Graph Theory and Optimization
Why is it useful?

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Outline

1. Combinatoric and Graph theory
2. Examples of applications
3. Objectives of this school
Combinatorics

Branch of mathematics concerning the study of finite or countable objects (existence, enumeration, structure).

Integer Linear Programming

Integer partition

Graphs
Graph Theory

Euler 1735: Koenisberg bridges.

Existe-t-il un parcours empruntant tous les ponts une fois et une seule ?

Is there a cycle going through each bridge exactly once?
Graph Theory

**Modeling:** city = **graph**, island = **vertex**, bridge = **edge**

**Question:** can we find an Eulerian cycle in this graph?

Cycle going through all edges once and only once
Graph Theory

an old story

**Modeling**: city = graph, island = vertex, bridge = edge

**Question**: can we find an **eulerian cycle** in this graph?

Cycle going through all edges once and only once
Graph Theory  
an old story

**Modeling:** city = graph, island = vertex, bridge = edge

**Question:** can we find an eulerian cycle in this graph?

**Solution:** Such cycle exists if and only if all nodes have even degree
Graph Theory

Modeling: city = graph, island = vertex, bridge = edge

Question: can we find an eulerian cycle in this graph?

Solution: Such cycle exists if and only if all nodes have even degree

An intriguing variant: find a cycle going through all vertices once and only once (Hamiltonian cycle) is very difficult

One million dollar (Clay price)!
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1st Example: roads’ network

What is the “best" road for reaching Oulu from Helsinki?
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Model geographical netowrk by a graph
1st Example: roads’ network

What is the “best" road for reaching Oulu from Helsinki?

Model geographical netowrk by a graph

Use powerful tools that deal with graphs
1st Example: roads’ network

More difficult setting

- traffic jam
- bus/subway schedule
- no-left, no-right and no U-turn signs at intersections.

Again, graph algorithm tools may help

That is how your GPS work!!
2nd Example: the Internet

Internet network (Autonomous Systems)
Optical networks (WDM)
  - node = IP routers
  - links = optical fiber
  - capacity on links

- How to compute "best" routes?
- Where to put Amplificators?
- Which links to be turned off to limit energy consumption?

...
3rd Example: Social Network

Model of social interaction
a user = a node
two friends = an edge

- structure of social networks?
- communities?
- how to do advertisement?
- how to prevent advertisement?
Combinatoric and Graph theory  Examples of applications  Objectives of this school

More Example: Web (google)

Google PageRank:
- sort search results
- node = web page
- link = hyperlink

1. finding pages with the word movies in it
2. determining the importance of a page.

<table>
<thead>
<tr>
<th>Showing search results in order of relevance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Movies.com: Everything Movies</strong></td>
</tr>
<tr>
<td>[8/10]</td>
</tr>
<tr>
<td>Movies.com: movie reviews, movie trailers, movie tickets and showtimes. Movie Night Right!</td>
</tr>
<tr>
<td><a href="http://movies.go.com/">View META Data - View Inbound Links - Analyze Links</a></td>
</tr>
<tr>
<td><a href="http://movies.go.com/">Cached Version - Similar Web Sites</a></td>
</tr>
</tbody>
</table>

| The Internet Movie Database (IMDb)            |
| [9/10]                                         |
| IMDb: The biggest, best, most award-winning movie site on the planet. |
| [View META Data - View Inbound Links - Analyze Links](http://www.imdb.com/) |
| [Cached Version - Similar Web Sites](http://www.imdb.com/) |
More Example: Web (google)

Google **PageRank:**
- sort search results
- node = web page
- link = hyperlink

1. finding pages with the word movies in it
2. build the graph of the Web
   - do a **random walk** on the graph or compute the eigenvector of a matrix
Outline

1. Combinatoric and Graph theory
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Objectives of this school

1. learn how to **model** problems (from many domains) using graphs
2. know the available tools to handle these problem
   - classical algorithms
   - Linear Programming
3. decide if a problem is "easy" or "difficult"
4. know what to do when facing a "difficult" problem
   - exact exponential algorithms
   - parameterized algorithms
   - approximation algorithms
   - heuristics