Part 2: Graph Algorithm

1. (1 point) Let Π be a maximization problem and let $c > 1$. What is a $\frac{1}{c}$-approximation algorithm for Π?

2. (1 point) Recall the definition of the Load-Balancing problem and explain why it may be interesting to have an approximation algorithm to solve it rather than an exact algorithm.
3. (1 point) Give a $3/2$-approximation algorithm to solve the Load-Balancing problem. (only explain it in few sentences, no proofs required.)

4. (2 points) Recall the definition of the Traveling Salesman Problem and give a 2-approximation algorithm to solve it. (only explain it in few sentences, no proofs required.)
5. (5 points) Let $G = (V,E)$ be a connected graph. Recall that the distance $dist_G(u,v)$ between two vertices $u$ and $v$ in $G$ is the length (number of edges) of a shortest path between $u$ and $v$ in $G$. The diameter $Diam(G)$ of $G$ is the maximum distance between two vertices, i.e.,

$$Diam(G) = \max_{u,v \in V} dist_G(u,v).$$

(a) (1 point) What are the diameters of each of the 3 graphs depicted in Figure 1?

(b) (1 point) Give an algorithm that computes the diameter of any graph. What is its time-complexity?

(You may assume that you are given an algorithm $A$ that computes the distance between one vertex and all other vertices in time $O(|E|)$.)
(c) (3 points) Consider the following algorithm.

**Algorithm:** Let $v_0$ be any vertex of $G$. Execute one Breadth First Search (BFS) from $v_0$ and let $\delta$ be the maximum obtained label. Return $\delta$.

*Recall that the BFS algorithm consists as follows: It starts from $v_0$ which is labelled with 0 and all other vertices are unlabelled. Then, while there are some unlabelled vertices, every unlabelled vertex adjacent to a vertex labelled $i$ becomes labelled $i+1$. 

(a) (1 point) Apply the algorithm on the tree in the center of Figure ?? with $v_0 = e$.

(b) (1 point) Prove that this is a $\frac{1}{2}$-approximation algorithm for computing the diameter of any graph.

(c) (1 point) Give an example of a graph $G$ where the value returned by the algorithm is half the diameter of $G$. 