble des transmissions valides induit un couplage de la grille. Le problème consiste à minimiser le nombre d'étapes nécessaires à la collecte de tous les messages par la station de base. Le meilleur algorithme connu était une $3/2$ approximation. Nous donnons un algorithme très simple qui approche l'optimum à 2 près, puis nous présentons un algorithme plus évolué qui est une $+1$ approximation. Nos résultats sont valides lorsque les appareils ne disposent d'aucune mémoire tampon et doivent retransmettre un message à l'étape suivant sa réception.

Keywords: Sensor Networks, gathering, makespan, grid

1 Introduction

We address here the challenging problem of gathering information in a Base Station (denoted BS) of a wireless multi hop grid network when interferences constraints are present. This problem is also known as data collection and is particularly important in sensor networks, but also in access networks. The communication network is modeled by a graph. Here we consider grid topologies as they model well both access networks and also random networks (which approximatively behave like if the nodes were on a grid [KLNP05]). We suppose the time is slotted and that during one time slot, or *step*, each node can transmit to one of its neighbors at most one data item (referred in what follows as a message). Each vertex of the grid may have any number of messages to transmit : zero if it is not concerned (sleeping station or no sensor at this node or failed device) one or many. We also suppose that each device (sensor, station,...) is equipped with an half duplex interface; so a node cannot both receive and transmit during a step. In particular, this is the case in a mono-frequency smart antennas radio system: at any step, each device can configure its antenna array to shape a beam to reach any of its neighbours, but sending a message would prevent it from receiving because, among other causes, of near-far effects. So we refer to this model as the *smart-antennas model*. During any step only non interfering transmissions can be done, thus the non interfering calls done during a step will form a matching (set of independent edges). Our aim is to design algorithms to do a gathering under such hypotheses, which minimize the minimum number of steps needed to send all messages to BS, a.k.a. *makespan* or *completion time*.

Related Work. In [FFM04], the smart antennas model is considered with the extra constraint that non buffering is allowed in intermediary node. That is, when a node receives a message at some step, it must transmit it during the next step. In this setting, optimal polynomial-time algorithms are presented for path and tree topologies [FFM04]. Their work has been extended to general graphs in [GR06] but in the uniform case where each node has exactly one message to transmit. The case of grids is considered in [RS07] where a 1.5-approximation algorithm is presented. When nodes can both emit and receive a message during the same step, the problem has also been studied when no buffering is allowed. This problem is known as the hot-potato routing problem.

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The case of omnidirectional antennas has been extensively studied (see [BKK$^+$09]). In this model, nodes can transmit to any of their neighbours. But, when a node v transmits, any node at distance at most $d_I \geq 0$ of v cannot receive during the same step. In [BGK$^+$06] no buffering is allowed and each node has at least one message to transmit; in this setting it is shown that computing the makespan is NP-hard and 4-approximation algorithm is provided. In [BP05], an optimal polynomial-time algorithm is provided. Continuous models ([GPRR08]) and online cases ([BKSS08]) have also been considered.

Our results. We deal with the gathering problem in grids. We propose a very simple algorithm that achieves makespan plus two, and a more involved $+1$ approximation algorithm. Our algorithms need no buffering and considerably improve existing algorithms. Furthermore, following our algorithms a message arrives at most one step (or two steps) after what will happen if we have no interferences (provided that one node can receive only one message per step). So the average time is also good. We present the results for the smart antennas model and when BS stands at some corner of the grid, but they can be extended to any binary distance-based interference model and to any position of BS.

One helpful idea is to actually study the related one–to-many personalized broadcast problem in which the BS wants to communicate different data items to some other nodes in the network. Solving the above dissemination problem is equivalent to solve data gathering in sensor networks. Indeed, let T denote the makespan (delay), that is, the largest step used by a personalized broadcast algorithm; a gathering schedule with delay T consists in scheduling a transmission from node y to x during slot t iff the broadcasting algorithm schedules a transmission from node x to y during slot $T-t+1$, for any t with $1 \leq t \leq T$.

2 Preliminaries

2.1. Notations. In the following, we consider the personalized broadcasting in a $n \times n$ grid $G=(V,E)$. The base station BS has coordinates $(0,0)$, and any vertex v has coordinates (x_v, y_v). We consider a set of $M \geq 0$ messages $\mathcal{M}$ that must be sent from the source BS to some destination nodes. Let $dest(m) \in V$ denote the destination of $m \in \mathcal{M}$. A message $m \in \mathcal{M}$ is *lower* (resp., *higher*) than $m' \in \mathcal{M}$ if $dest(m)$ is below (resp., above) $dest(m')$; m is *righter* (resp., *lefter*) than m', if $dest(m)$ is to the right (resp., to the left) of $dest(m')$. We use $d(m)$ to denote the distance between $dest(m)$ and BS We suppose in what follows that the messages are ordered by non increasing distance of their destination nodes, and we note the ordered sequence $O=(m_1,\cdots,m_M)$ that is $d(m_1) \geq d(m_2) \geq \cdots \geq d(m_M)$. $S \odot S'$ denotes the sequence obtained by concatenation of two sequences S and S'.

2.2. Lower bound. Consider a model where nodes may transmit and receive simultaneously, but where the source can only send one message per step. Whatever be the broascasting scheme, a message m sent at step $t \geq 1$ will be received at step $t' \geq d(m)+t-1$. A scheme is *greedy* if, given an ordered sequence $\mathcal{S}$ of the messages, the source sends one message per step, in the ordering $\mathcal{S}$, and each message follows a shortest path toward its destination node. In the smart antennas model, if the messages follow shortest paths, a vertex will never receive more than one message per step.

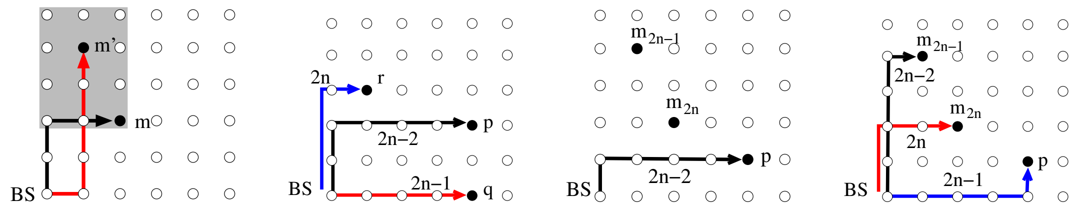
Lemma. If nodes can emit (to a single node) and receive simultaneously, a greedy algorithm following the ordered sequence of messages $O=(m_1,m_2,\cdots,m_M)$ is optimal, with makespan $LB = \max_{i \leq M} d(m_i)+i-1$.

Proof. Clearly, sending the messages in the ordering of the sequence O along shortest paths achieves LB. Let $(s^*_i)_{i \leq M}$ be an optimal schedule of the messages and let $i \geq 1$ be the smallest integer such that $s^*_i \neq m_i = s^*_j$ ($j > i$). Sending the messages in the ordering of the sequence $(s^*_1,\cdots,s^*_{i-1},s^*_j,s^*_{i+1},\cdots,s^*_{j-1},s^*_i,s^*_{j+1},\cdots,s^*_m)$ does not increase the makespan: only the i^{th} and j^{th} messages differ and $\max\{d(s^*_j)+i-1, d(s^*_i)+j-1)\} \leq d(s^*_j)+j-1)$ ($d(s^*_j) \geq d(s^*_i)$) and $j > i$. Iterating the process, we get that the ordering of the sequence O is optimal. □

Hence, in the smart antennas model (more constraint), no algorithm can achieve a makespan less than LB. Moreover, there exist configurations for which no gathering protocol can achieve better makespan than $LB+1$. It is the case when there are 3 destinations a, b and c with coordinates $(1,0),(1,1)$ and $(1,2)$.

3 Algorithms

Given a message whose destination node v has coordinates (x,y), the message is sent *horizontally* to v if it follows the shortest path from BS to v passing through $(x,0)$. The message is sent *vertically* if it follows



(a) Configuration when consecutive messages interfer.

(b) $\{q,r\} = \{m_{2n-1}, m_{2n}\}$ and q is strictly lower than r.

(c) Both m_{2n-1} and m_{2n} are higher than p.

(d) New scheduling in Case of Figure 1(c)

Fig. 1: $2n-2$ messages have been scheduled, finishing with the one to $x \in \{m_{2n-2}, m_{2n-3}\}$. When the next two messages must be scheduled, two cases occur according to the position of m_{2n-1} and m_{2n} relatively to x. In the figures, an arrow with label i represents the route of the i^{th} message.

Algorithm OneApprox
Input: $\mathcal{M} = \{m_1, \cdots, m_M\}$, the set of messages ordered in non increasing distance order
Output: $(s_1, \cdots, s_M)$ an ordered sequence of $\mathcal{M}$ such that $m_i \in \{s_{i-1}, s_i, s_{i+1}\}$ for any $i \leq M$
Case $M = 0$ **return** $\emptyset$ **Case** $M = 1$ **return** (m_1)
Case $M \geq 2$ Let $O \odot p = OneApprox(\{m_1, \cdots, m_{M-2}\})$
Let q be the lowest message in $\{m_{M-1}, m_M\}$ and let r be the other one
1) **if** p is higher than q **return** $O \odot (p, q, r)$ **else if** $p = m_{M-2}$ **return** $O \odot (m_{M-1}, p, m_M)$
2) **else** Let $(s_1, \cdots, s_{M-4}) \odot (m_{M-2}, m_{M-3}) = OneApprox(\{m_1, \cdots, m_{M-2}\})$
3) **return** $MakeValid((s_1, \cdots, s_{M-4}) \odot (m_{M-3}, m_{M-1}, m_{M-2}, m_M), 2)$

Algorithm MakeValid
Input: An integer j, $1 < j \leq \lfloor M/2 \rfloor$, and a sequence $O = (s_1, \cdots, s_M)$ of $\mathcal{M}$
Output: An ordered sequence of $\mathcal{M}$
if s_{M-2j} and s_{M-2j+1} do not interfer **return** O
else if $s_{M-2j} = m_{M-2j}$ **return** $(s_1, \cdots, s_{M-2j-2}) \odot (s_{M-2j-1}, s_{M-2j+1}, s_{M-2j}, s_{M-2j+2}) \odot (s_{M-2j+3}, \cdots, s_M)$
else return $MakeValid((s_1, \cdots, s_{M-2j-2}) \odot (s_{M-2j}, s_{M-2j+1}, s_{M-2j-1}, s_{M-2j+2}) \odot (s_{M-2j+3}, \cdots, s_M), j+1)$

the shortest path from BS to v passing through $(0, y)$. A *Horizontal-Vertical broadcasting scheme*, or *HV-scheme*, takes an ordering $\mathcal{S}$ of $\mathcal{M}$ as an input and proceeds as follows. A direction, horizontal or vertical, is chosen for the first message. Then, the source sends one message every step in the ordering $\mathcal{S}$ alternating horizontal and vertical messages. Let us do some easy remarks about any HV-scheme. Consider two distinct messages sent by the source x time-slots apart. Since these messages follow shortest paths, while the first message has not reached its destination, both messages are separated by a distance at least x. Hence,

Claim 1. In a HV-scheme, only consecutive messages may interfer.

Let us characterize forbidden and acceptable configurations in HV-scheme. Assume that two messages are sent consecutively. It is possible to guess the respective positions of their destination nodes by knowing whether both messages interfer or not. In Figure 1(a), nodes in the grey part are the nodes that are higher and lefter than y. Figure 1(a) illustrates the following Claim.

Claim 2. Let m, m' be two messages sent consecutivelly by a HV-scheme, with m sent vertically and m' sent horizontally. m and m' interfer iff their destinations are distinct and m' is higher and lefter than m.

In what follows, we present algorithms for computing an efficient ordering of $\mathcal{M}$ to be used by HV-schemes. Our main result is the algorithm $OneApprox$, for computing an ordering of $\mathcal{M}$ with the following two properties: (1) HV-scheme($\mathcal{S}$) broadcasts the messages without collisions, **sending the last message vertically**, and (2) $m_i \in \{s_{i-1}, s_i, s_{i+1}\}$ for any $i \leq M$. Both properties implies:

Theorem 1. $OneApprox$ computes an ordering $\mathcal{S}$ of the messages, s.t. HV-scheme($\mathcal{S}$) achieves $LB + 1$.

Because of space restriction, the proof of Theorem 1 is omitted and can be found in [BNRR09]. We only prove the correctness of a simpler algorithm *TwoApprox* computing an ordering of $\mathcal{M}$ such that the second above property is replaced by: (2') $m_i \in \{s_{i-2}, s_{i-1}, s_i, s_{i+1}, s_{i+2}\}$ for any $i \leq M$, and $s_M \in \{m_{M-1}, m_M\}$. Algorithm *TwoApprox* is obtained by replacing in Algorithm *OneApprox* the three lines 1,2,3) by the following instruction:

if p is higher than q **return** $O \odot (p,q,r)$ **else return** $O \odot (m_{M-1}, p, m_M)$

Theorem 2. *TwoApprox* computes an ordering $\mathcal{S}$ of the messages, s.t. HV-scheme($\mathcal{S}$) achieves $LB+2$.
Proof. We proceed by induction on M. If $M \leq 2$, the result holds obviously. Let us assume that the ordering of the sequence computed by *TwoApprox*($\{m_1, \cdots, m_{M-2}\}$) satisfies the two properties. Let p be the last message of this sequence. By the induction hypothesis, $p \in \{m_{M-3}, m_{M-2}\}$ is sent vertically. Let t be the next to last message in this sequence, if any. By Claim 2,p must be higher or lefter than t.

Let q be the lowest message in $\{m_{M-1}, m_M\}$ and let r be the other one. If p is higher than q, it is sufficient to send q horizontally at step $M-1$, and r vertically at step M. This case is depicted in Figure 1(b). Indeed, by Claim 1 only p and q, or q and r may interfer. By Claim 2, there are no interferences. It is easy to check that $O \odot (p,q,r)$ satisfies the properties. Otherwise, q and r are higher than p. Moreover, since q,r are nearer than p, they are higher and lefter than p. This case is depicted in Figure 1(c). In this case, instead of sending p at step $M-2$, the source sends m_{M-1} vertically at step $M-2$, then p horizontally at step $M-1$, and then m_M vertically at step M. The transformation is depicted in Figure 1(d). Clearly, $O \odot (m_{M-1}, p, m_M)$ satisfies the properties. By Claim 1 only t and m_{M-1}, or m_{M-1} and p, or p and m_M may interfer. Since m_{M-1} is higher and lefter than p that is higher or lefter than t, by Claim 2, m_{M-1} interfers neither with t nor with p. Similarly, m_M is higher and lefter than p and these messages do not interfer. □

Theorem 3. The time complexity of both algorithms is $O(M)$ (see [BNRR09]).

4 Future works

In this paper, we have presented centralized algorithm for the minimum makespan personalized broadcasting in grid networks. In these settings, the problem is strictly equivalent to the data gathering problem. In [BNRR09], we have developed a distributed version of the algorithms. One can note that our network model assumes that an optimal MAC layer is available. It would be interesting to investigate on the behavior of the problems under weaker assumptions. Another direction to investigate is the online version of the problems. It is worth pointing out that, in this case, personalized broadcasting and gathering are no longer equivalent.

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