# Computing Tree-decompositions of Graphs

Julien Bensmail julien.bensmail@unice.fr , http://www-sop.inria.fr/members/Julien.Bensmail/
Nicolas Nisse nicolas.nisse@inria.fr, http://www-sop.inria.fr/members/Nicolas.Nisse/

**Graph decompositions and treewidth.** *Tree-decompositions* of graphs are a way to decompose a graph into small "pieces" (called bags) with a tree shape. To every tree-decomposition of a graph a measure is associated (its *width*). In the last decades, graph decompositions have been studied a lot for their algorithmic applications (among others, they are an important ingredient of the celebrated Robertson and Seymour work on Graph Minors). Indeed, once a graph admits a good decomposition (the smaller the width is, the better the decomposition is), many hard problems can be solved efficiently using dynamic programming (e.g., the famous Courcelle's Theorem). Unfortunately, computing a good decomposition is a hard task by itself. More precisely, computing an optimal tree-decomposition of a graph is an NP-hard problem. There are algorithms that decide if a graph has *treewidth* at most $k$ in polynomial time (for a fixed $k$) but none of them is practical even for small graphs. Therefore, a research effort has to be done to find efficient algorithms to compute tree-decompositions.

One way to tackle this question is to focus on different measures of tree-decompositions [KLNS15, Seymour16]. For instance, the *treelength* (measuring the diameter of the bags) [DG07,CDN16] and the *treebreadth* (corresponding to the radius of the bags) are known to be NP-hard to compute but admit efficient approximation algorithms in general graphs (e.g., [DLN16]). Moreover, the treelength and the treewidth are equivalent (up to a constant ratio) in large graph classes [CDN16]. For these reasons, it is important to investigate the computational complexity of treebreadth in such graphs.

**Objectives of the internship.** The ambitious objectives of the internship may be some of the following:

- Study the computational complexity of treelength (or treebreadth) in the class of planar graphs.
- Propose better approximation algorithms for treelength or treebreadth in general graphs.
- Turn the result of [CDN14] into an algorithmical one (i.e., find an algorithm that, given a tree-decomposition with small length, computes one with small width).

**References:**

**CDN16** David Coudert, Guillaume Ducoffe and Nicolas Nisse, To Approximate Treewidth, Use Treelength! SIAM J. Discrete Math. 30(3): 1424-1436 (2016)

**DG07** Yon Dourisboure, Cyril Gavoille: Tree-decompositions with bags of small diameter. Discrete Mathematics 307(16): 2008-2029 (2007)

**[LN16** Guillaume Ducoffe, Sylvain Legay, Nicolas Nisse: On the Complexity of Computing Treebreadth. in Proceedings of 27th International Workshop on Combinatorial Algorithms (IWOCA) 2016: 3-15

**KLNS15** Adrian Kosowski, Bi Li, Nicolas Nisse, Karol Suchan: k-Chordal Graphs: From Cops and Robber to Compact Routing via Treewidth. Algorithmica 72(3): 758-777 (2015)

**Seymour16** Paul D. Seymour: Tree-chromatic number. J. Comb. Theory, Ser. B 116: 229-237 (2016)

Project team COATI, INRIA, I3S, CNRS, University of Nice Sophia
2004, route des Lucioles – B.P. 93 – F-06902 Sophia Antipolis Cedex – France