University Côte d'Azur — Graphs
Master 2 : Informatique Fondamentale — September, 2024

Short exam : 40 minutes

No electronic/manuscript document is allowed. You can answer in french or english. Most of the questions may be answered in at most one line or one drawing.

1 Planar graphs

Question 1 *Give the definition of a planar graph.*

Question 2 *For each of the following graphs, say if it is planar or not planar. Explain your answer.*

K_4, $K_{3,3}$ and G_3 are depicted in Figure 1.

1. *K_4 : the complete graph with 4 vertices (i.e., 4 vertices and all possible edges) ;*
2. *K_6 : the complete graph with 6 vertices (i.e., 6 vertices and all possible edges) ;*
3. *$K_{2,3}$: the complete bipartite graph with three vertices in one part and two vertices in the other, and all possible edges between the two parts. That is, $V = \{a, b, c\} \cup \{x, y\}$ and $E = \{uv \mid u \in \{a, b, c\}, v \in \{x, y\}\}$;*
4. *$K_{3,3}$: the complete bipartite graph with three vertices in each part, and all possible edges between the two parts. That is, $V = \{a, b, c\} \cup \{x, y, z\}$ and $E = \{uv \mid u \in \{a, b, c\}, v \in \{x, y, z\}\}$;*
5. *Let $k \geq 2$ be an integer. Let G_k be the graph defined with $V = \{v_0, v_1, v_2, \cdots, v_{5k-1}\}$ and, for every $0 \leq i < 5k$, the neighbours of v_i are $v_{i+1 \mod 5k}$, $v_{i-1 \mod 5k}$ and $v_{i+\alpha k \mod 5k}$ for every $\alpha \in \{1, 2, 3, 4\}$.*

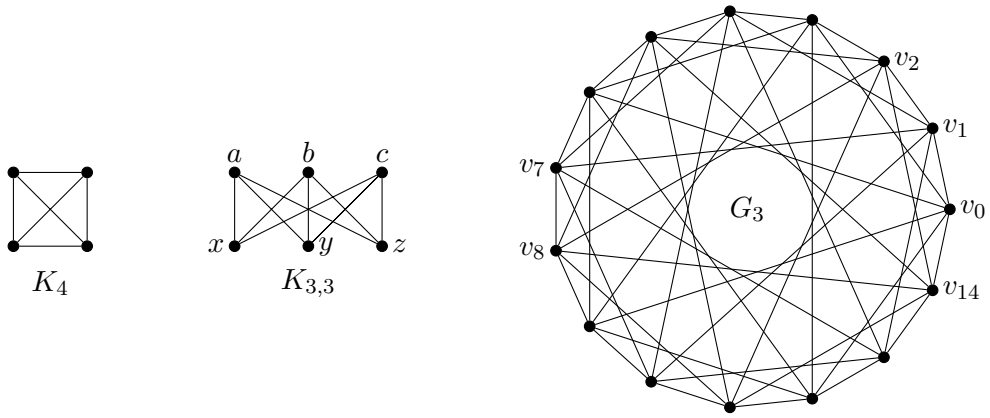

FIGURE 1 – Some graphs of question 2.

Question 3 *Give the Euler formula for a connected planar graph. Explain/Explicit all terms.*

Question 4 1. *Give the definition of a k-proper colouring of a graph $G = (V, E)$;*

2. *Give the definition of the chromatic number $\chi(G)$ of a graph G ;*
3. *State the four-colour theorem.*

A *regular n-gone* is a geometric figure (a polygone) with n sides of equal sizes and n equal angles. For instance, regular k-gones for $k = 3, 4, 5$ are respectively : equilateral triangle, square, regular pentagone.

Here, we study some tilings of surfaces by regular polygon faces. We add the **restriction** that, if two polygones intersect, either they intersect in exactly one vertex, or they share an edge.

Question 5 *Prove that the plane cannot be tiled using only regular heptagones (7-gones).*
Hint : give the value of angles in a regular 7-gone.

Question 6 *Prove that, in any tiling of a sphere using regular hexagones and regular pentagones, there are exactly 12 pentagones.*
Hint : assume you have such a tiling of the sphere with x hexagones and y pentagones (Here, you may assume that every vertex has degree 3). Express the number of edges as a function of x and y. Express the number of vertices as a function of x and y. Finally, use Euler's formula.

2 General problems in graphs

Question 7 *What is the class P ? the class NP ? Give an intuitive explanation of what means to be NP-complete.*

Question 8 *Give the definition of a matching in a graph $G = (V, E)$.*

Question 9 *Give the definition of a dominating set in a graph $G = (V, E)$.*

3 Bonus ?

Question 10 *Draw K_5 (5 vertices and all possible edges) on the torus (i.e., on a donuts) without crossing edges.*

Question 11 *Assume that you are the driver of a train (or a long truck) and want to go from A to B. Since you are driving a train, you cannot take a sharp turn. Could you find such a walk from A to B ? That is : In the picture on Figure 2, find a walk following the lines (you can use several times a same segment) from A to B without acute angles.*

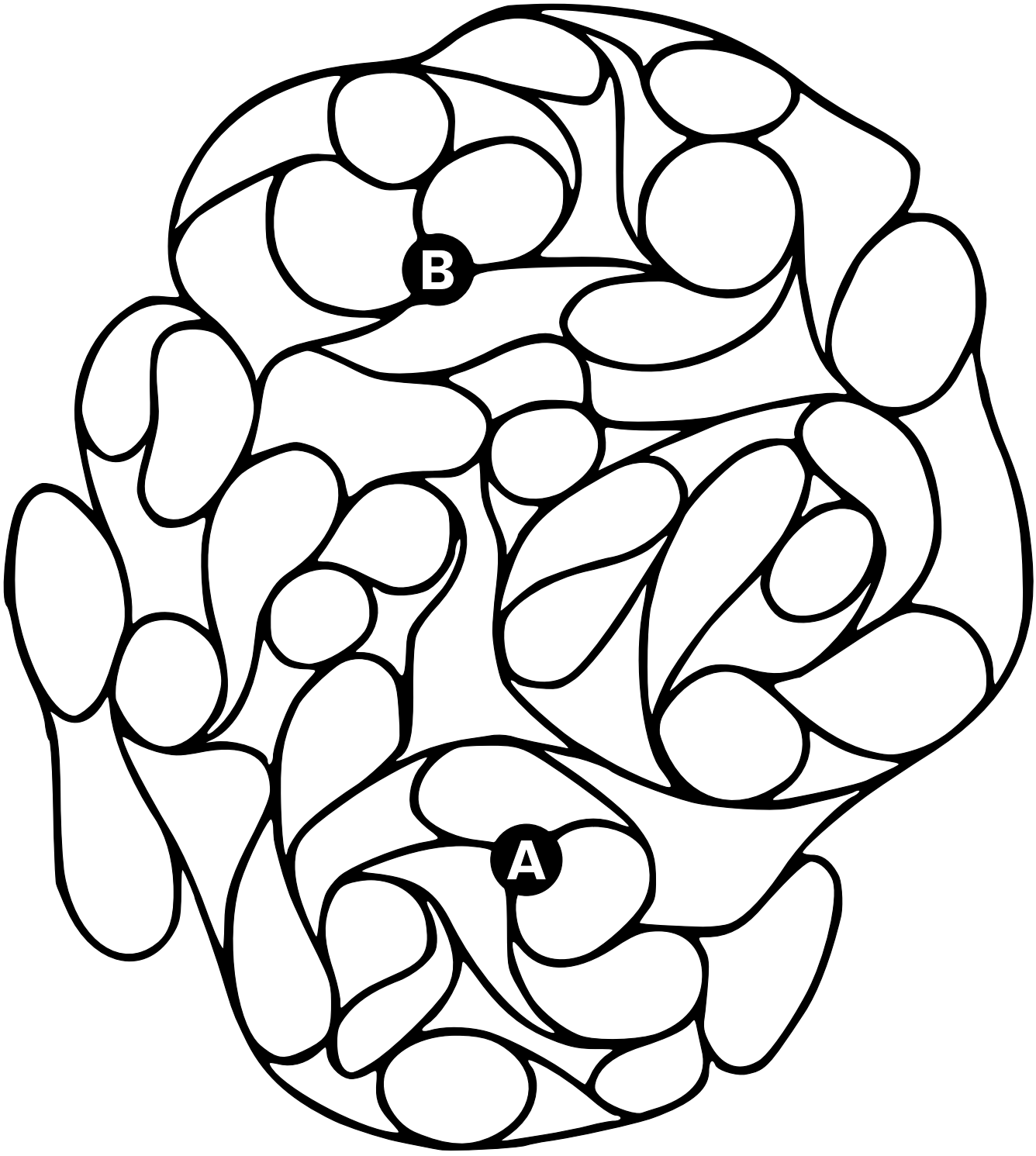

FIGURE 2 – The train track.