To Satisfy Impatient Web Surfers is Hard

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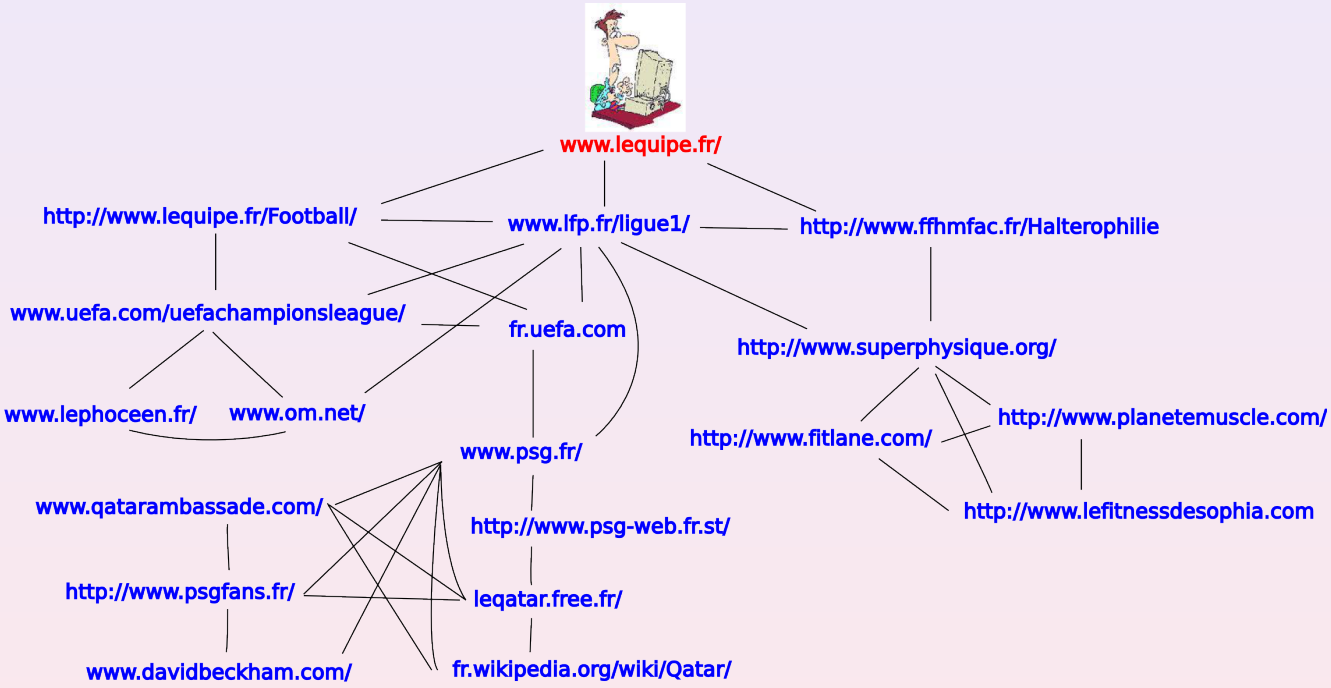
Prefetching for faster data access

Web: pre-loading web pages before the web Surfer accesses it
Goal: Avoid the web Surfer to wait

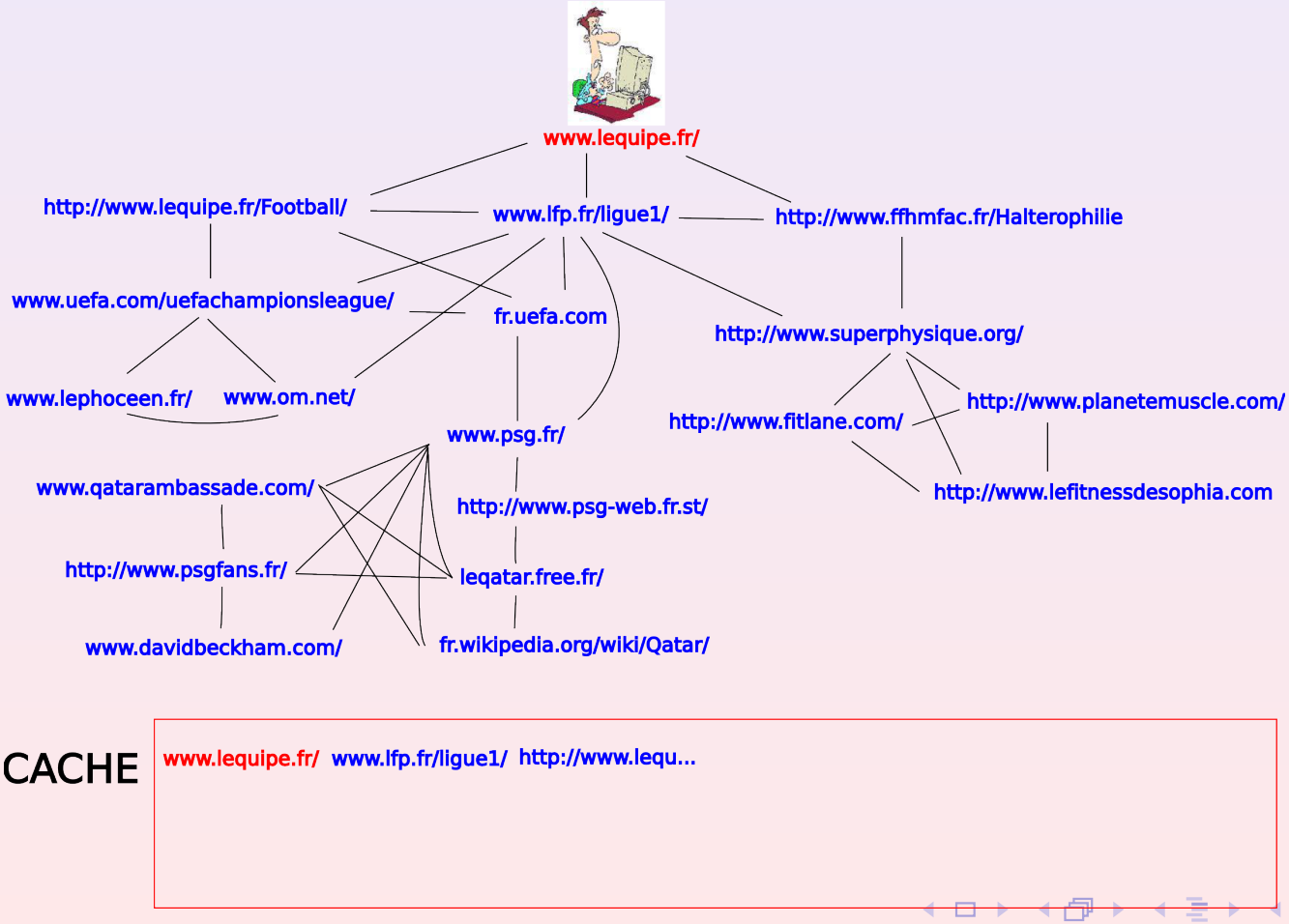


CACHE

www.lequipe.fr/

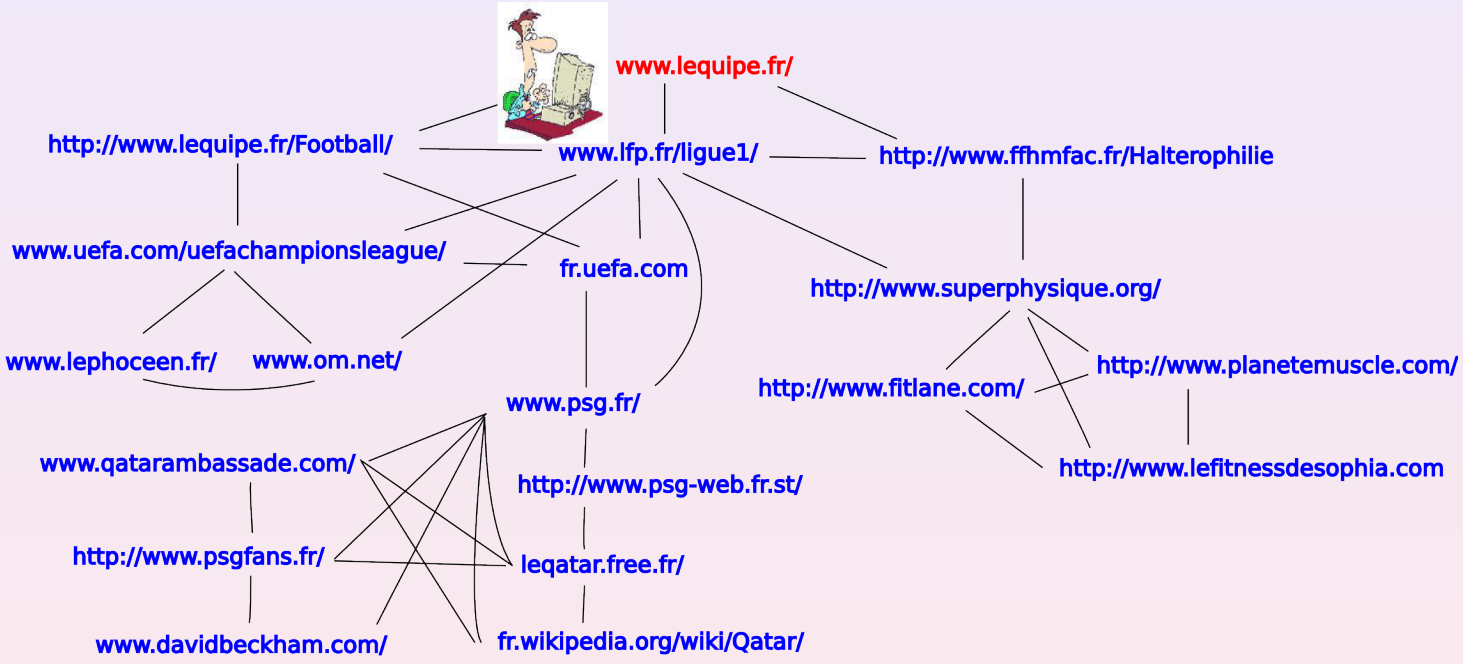
Prefetching for faster data access

the web Surfer starts from a given web page in cache
try to load web pages in cache →TAKES TIME



Prefetching for faster data access

at some point, web Surfer moves
if web page reached already in the cache OK

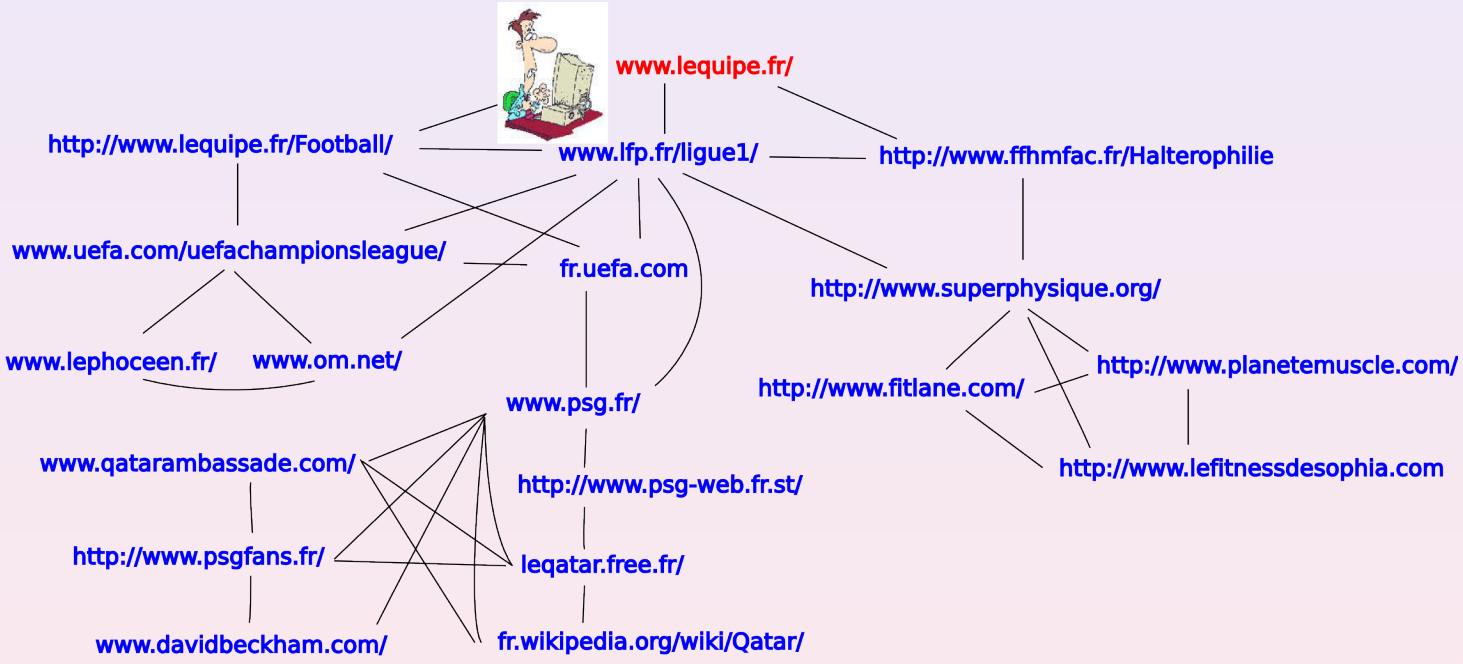


CACHE

www.lequipe.fr/ www.lfp.fr/ligue1/ http://www.lequipe.fr/Football/ http://www.ffhmfac.fr/Halterophilie
www.uefa.com/uefacham...

Prefetching for faster data access

web Surfer follows the hyperlinks in an unpredictable way
web pages to be loaded may be "guessed"



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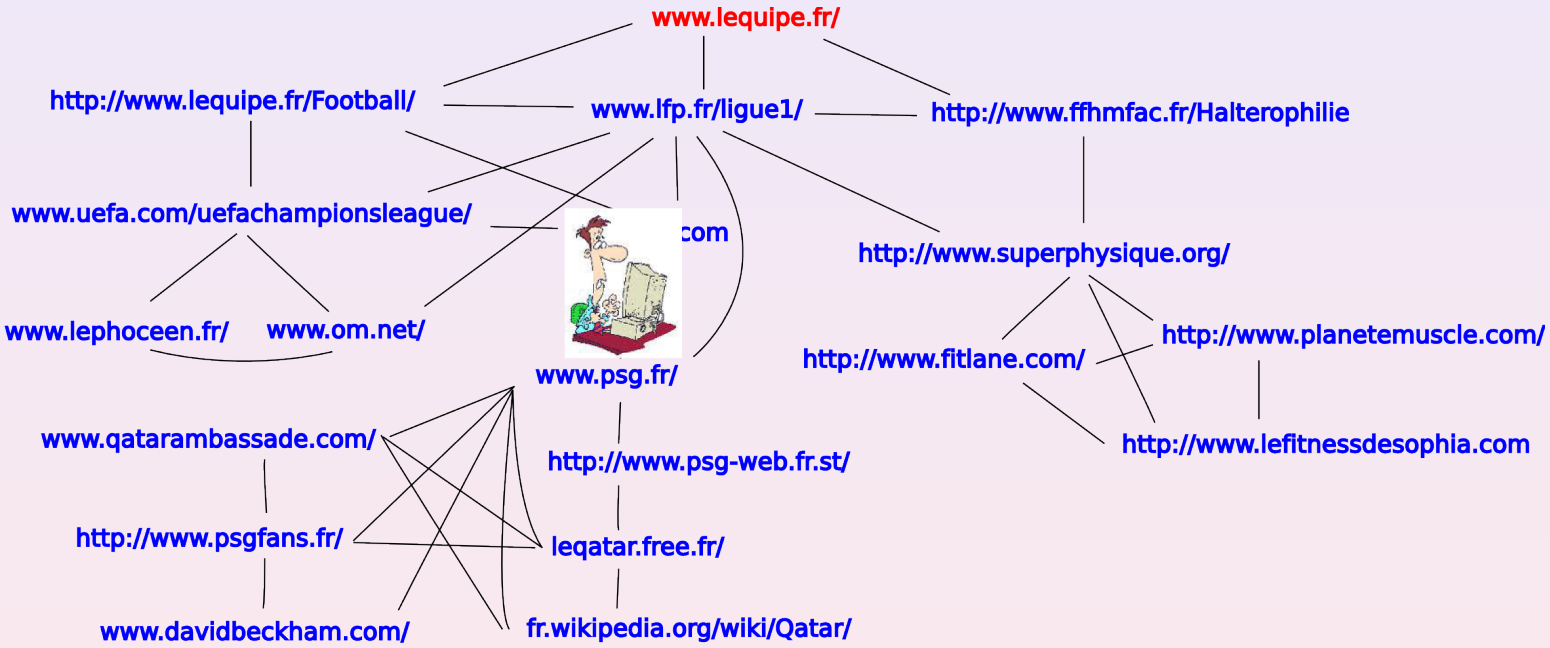
www.uefa.com/uefachampionsleague/ fr.uefa.com www.om.net/ www.psg.fr/
http://www.superp...

Prefetching for faster data access

even if we guessed well
choices might be too numerous



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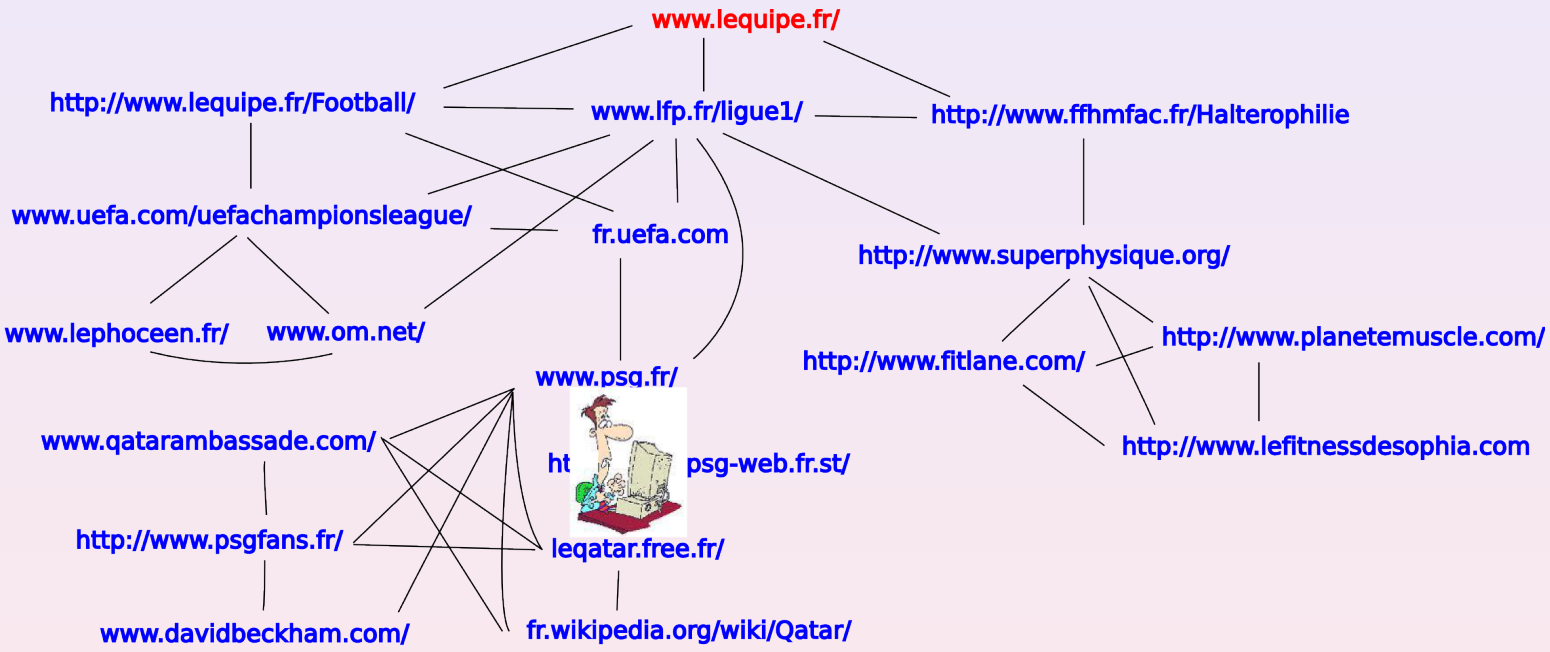
www.uefa.com/uefachampionsleague/ fr.uefa.com www.om.net/ www.psg.fr/
http://www.superp...

Prefetching for faster data access

web Surfer may access a page not in cache
and has to wait !!



CACHE

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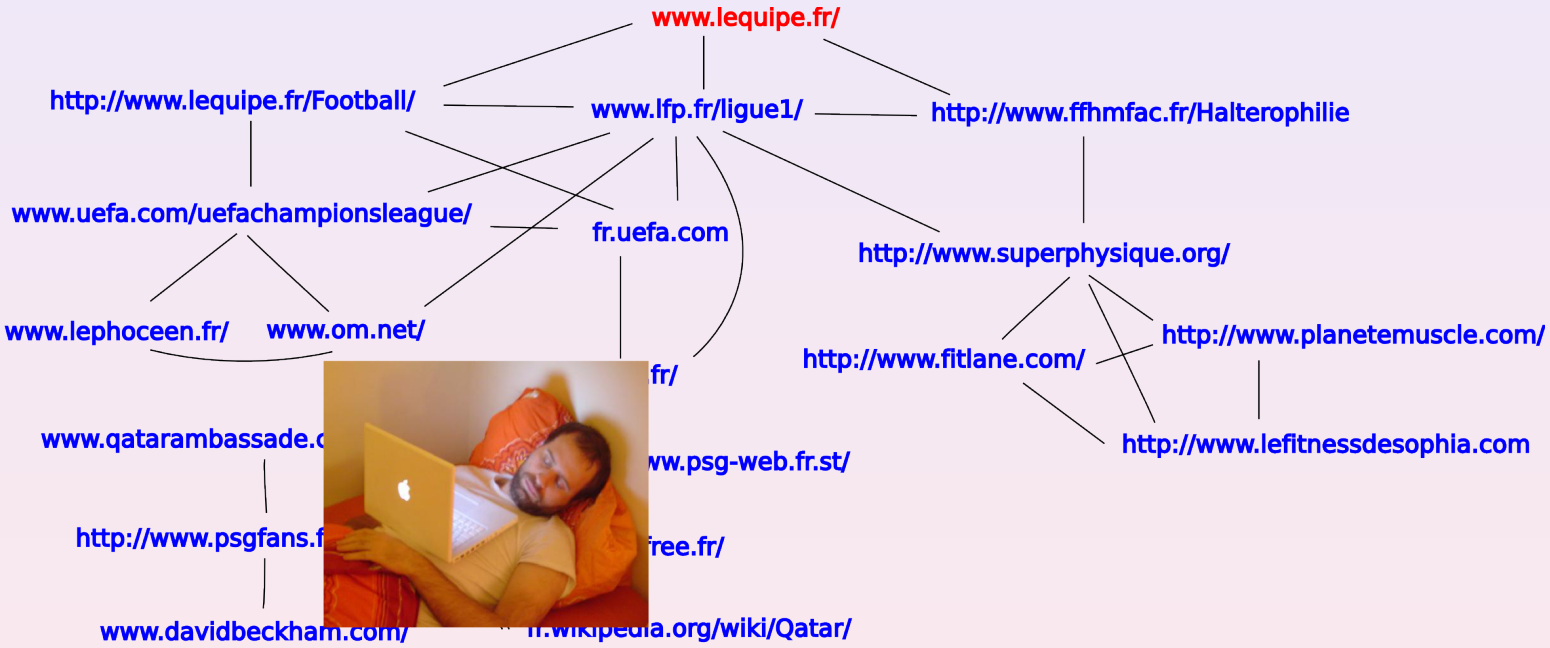
www.qatarambassade.com/ http://www.psgfans.fr/ http://www.psg-web.fr.st/ www.davidbec...

Prefetching for faster data access

web Surfer may access a page not in cache

not good for research...



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www.uefa.com/uefachampionsleague/ fr.uefa.com www.om.net/ www.psg.fr/
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Prefetching for faster data access

Issue: download speed, NOT size of cache
Instance: network KNOWN ((di)graph)

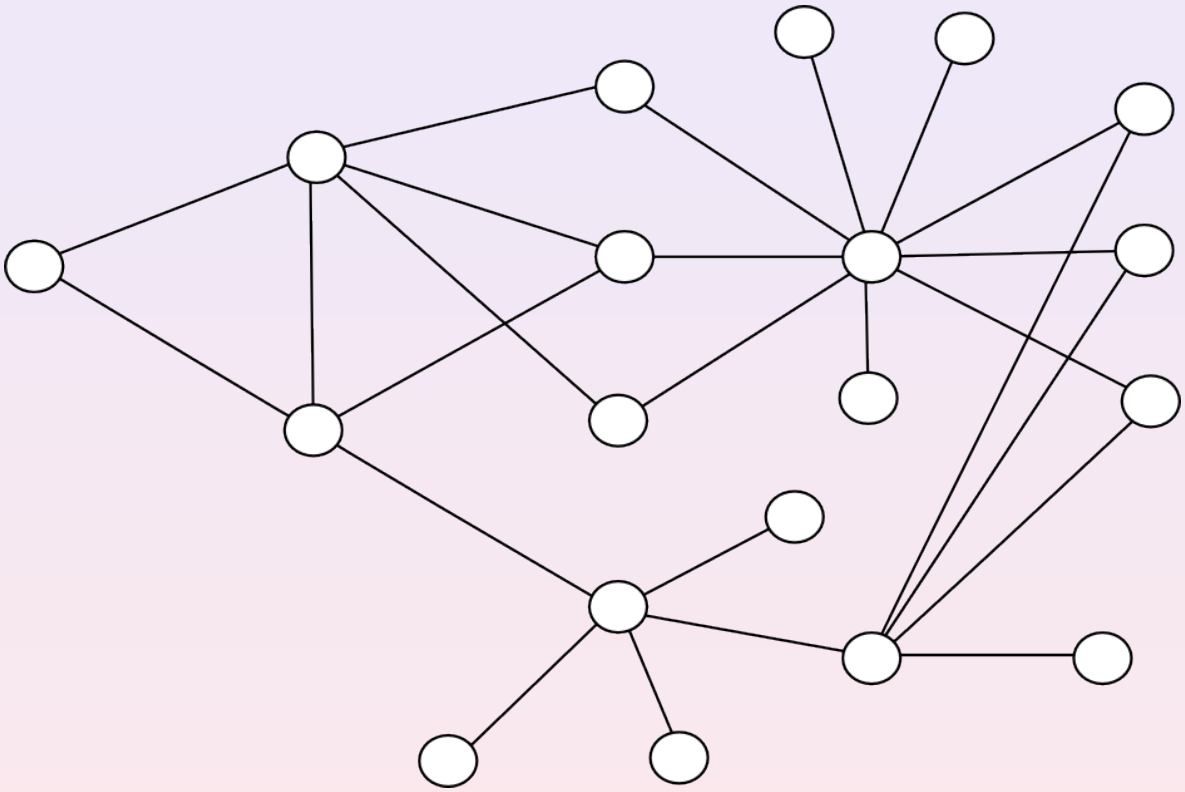
Related work: Probabilistic algorithms
(arcs + transition probabilities)

- Markovian model [Vitter,Krishnan. JACM'96]
  [Morad,Jean-Marie. ROADEF'10]
- Stochastic Dynamic Programming framework
  [Joseph,Grunwald. ISCA'97]
  [Grigoras,Charvillat,Douze. ACM Multimedia'02]

Our work:

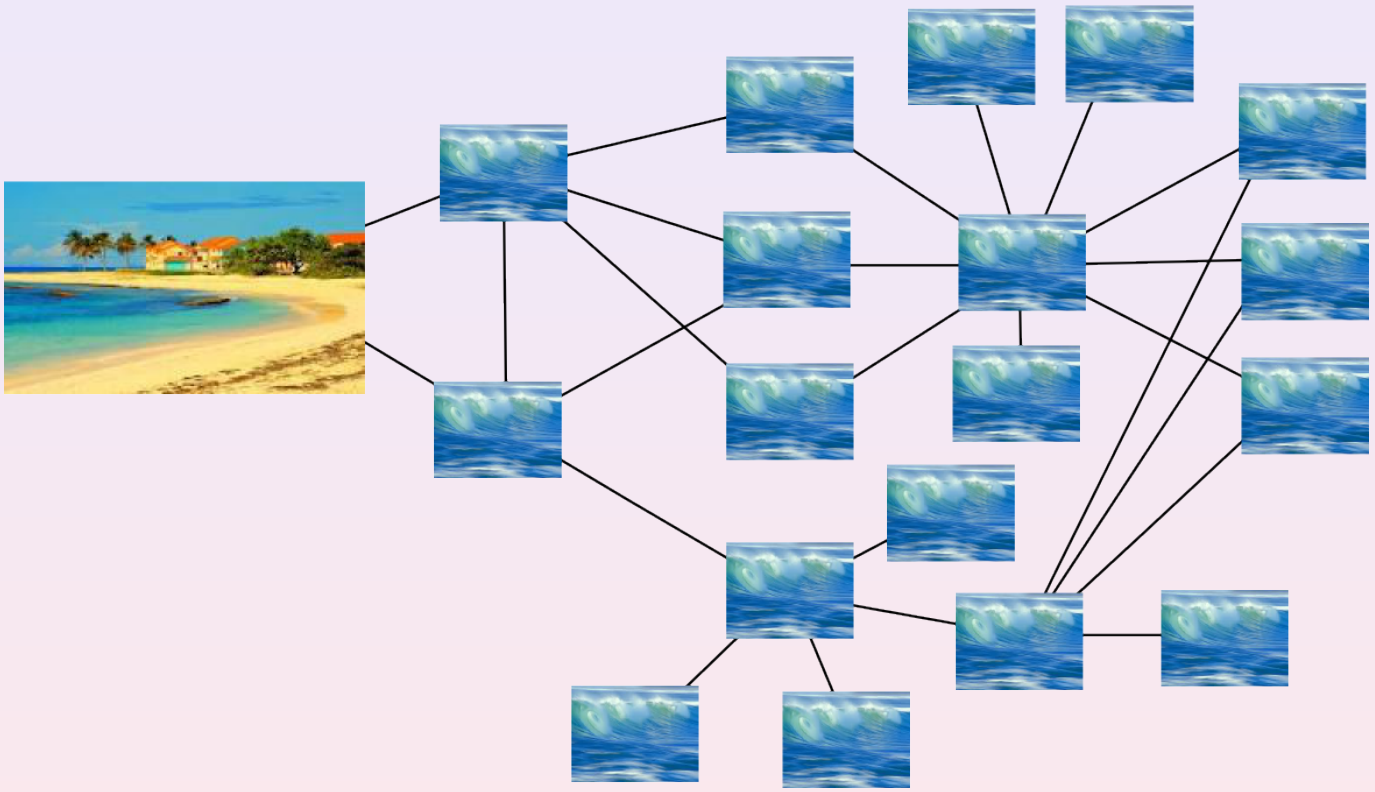
minimize download speed
to insure web Surfer never waits
(worst case, deterministic)

Model



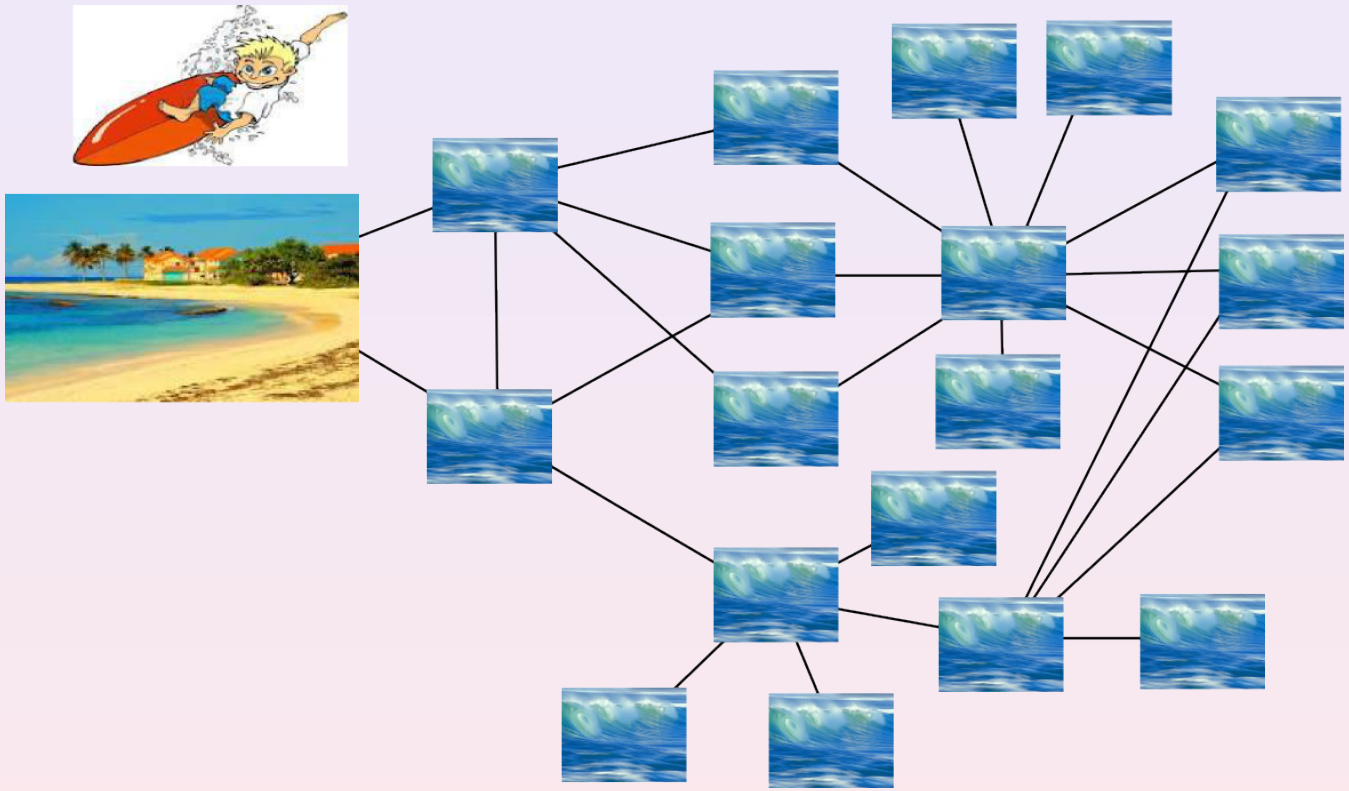
Web = connected (di)graph

Model



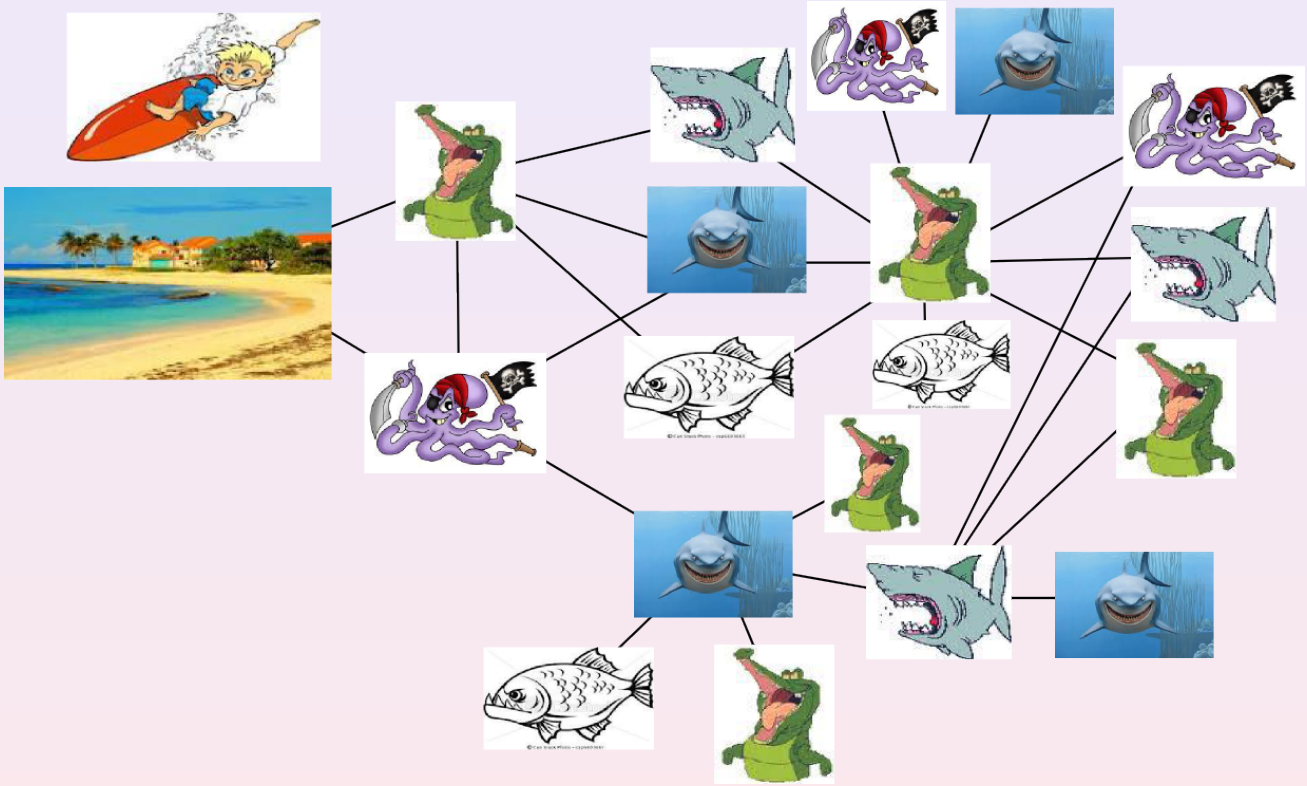
Web = (di)graph with a specified starting point (beach)

Model



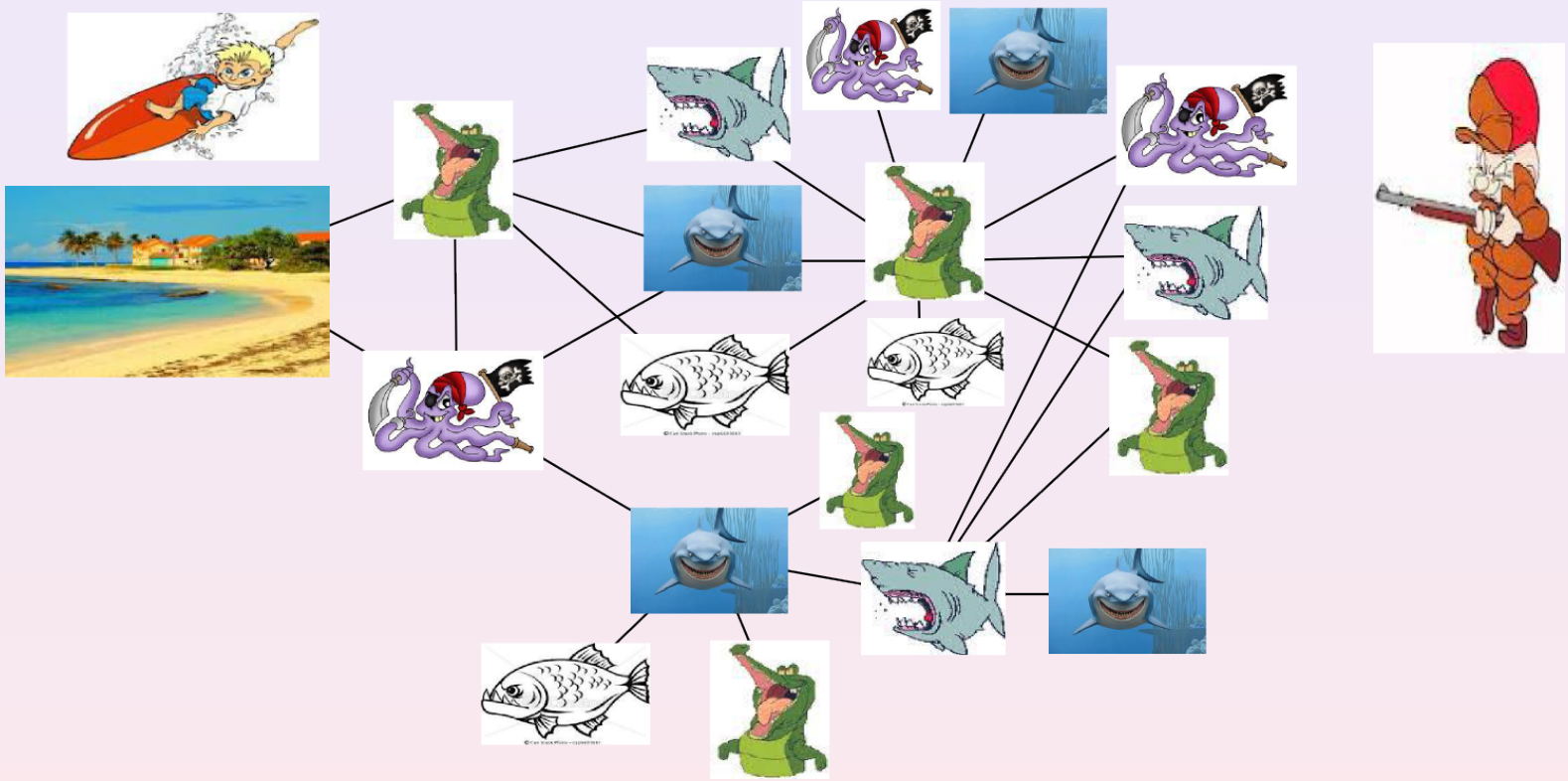
then, we need a (web) Surfer

Model



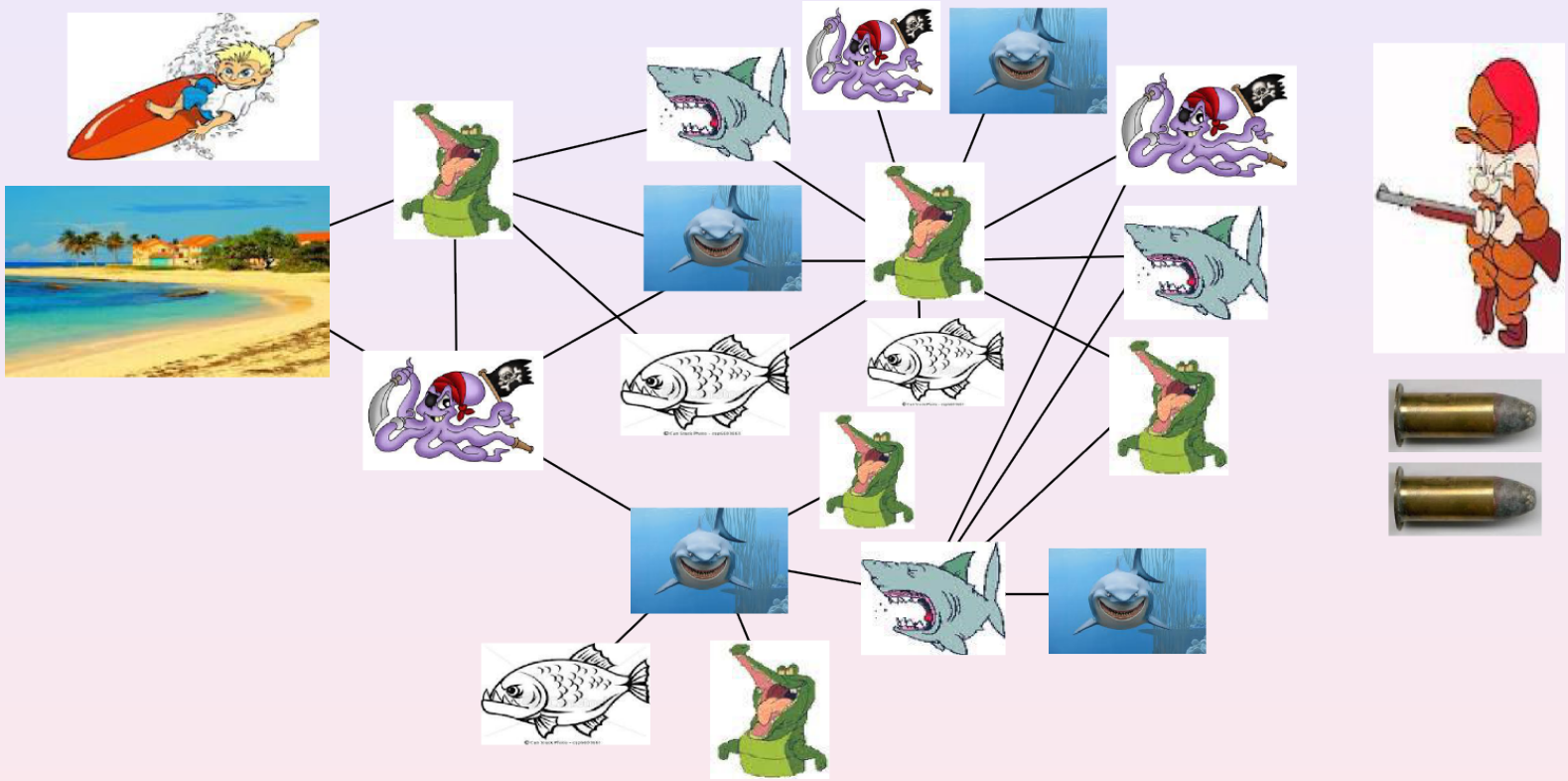
unloaded page = dangerous node

Model



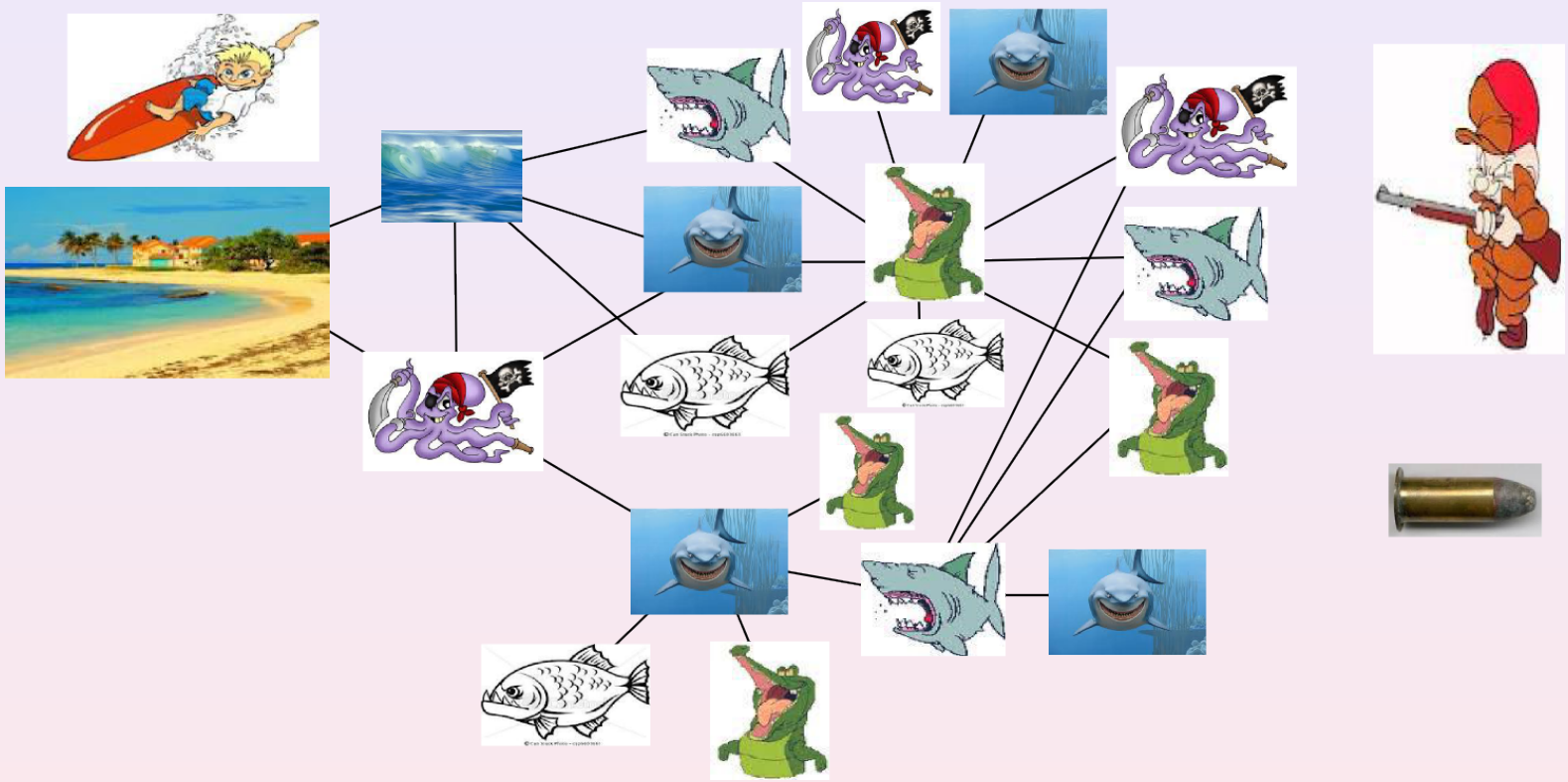
we need someone to help the Surfer let's call it *the Guard*

Model



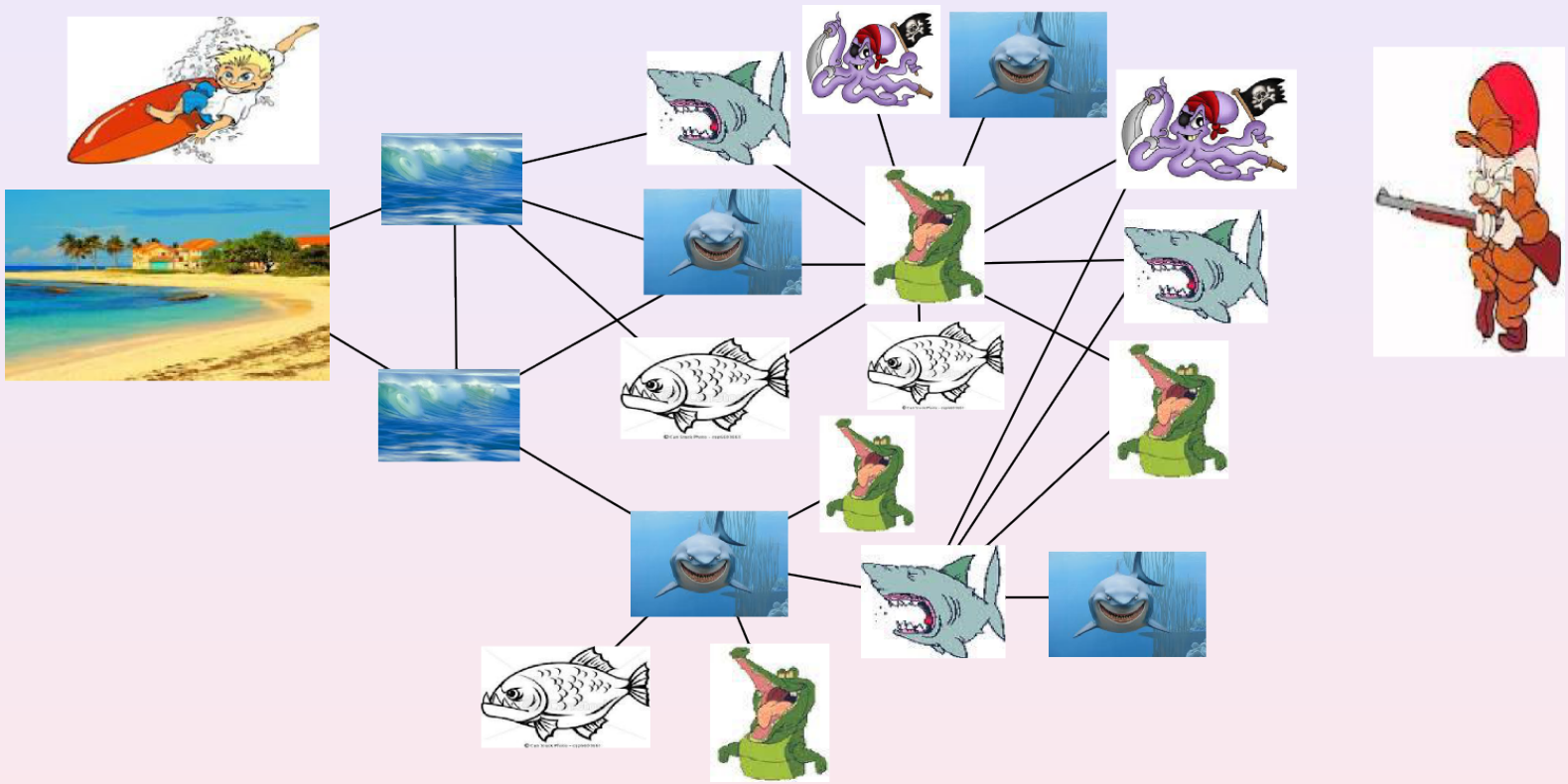
download speed = amount of bullets

Model



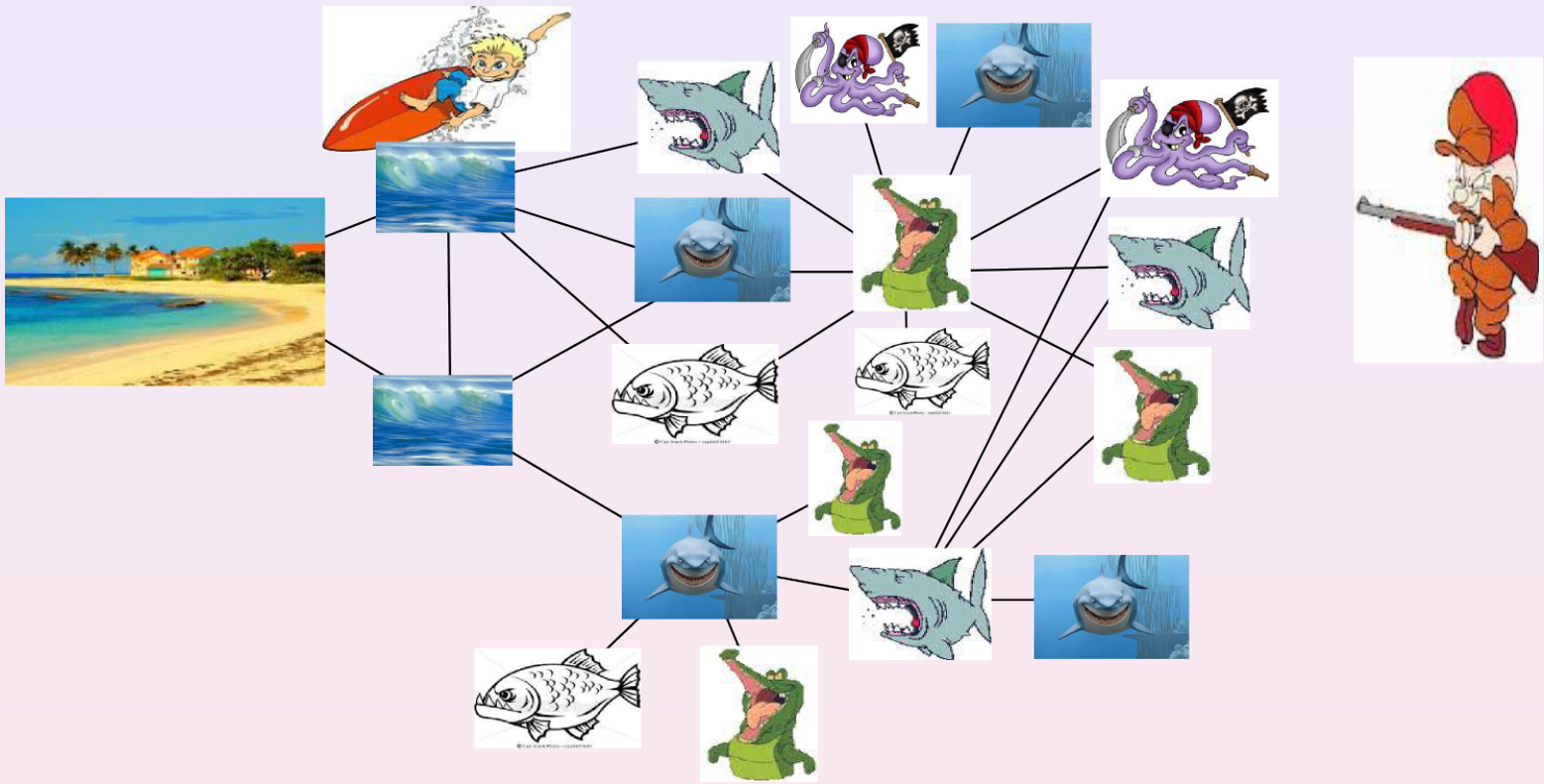
Guard uses one bullet to secure one node

Model



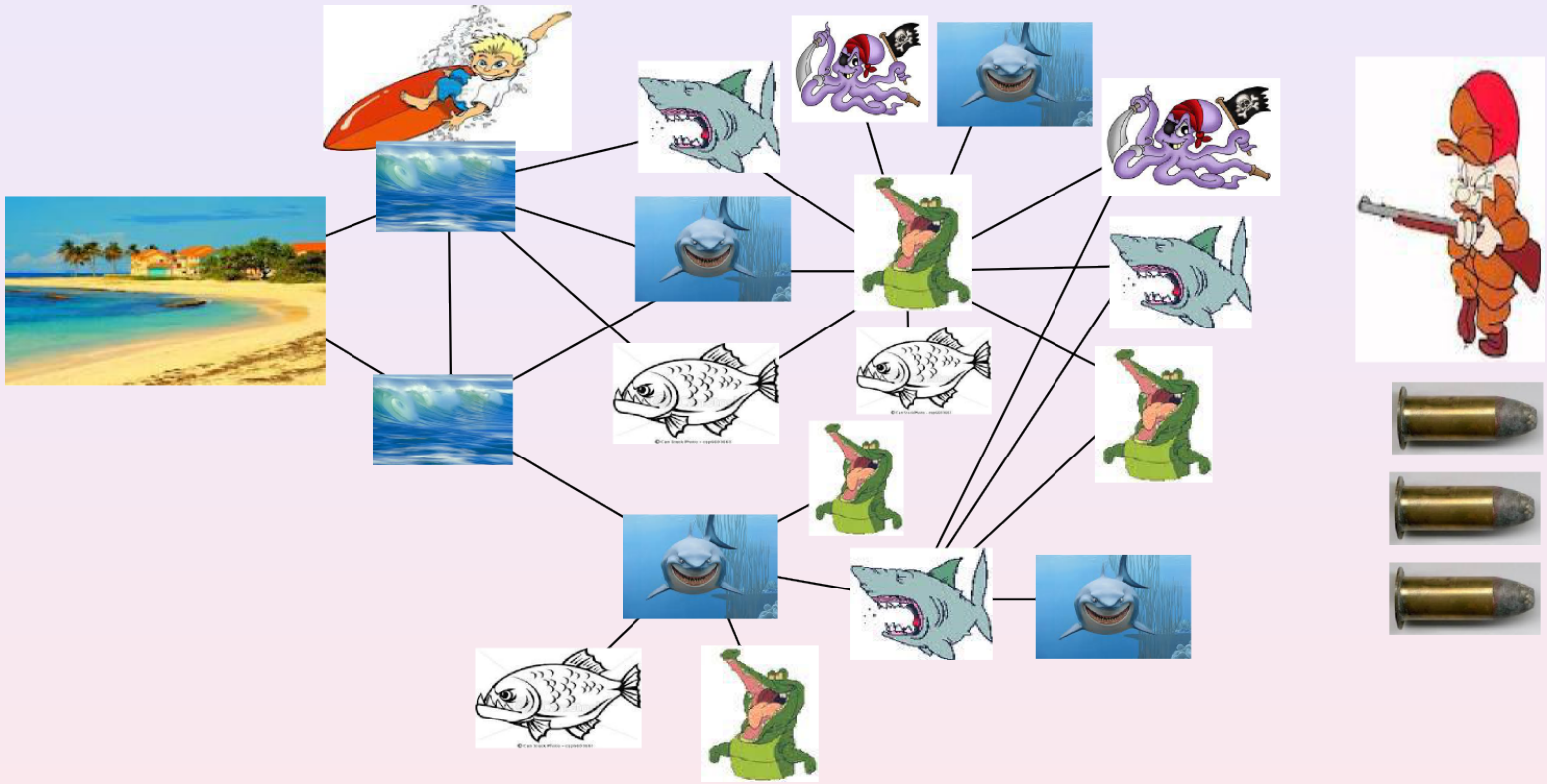
clearly, $degree(beach) \leq \#$ of bullets required to save Surfer

Model



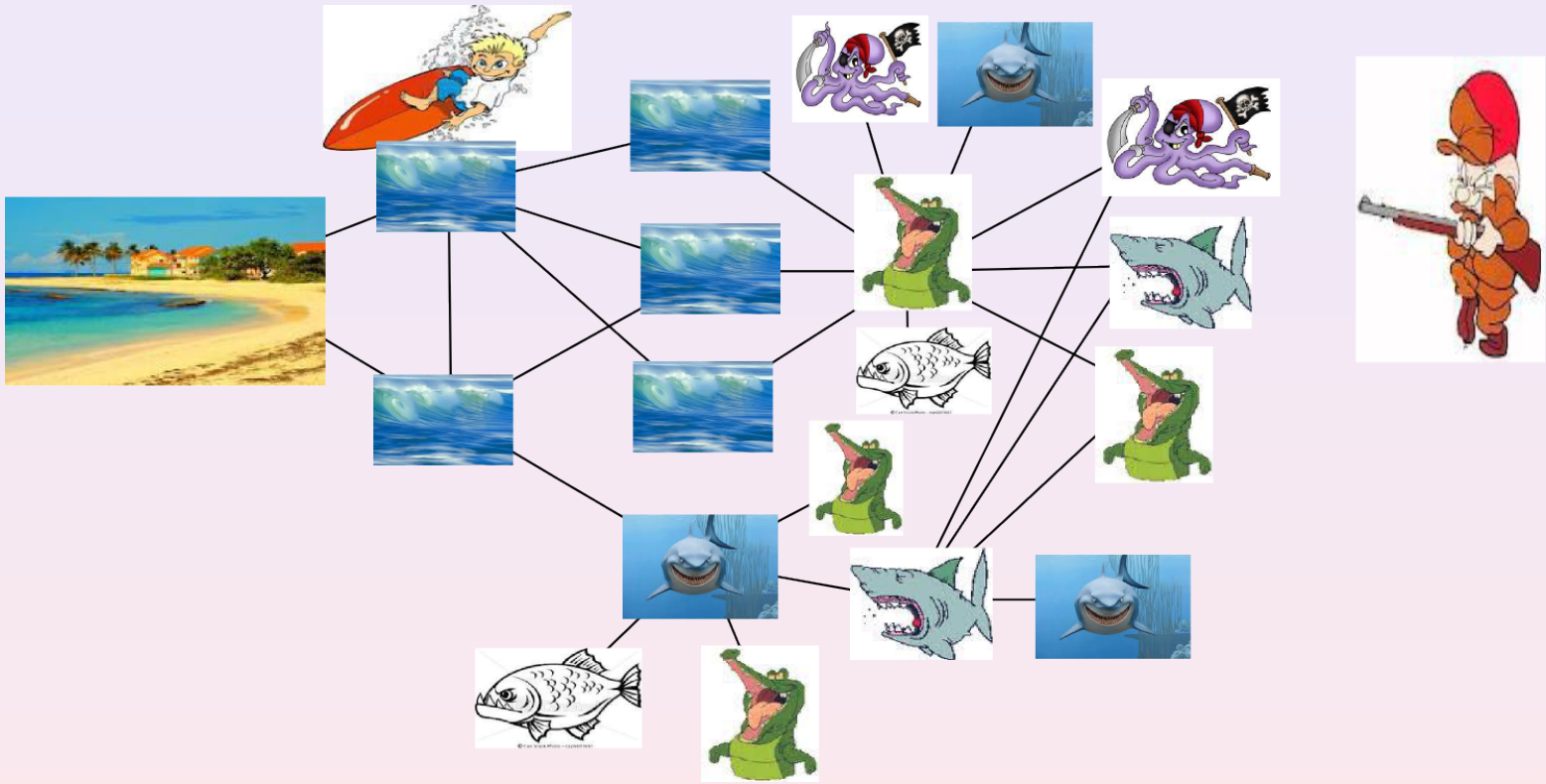
then, Surfer may move

Model



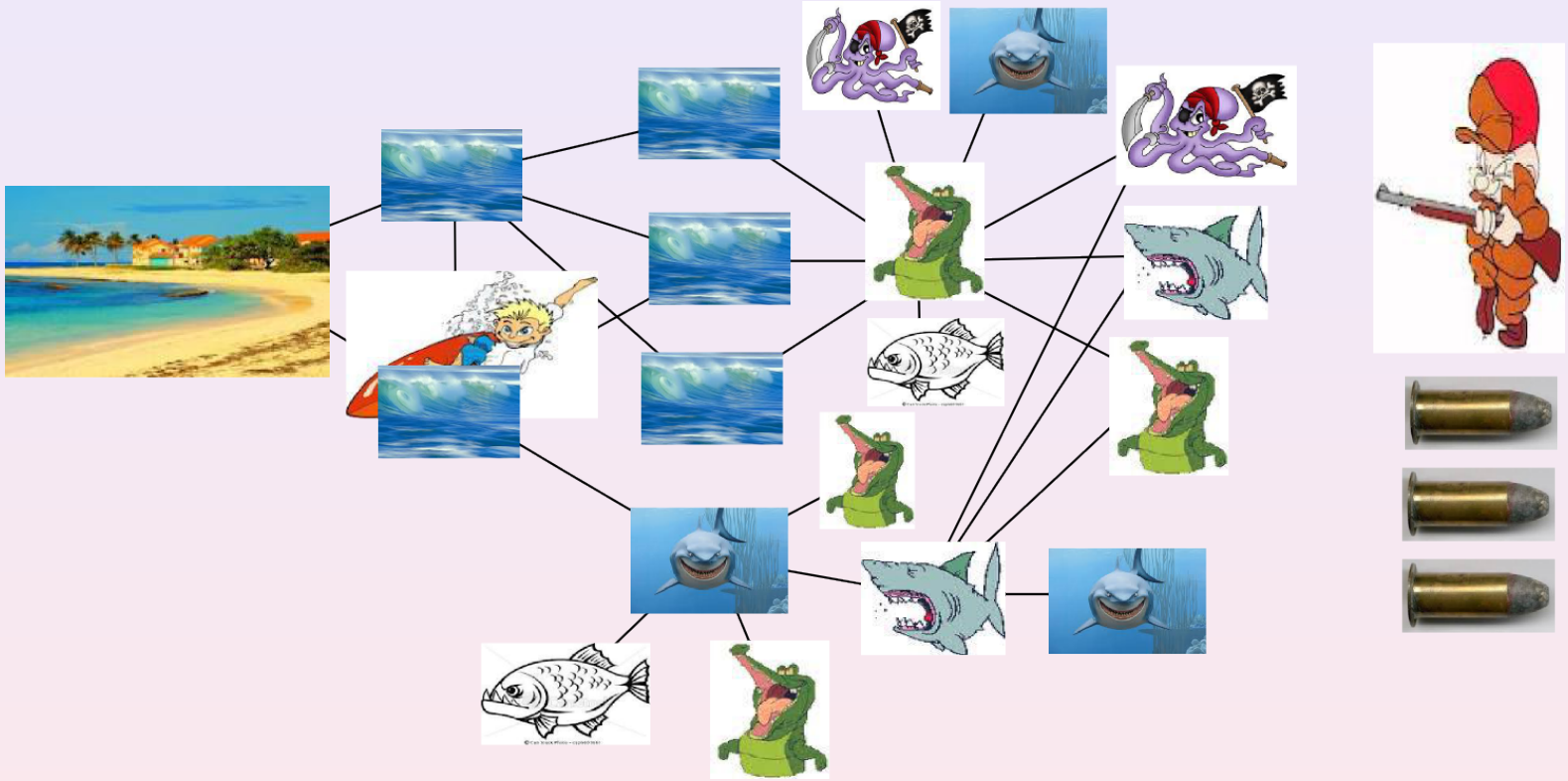
here one more bullet is needed

Model



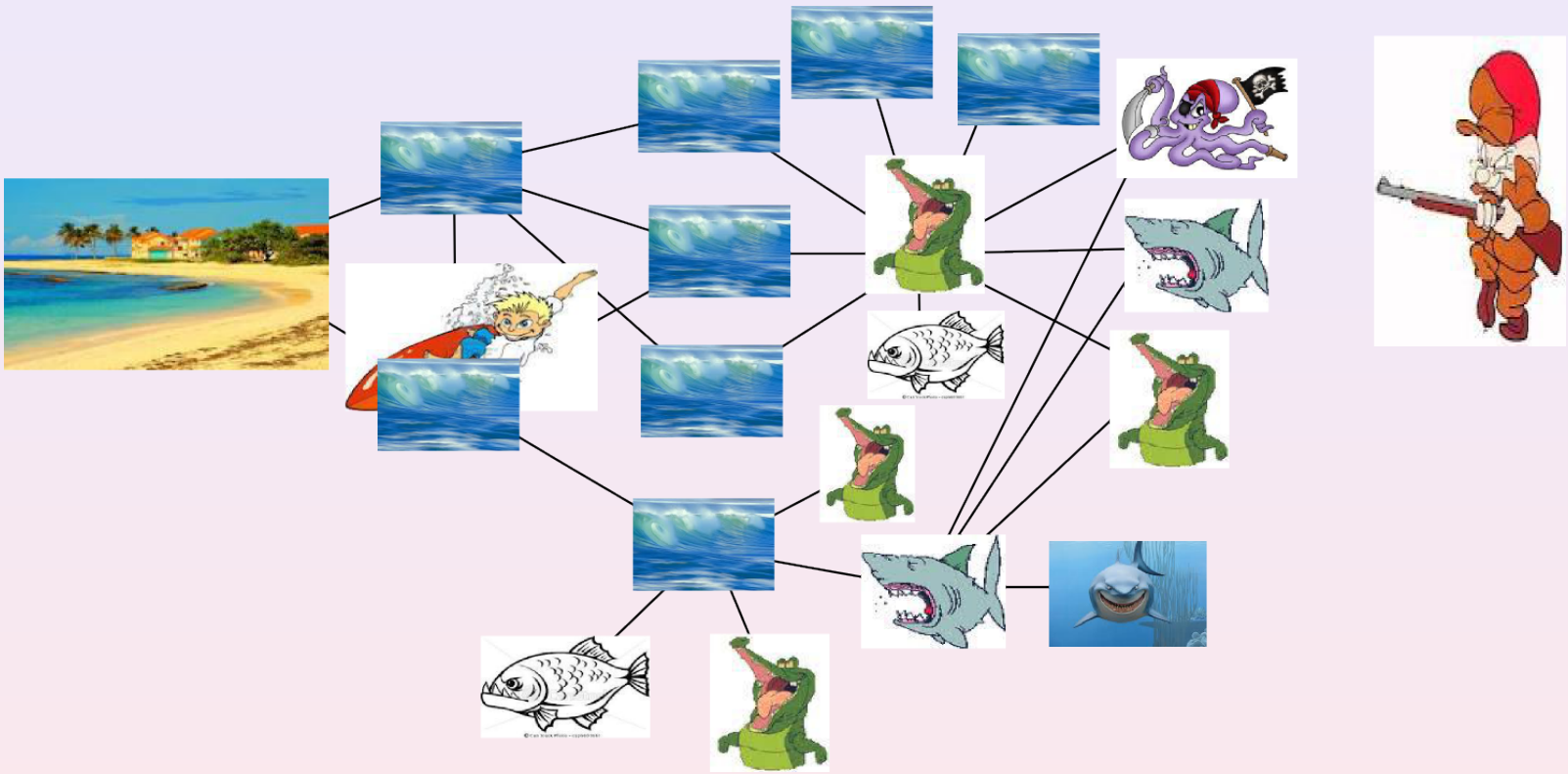
$degree(beach) \leq$ amount of bullets $\leq max\ degree$

Model



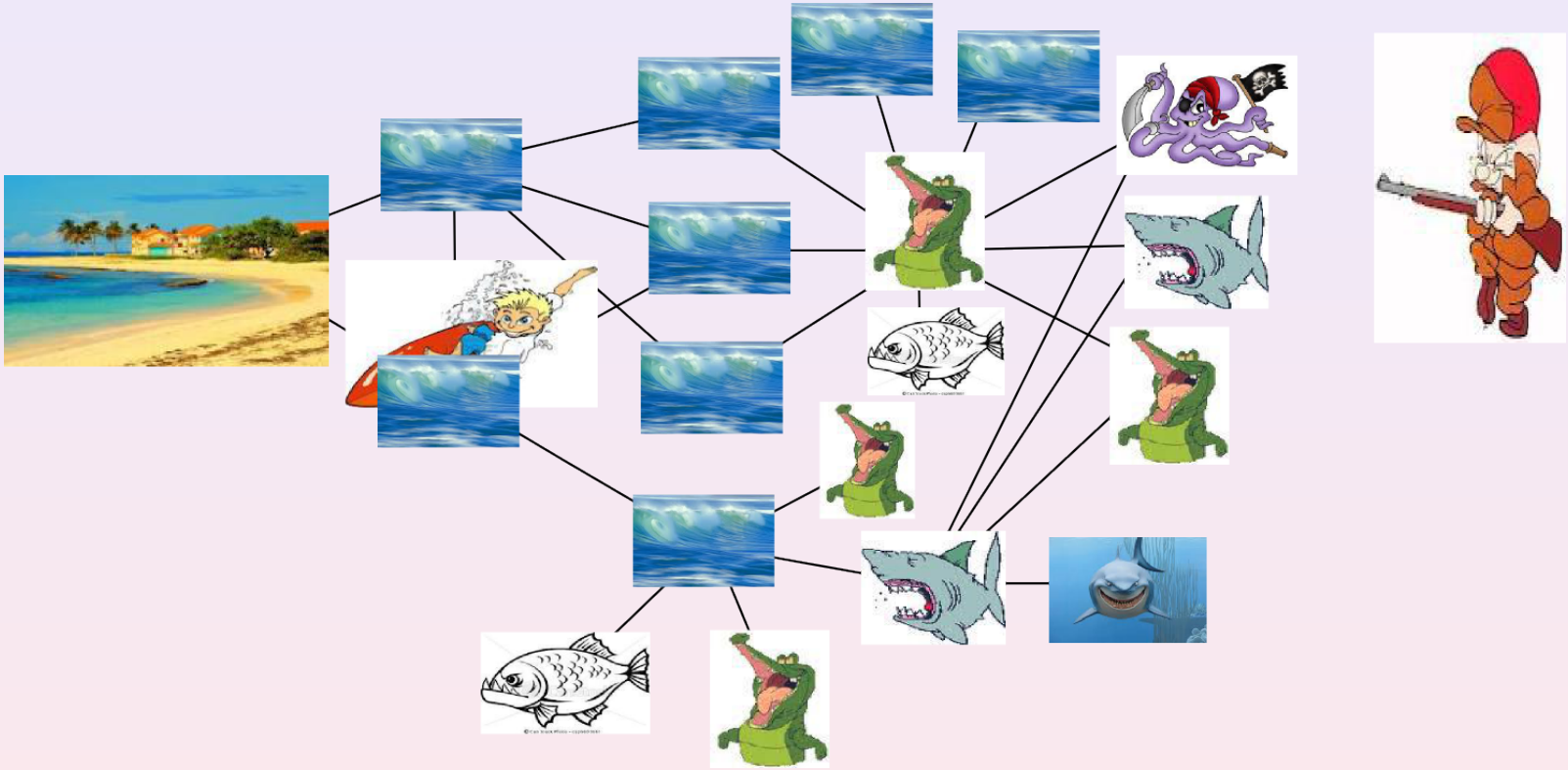
Surfer may move anywhere in its neighborhood

Model



bullets may be used to prevent future moves

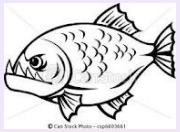
Model



are 3 bullets sufficient to make sure Surfer never eaten?

Model: a Two players game

- a *Surfer*  starts from safe homebase v_0 

  in G, a dangerous graph     

- a *Guard*  with some amount k of bullets 

Turn by turn:

1. the guard secures $\leq k$ nodes;
2. then, the Surfer may move to an adjacent node.

Defeat: Surfer in unsafe node **Victory:** G safe

Minimize amount of bullets to win for any Surfer's trajectory

Surveillance number of G (**connected**) from v_0: $sn(G, v_0)$

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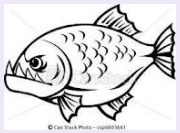
  in G, a dangerous graph     

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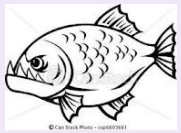
  in G, a dangerous graph     

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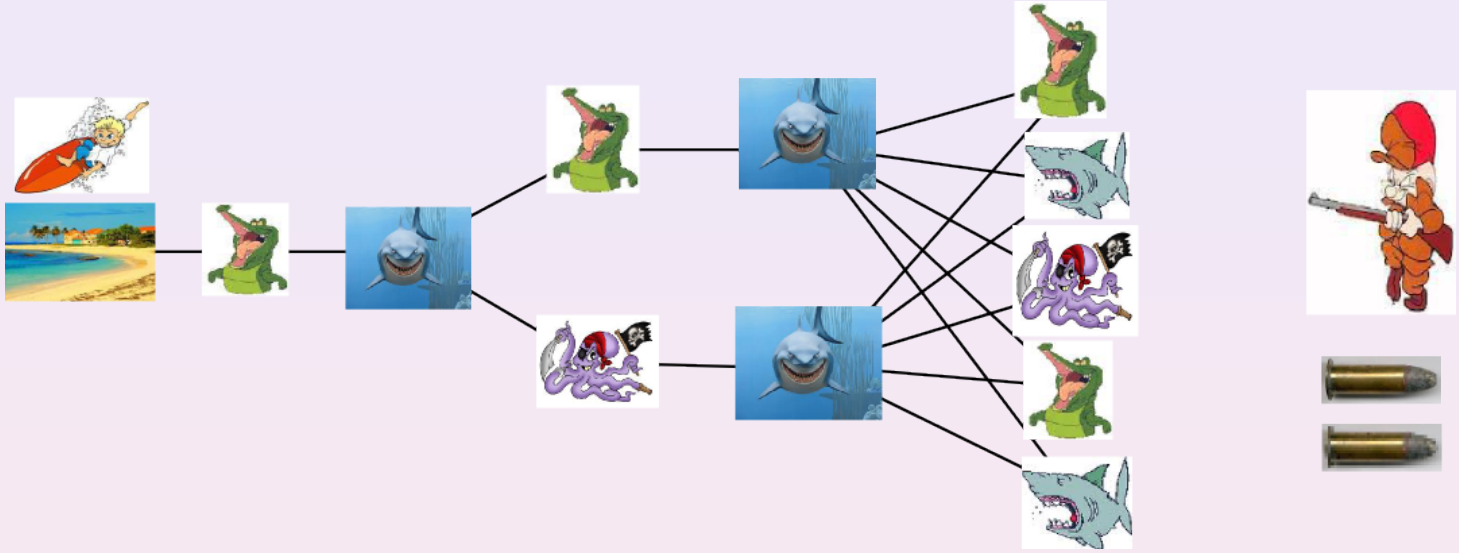
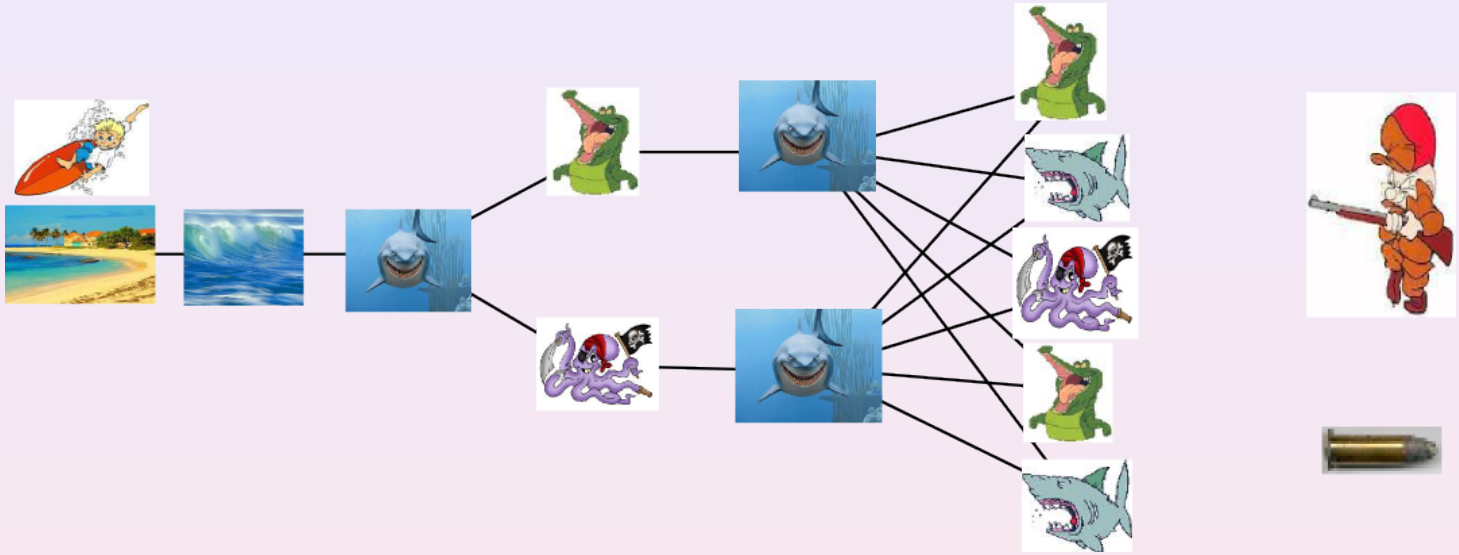


with 1 bullet: after 2 steps, Surfer faces 2 dangerous nodes!!

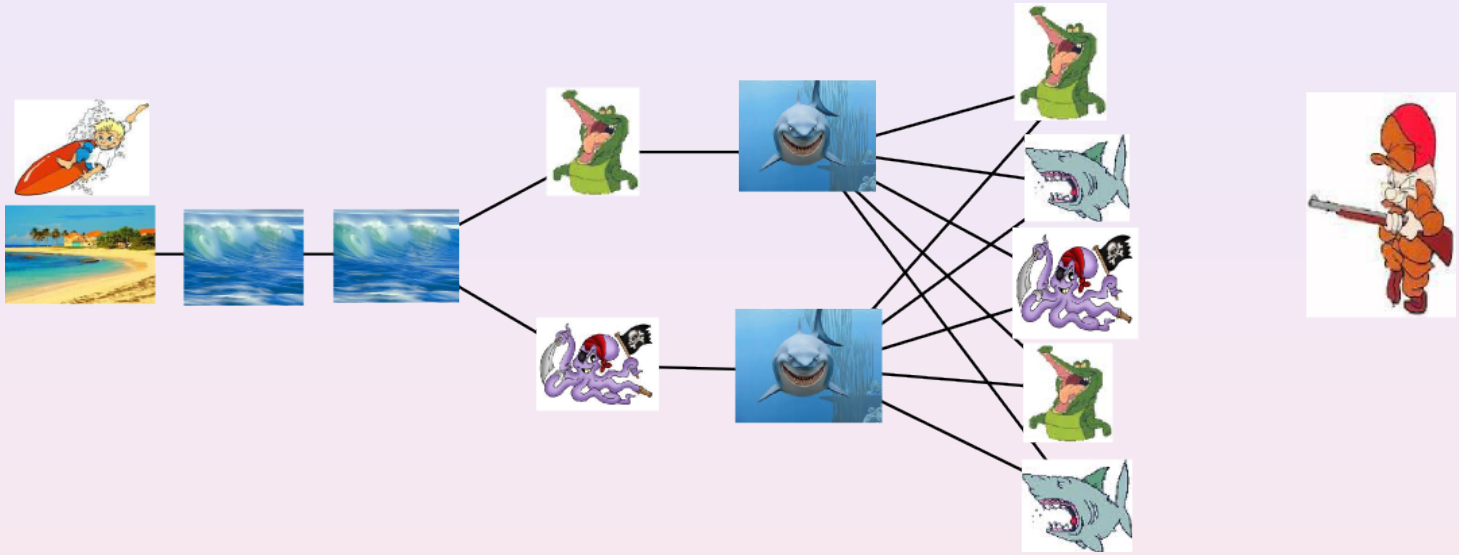
$$sn(G, v_0) > 1$$

First Example



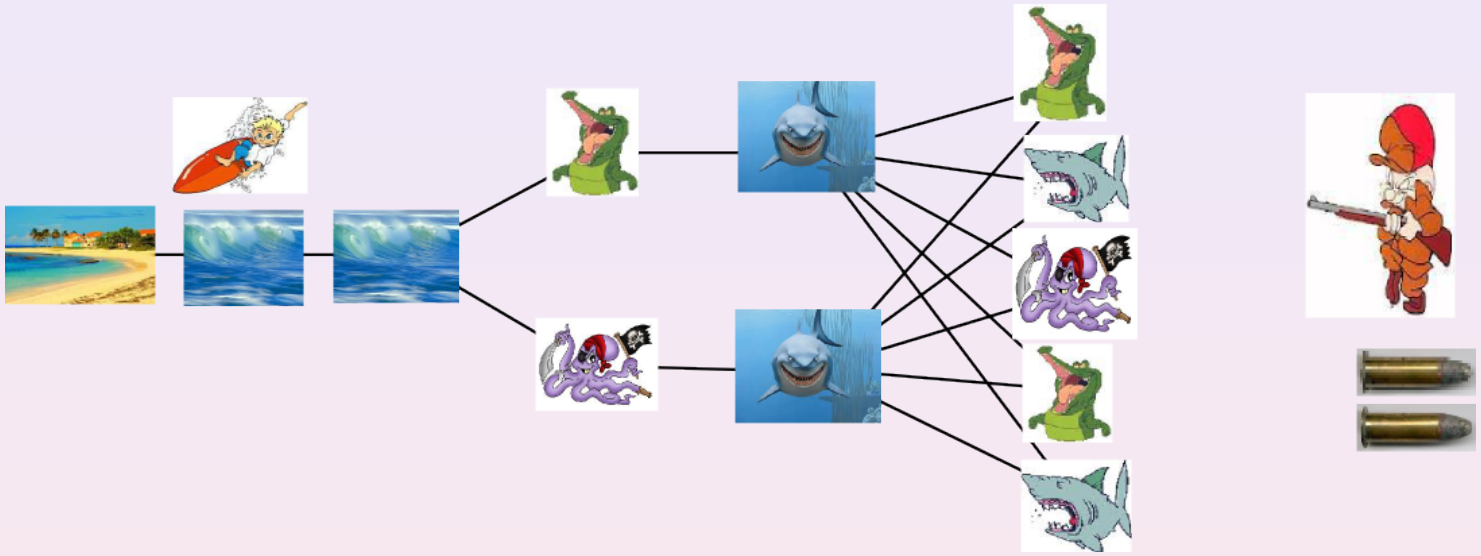
Guard uses his bullets

First Example



Guard uses (all) his bullets

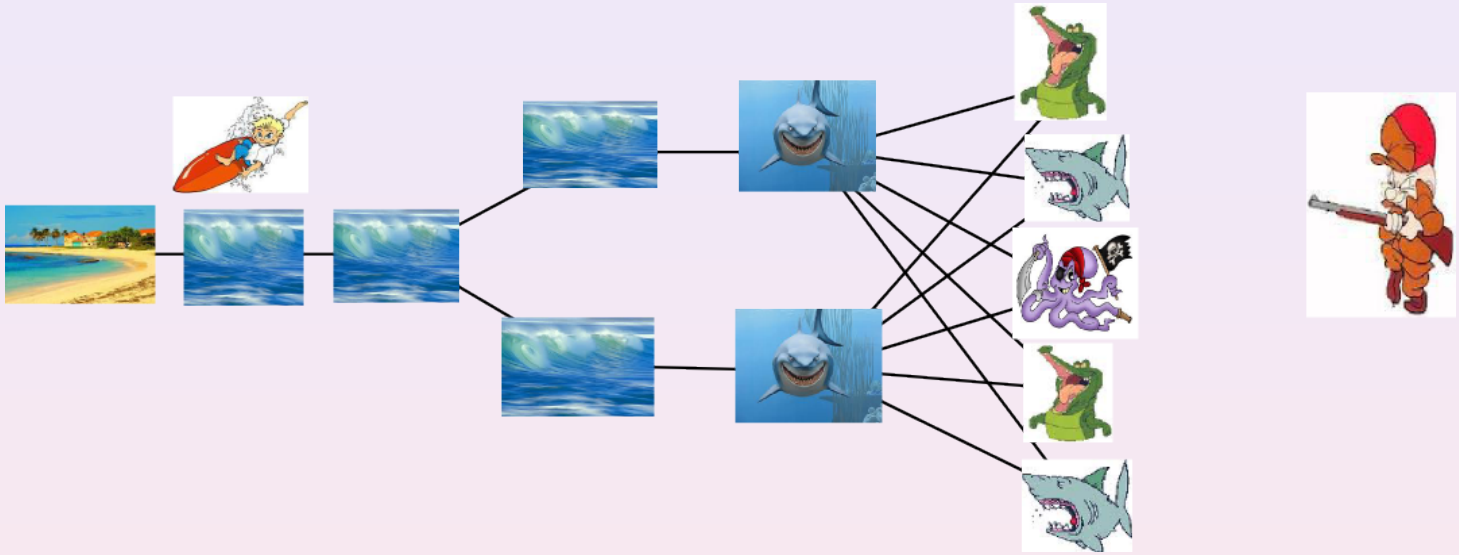
First Example



Guard uses (all) his bullets, **then** Surfer may move

Clearly: worst case if Surfer always move

First Example



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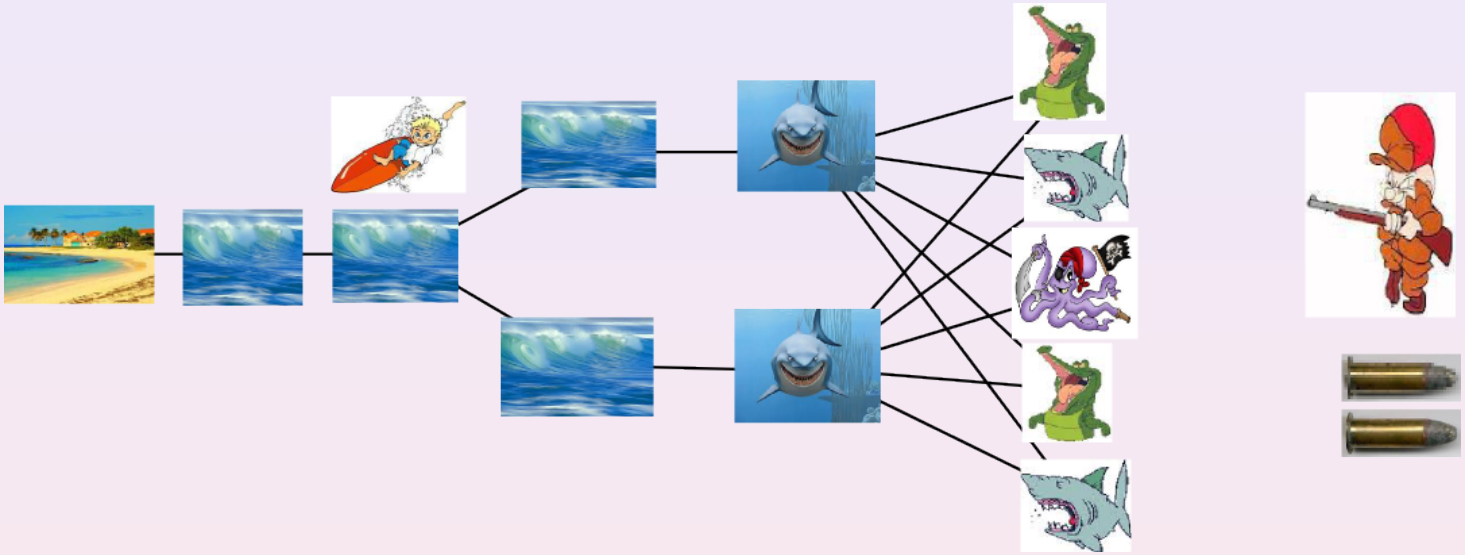

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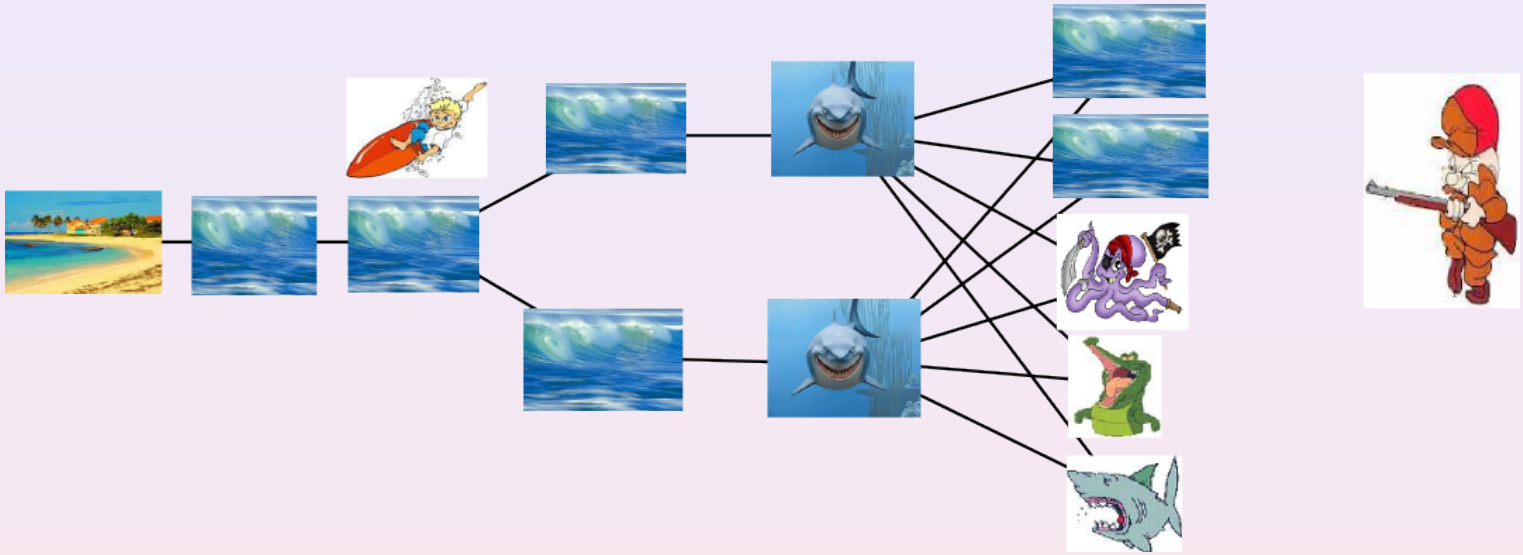
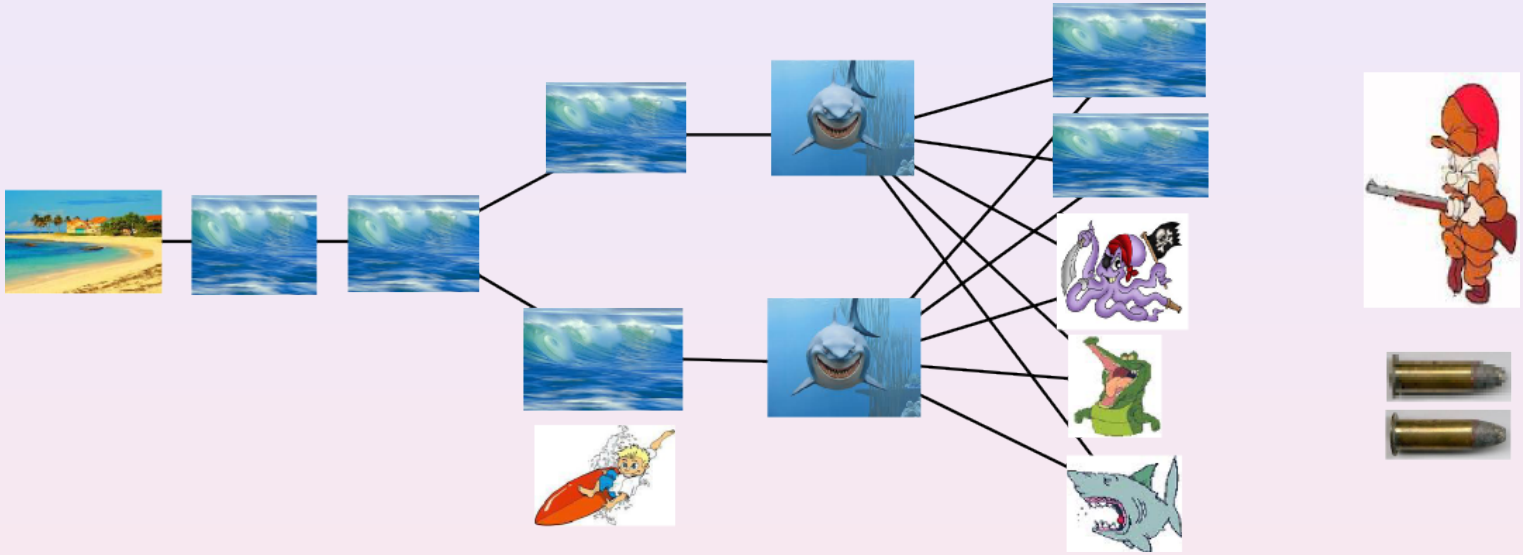


Guard uses (all) his bullets, **then** Surfer moves

Guard may secure any node in the graph

First Example



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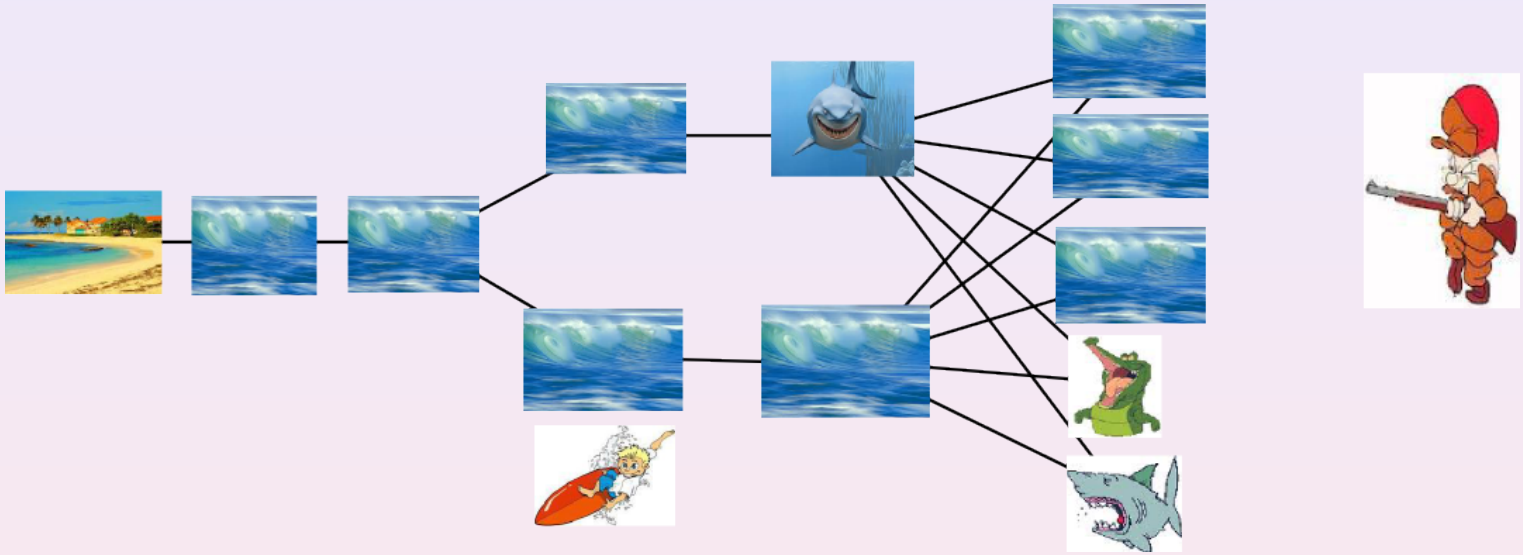

Guard uses (all) his bullets, **then** Surfer moves

strategy: safe nodes + Surfer's node $\Rightarrow \leq k$ nodes to secure

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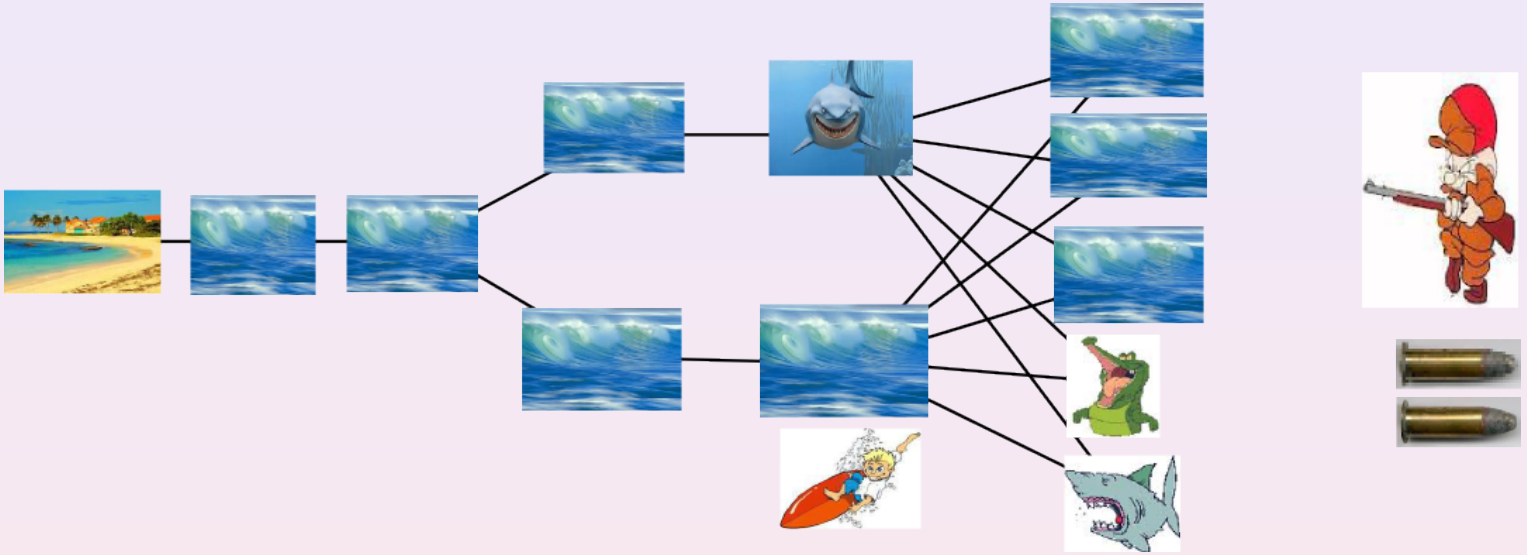

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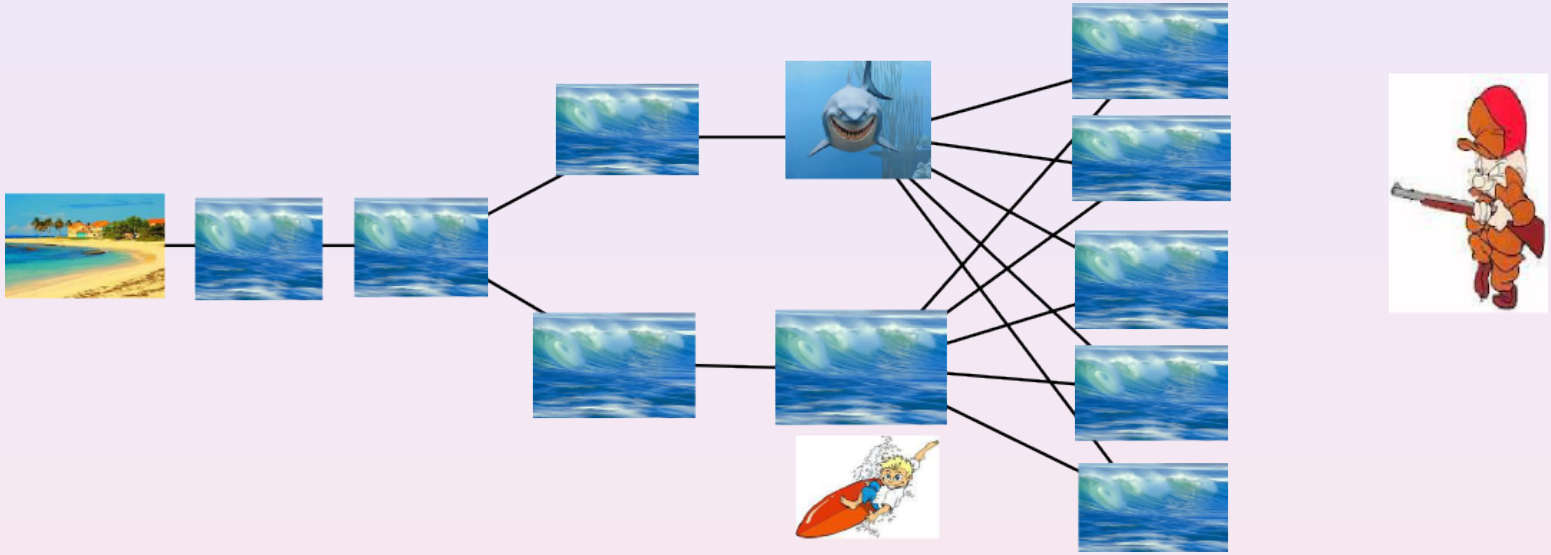

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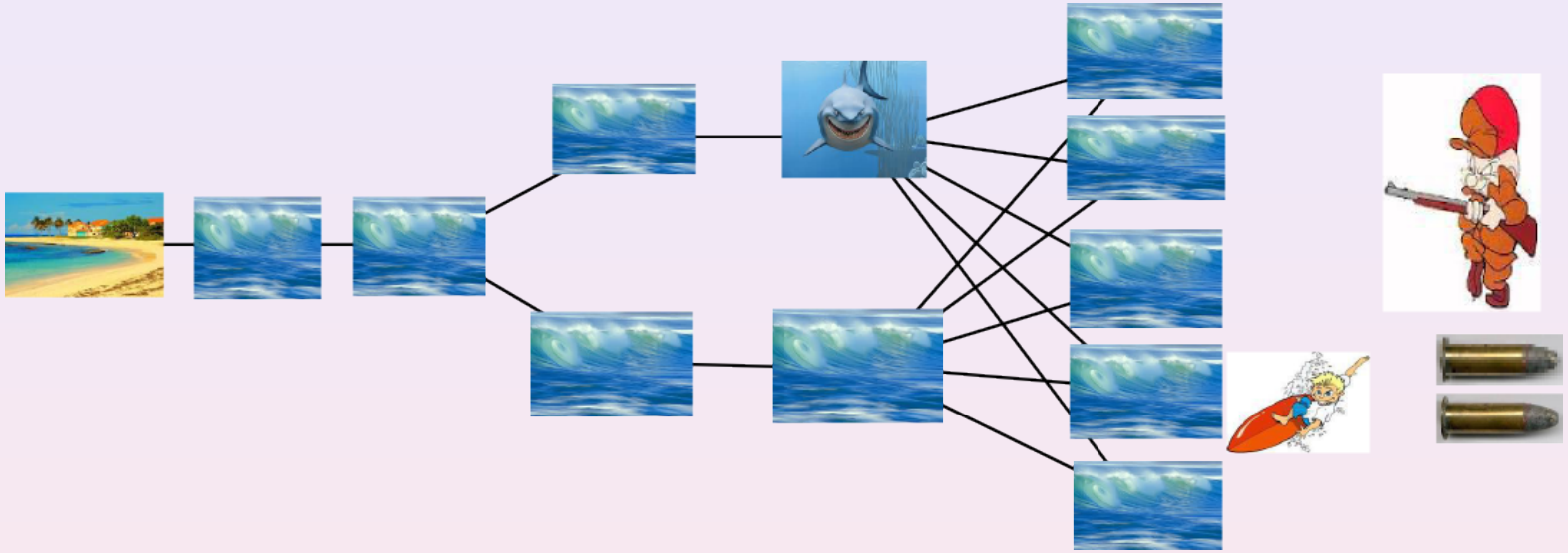

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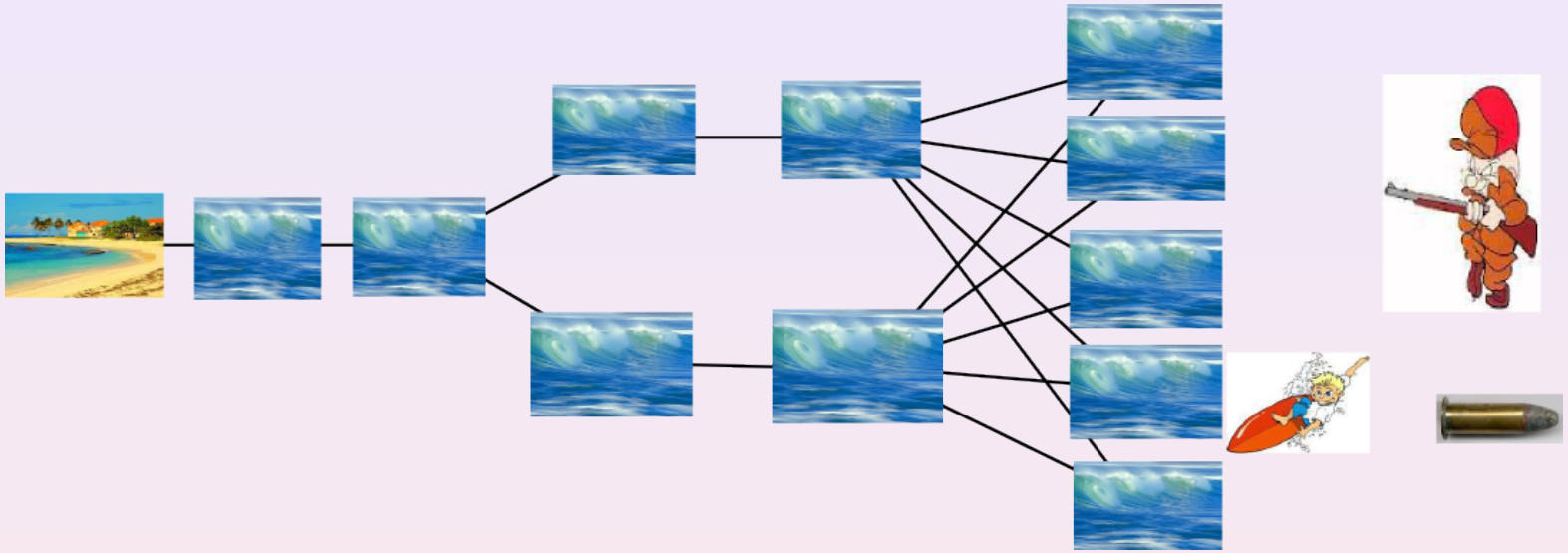
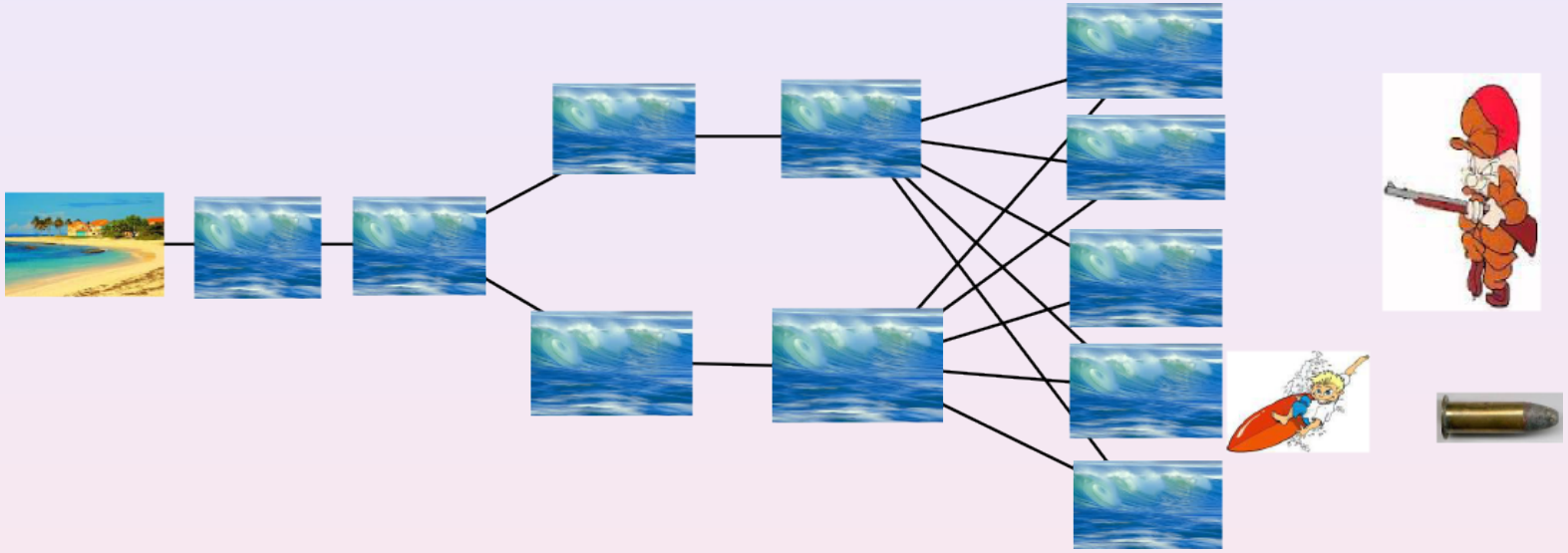


Guard uses (all) his bullets, **then** Surfer moves

All nodes safe: Victory against this trajectory of the Surfer

First Example

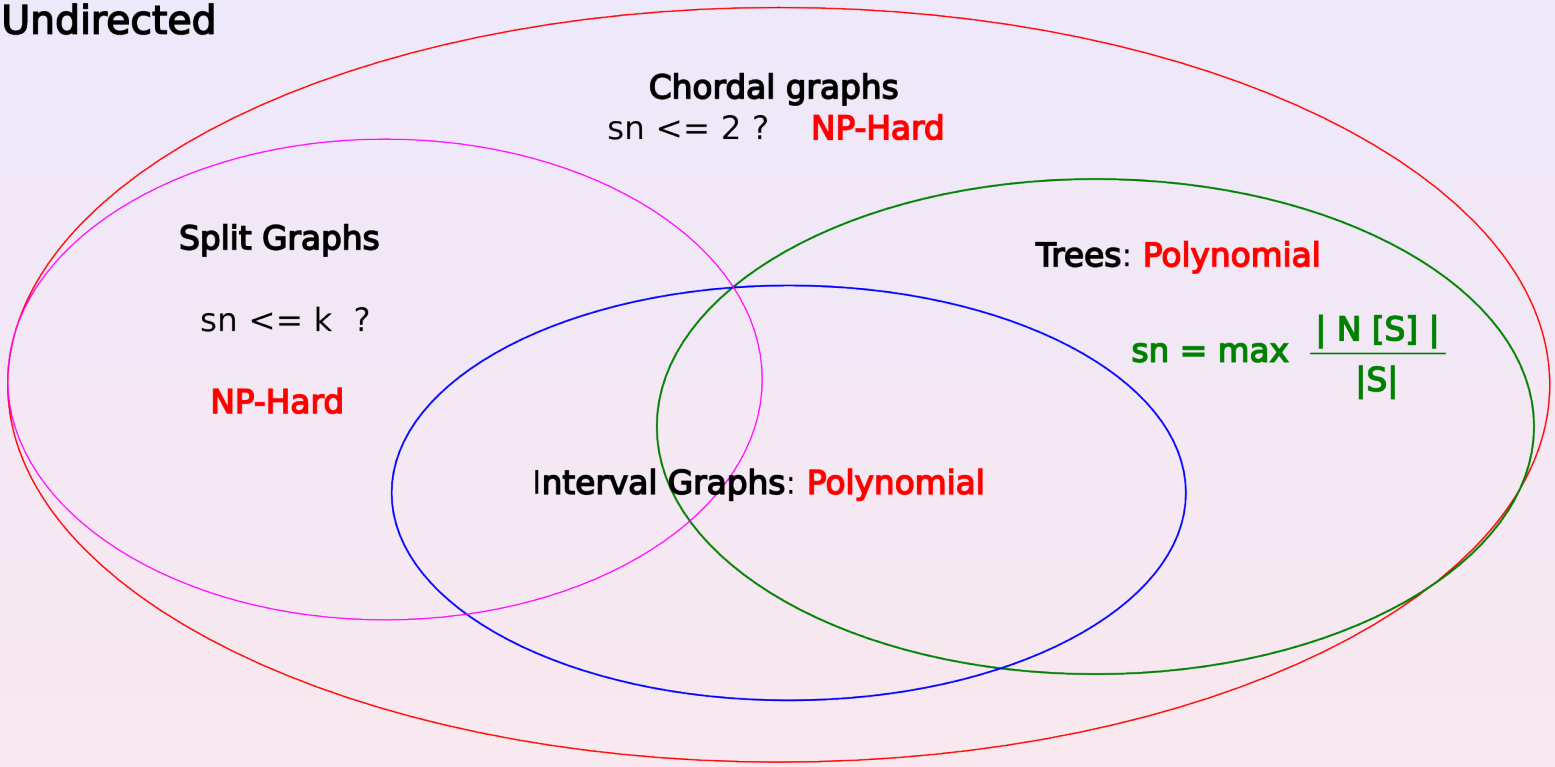


In this example, all Surfer's trajectory similar (by symmetry)

Victory whatever Surfer's trajectory $\Rightarrow sn(G, v_0) = 2$

Results: Complexity, Algorithms and Combinatoric

Undirected



Directed

DAGs: sn <=4 ? **P-SPACE-Complete**

DAGs: sn <=2 ? **NP-Hard**

General

(out)-**degree (v0)** $\leq$ **sn** $\leq$ max (degree (v0) / **max degree** -1 / max out-degree)

$O(2^n)$ exact algorithm

Positive Results

Combinatorial characterization in Trees

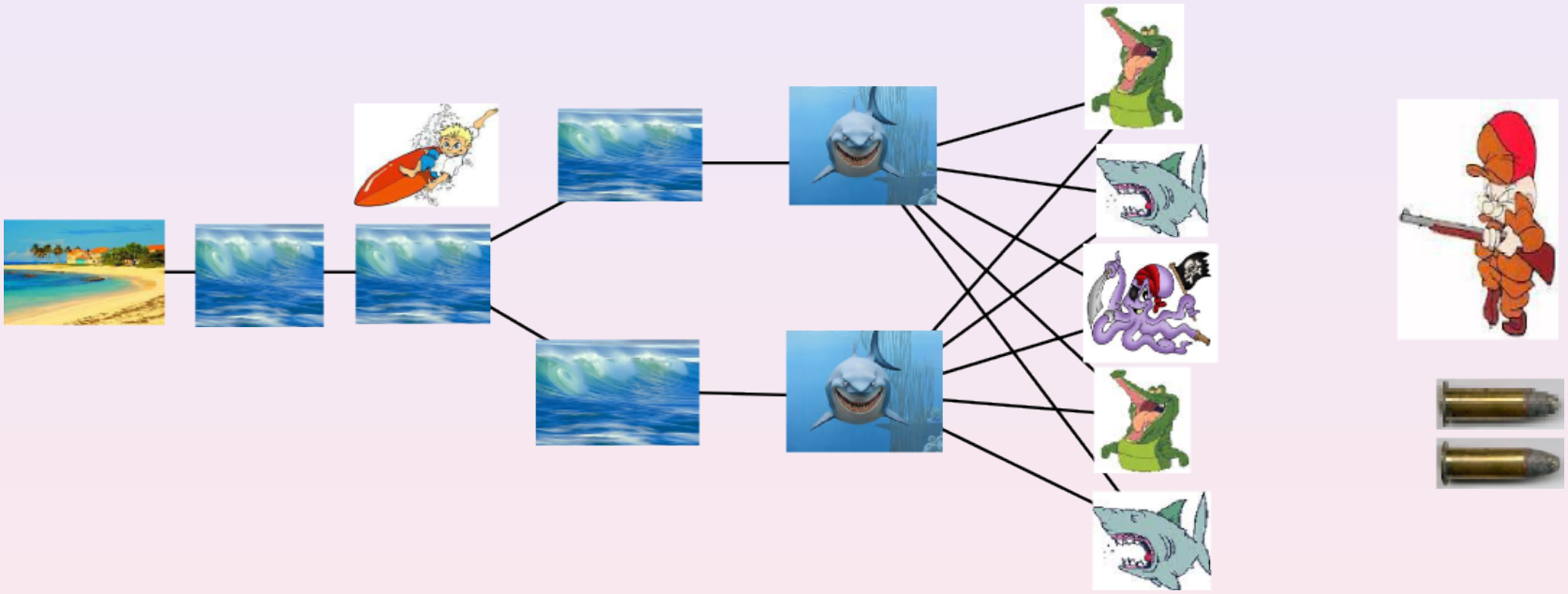
For any tree T, $v_0 \in V(T)$, $sn(T, v_0) = \max\lceil \frac{|N[S]|-1}{|S|} \rceil$, taken for any subtree S containing v_0.

Exact Algorithms

- $O(2^n)$ algorithm in n-node graphs;
- $sn(T, v_0)$ can be computed in time $O(n \log n)$ in any n-node tree T and for any $v_0 \in V(T)$;
- $sn(G, v_0)$ can be computed in time $O(n \cdot \Delta^3)$ in any n-node interval graph G with maximum degree Δ and for any $v_0 \in V(T)$.

Further Work: Connected Variant

Constraint: safe vertices must induce a connected subgraph



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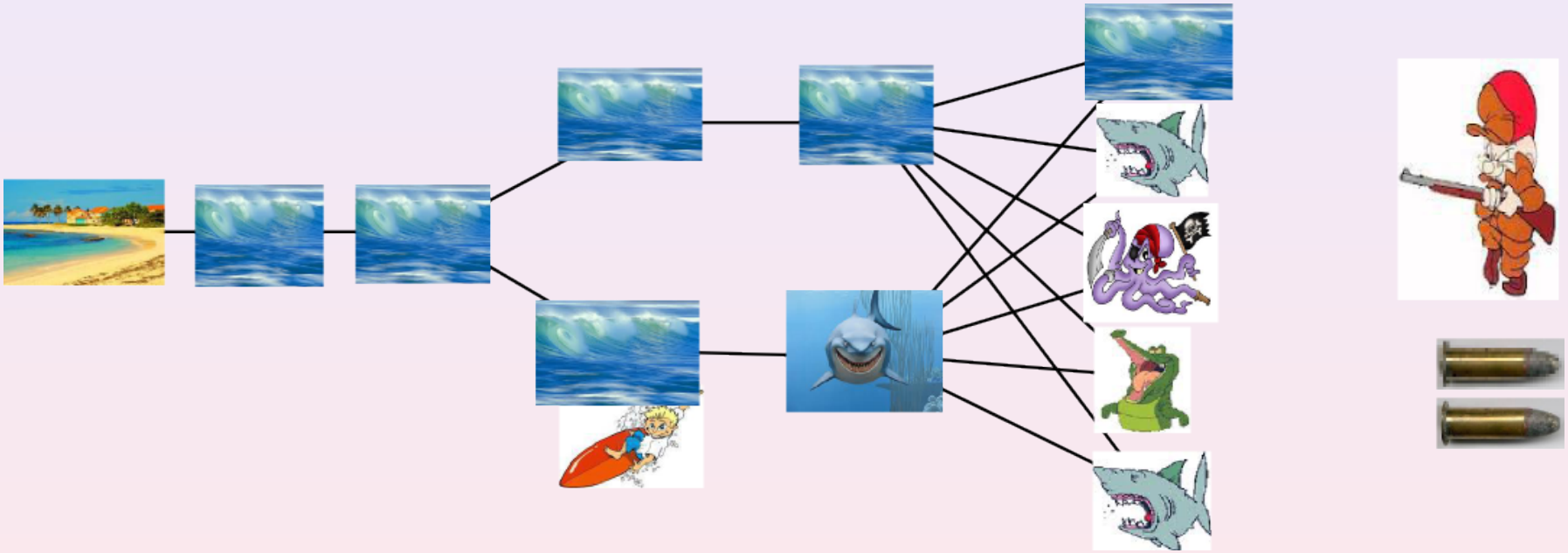

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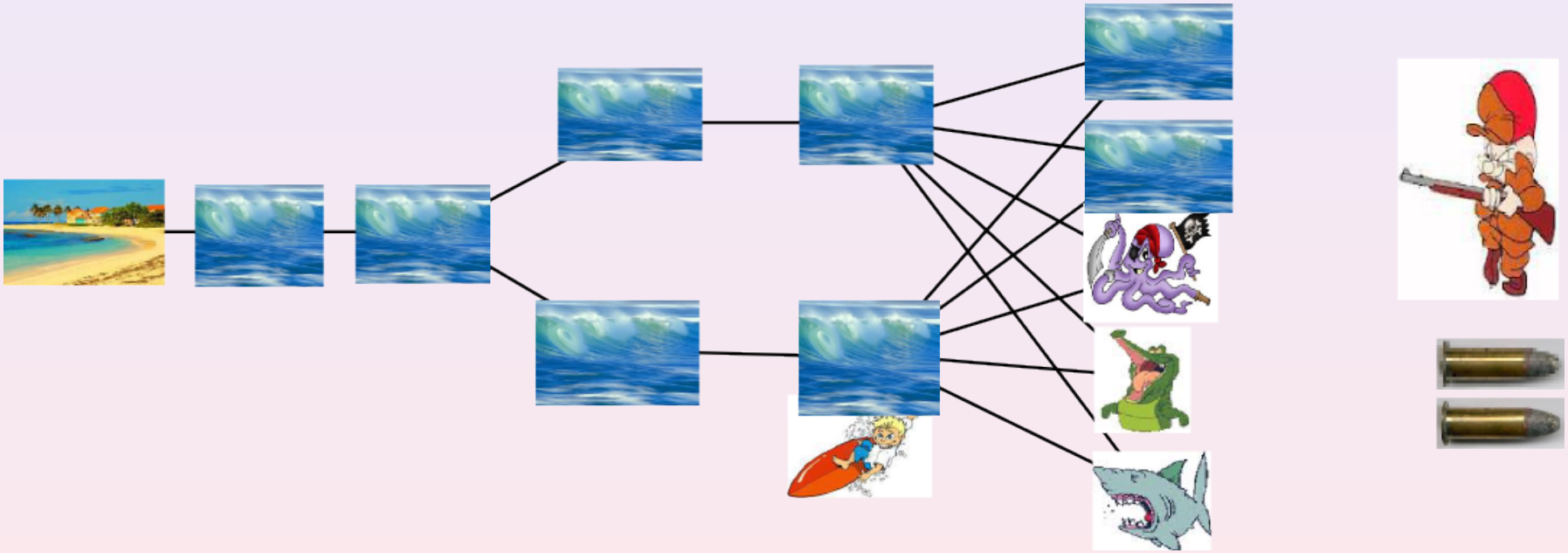

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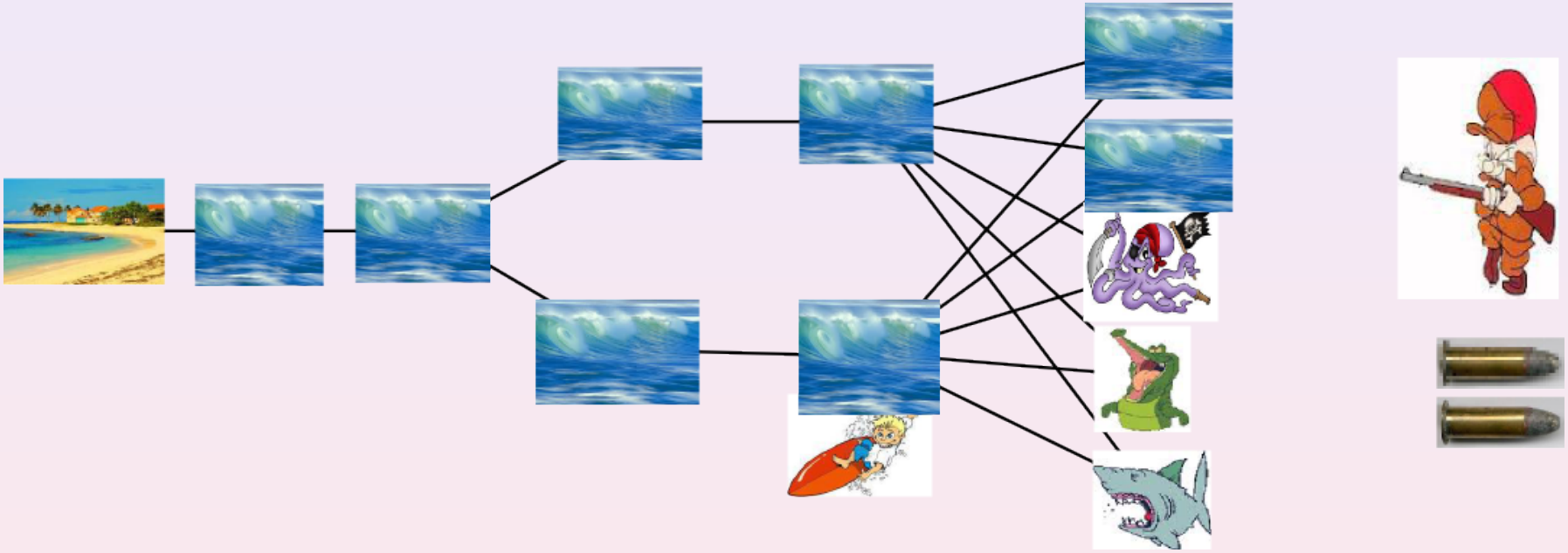

Connectivity costs:

connected-$sn(G, v_0) = 3 > sn(G, v_0) = 2$

All previous results hold for the connected variant

Further Work: Connected Variant

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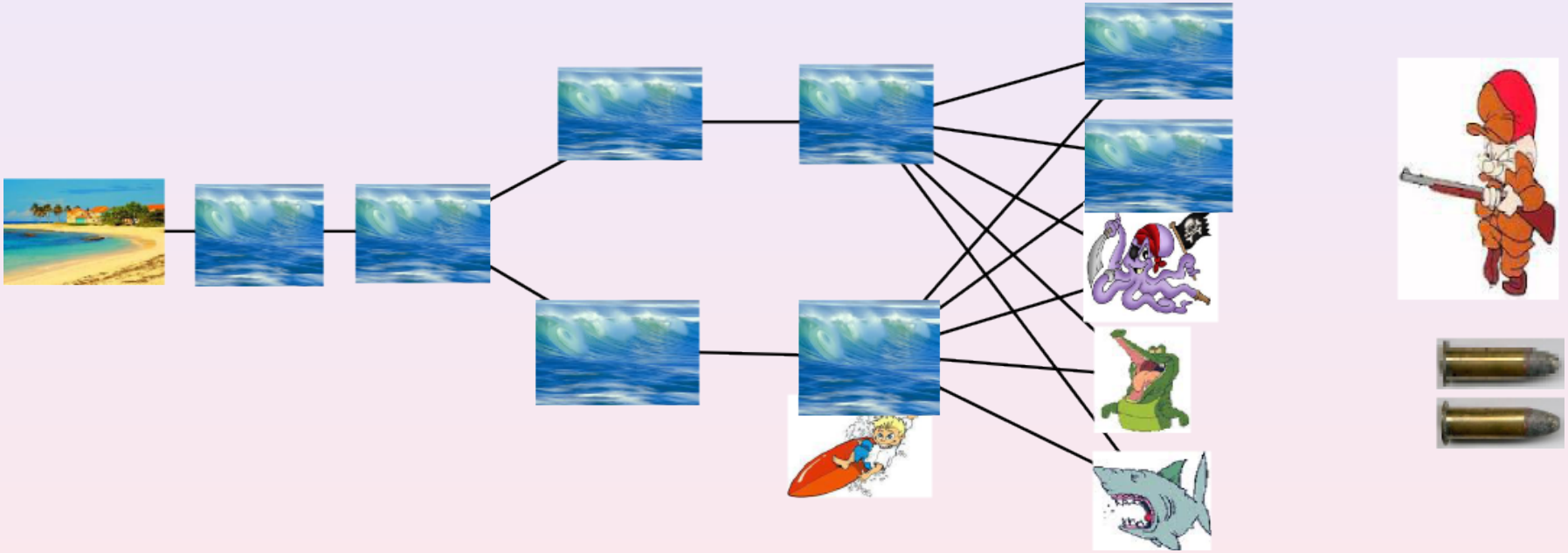

Connectivity costs: connected-$sn(G, v_0) = 3 = sn(G, v_0)+1$

$\exists$? G and v_0 such that $c\text{-}sn(G, v_0) \geq sn(G, v_0) + 2$????

Open Questions

- complexity in bounded degree graphs?
  (polynomial if $\Delta \leq 3$)
- complexity in bounded treewidth graphs?
- $\exists ? c < 2$ and $O(c^n)$ algorithm in n-node graphs?
- $sn(G, v_0) = \max \lceil \frac{|N[S]|-1}{|S|} \rceil$, taken for any connected subgraph S containing v_0?
- cost of connectivity? $\frac{connected\ \ sn}{sn} \leq cte$?
  $\exists ?$ G and v_0 such that $c\text{-}sn(G, v_0) \geq sn(G, v_0) + 2$????
- ...

Thank you for your attention[1]

[1] No seafood... no animal has been hurt when preparing this talk.