

Exercises : Duality

1 Definitions and Optimality Certificates

Exercise 1 Write the dual of the following LP :

$$\begin{aligned} & \text{Maximize} && 7x_1 + x_2 \\ & \text{Subject to :} && \\ & && 4x_1 + 3x_2 \leq 3 \\ & && x_1 - 2x_2 \leq 4 \\ & && 5x_1 + 2x_2 \leq 3 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

Exercise 2 Consider the following linear program :

$$\begin{aligned} & \text{Minimize} && -2x_1 - 3x_2 - 2x_3 - 3x_4 \\ & \text{Subject to :} && \\ & && -2x_1 - x_2 - 3x_3 - 2x_4 \geq -8 \\ & && 3x_1 + 2x_2 + 2x_3 + x_4 \leq 7 \\ & && x_1, x_2, x_3, x_4 \geq 0 \end{aligned} \tag{1}$$

- Write the program (1) under the standard form.
- Write the dual (D) of program (1).
- Give a graphical solution of the dual program (D).
- Carry on the first iteration of the simplex on the program (1).
After three iterations, one find that the optimal solution of this program is $x_1 = 0$, $x_2 = 2$, $x_3 = 0$ et $x_4 = 3$.
- Verify that the solution of (D) obtained at Question c) is optimal.

Exercise 3 [Optimality certificates]

We consider the following linear program :

$$\begin{aligned} & \text{Maximize} && x_1 - 3x_2 + 3x_3 \\ & \text{Subject to :} && \\ & && 2x_1 - x_2 + x_3 \leq 4 \\ & && -4x_1 + 3x_2 \leq 2 \\ & && 3x_1 - 2x_2 - x_3 \leq 5 \\ & && x_1, x_2, x_3 \geq 0 \end{aligned}$$

If the solution $x_1^* = 0$, $x_2^* = 0$, $x_3^* = 4$ optimal?

Exercise 4 [Optimality certificates] We consider the following linear program.

$$\begin{array}{rllllll} \text{Maximize} & 7x_1 & + & 6x_2 & + & 5x_3 & - & 2x_4 & + & 3x_5 \\ \text{Subject to :} & & & & & & & & & \\ & x_1 & + & 3x_2 & + & 5x_3 & - & 2x_4 & + & 2x_5 & \leq & 4 \\ & 4x_1 & + & 2x_2 & - & 2x_3 & + & x_4 & + & x_5 & \leq & 3 \\ & 2x_1 & + & 4x_2 & + & 4x_3 & - & 2x_4 & + & 5x_5 & \leq & 5 \\ & 3x_1 & + & x_2 & + & 2x_3 & - & x_4 & - & 2x_5 & \leq & 1 \\ & & & & & & & x_1, x_2, x_3, x_4, x_5 & & & \geq & 0. \end{array}$$

Is the solution $x_1^* = 0, x_2^* = \frac{4}{3}, x_3^* = \frac{2}{3}, x_4^* = \frac{5}{3}, x_5^* = 0$, optimal?

Exercise 5 1. Because of the arrival of new models, a salesman wants to sell off quickly its stock composed of eight phones, four hands-free kits and nineteen prepaid cards. Thanks to a market study, he knows that he can propose an offer with a phone and two prepaid cards and that this offer will bring in a profit of seven euros. Similarly, we can prepare a box with a phone, a hands-free kit and three prepaid cards, yielding a profit of nine euros. He is assured to be able to sell any quantity of these two offers within the availability of its stock. What quantity of each offer should the salesman prepare to maximize its net profit?

2. A sales representative of a supermarket chain proposes to buy its stock (the products, not the offers). What unit prices should he negotiate for each product (phone, hands-free kits, and prepaid cards)?