American Basket Option Pricing on a multi GPU Cluster

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ABSTRACT

This article presents a multi GPU adaptation of a specific Monte Carlo and classification based method for pricing American basket options, due to Picazo [1]. The first part relates how to combine fine and coarse grained parallelization to price American basket options. In order to benefit from different GPU devices, a dynamic strategy of kernel calibration is proposed, and contributes to the dynamic split of GPU calculus. Our implementation achieves a realistic size option pricing in less than one hour against more than 7 for a multi CPU cluster-based solution. After an analysis of possible bottleneck effects, we distribute the sequential bottleneck due to the training phase. For this we rely upon Random Forests classification method which is suited to parallelization. We show through tests that the obtained parallel pricing algorithm is scalable.

Keywords

Distributed and parallel computing; Grid; Cloud; GPU; OpenCL; machine learning; mathematical finance; option pricing

1. INTRODUCTION: GPUS IN FINANCE

Many financial measures require huge resources to be computed in acceptable time. “Acceptable” is related to specific context: Value at Risk may be performed to forecast the maximum loss of a given portfolio at a two weeks horizon whereas computing hedging portfolios is often dedicated to intraday operations. The difficulty not necessarily depends on computation methods but on engaged financial instruments. For instance, a portfolio can be composed of several financial instruments and which can vary from a simple asset to option on several assets. In this paper, we focus on pricing one instrument: an American option, which for being realistic, is based upon a basket of up to 40 assets. The difficulty to price an American option is to predict an exercise frontier to consider all possible exercises times until the maturity date. Furthermore, model parameters such as discretization, number of simulations, complicate computation time. Our previous work [2] highlights the necessity to target GPU rather than distributed CPUs to provide the same performance with alleviated resources. By this way we price complex American basket options, in the same order of time than a 64 cores cluster of CPU implementation [3], which is around 8-9 hours. However a single GPU is limited for such complex problems. Targeting cluster of GPUs is the natural following step to benefit of both aggregated memory of their host CPUs, and high parallelism of SIMT architectures.

The paper makes the following contributions. First we propose a two-level CPU/GPU parallelization of the Picazo pricing algorithm. Then we perform a dynamic load balancing strategy to exploit heterogeneous multi GPU clusters. Finally we show how to integrate Random Forests [4] in our pricing engine to make it better scale: we propose a distribution of the classifier training and a GPU based implementation of the classification.

We will describe in section 2 a multi GPU implementation to price such financial instruments through Picazo method. At a coarse-grained level, we will focus on the parallelism orchestration across the cluster nodes. Then we will explain our fast dynamic strategy to calibrate kernel parameters in parallel, and use them in a load balancing solution for heterogeneous multi-GPU clusters. Finally at a fine-grained level, we will detail the SIMT oriented implementation. In section 3, we will expose our strategy to tackle the bottleneck effect of the sequential learning phase, by using Random Forests rather than AdaBoost or SVM (Support Vector Machine). We are able to parallelize it over CPU nodes, each node training a small Random Forest. Doing so, we obtain a fully parallel pricing algorithm. The two approaches will be compared through several tests.

2. A GPU CLUSTER BASED OPTION PRICING ENGINE

Here we describe a Java implementation of the selected pricing method due to Picazo. We use the JOCL [5] and OpenCL [6] libraries to exploit distributed GPUs. Through a dynamic strategy we recognize GPUs over nodes and adapt kernel parameters before load balancing main computation phases. Tests reveal bottleneck effect due to building phases of classifiers and necessity to parallelize them as exposed in section 3.

2.1 Picazo pricing algorithm

High dimensional American basket call/put option is a contract allowing the owner to buy/sell at a specified strike price *K*, a possibly high size (e.g. 40) set of underlying assets S^i_t (numbered *i*) at any time *t* until a maturity date *T*. So a call option owner expects the basket of assets price on the market to raise over strike, as in this case and according to the option contract, the owner will have to spend less money to buy these assets, i.e. to exercise the option. There is no analytic solution to price this financial instrument but Monte Carlo (MC) methods, based on the law of large number and central limit theorem, allow a simplified approach for high complex problems, reaching good accuracy in reasonable time. Consider $S_t^{(s)}$ as independent price trajectories of the basket of assets following geometric Brownian motion processes, $\Psi\ (f\ (S_t^{(s)}),\ t)$ as the option pay-off, *f* as the arithmetic or geometric mean function, *r* as the risk free rate. European option price *V* at time zero can be estimated, through a number of MC simulations *nbMC*, as follows

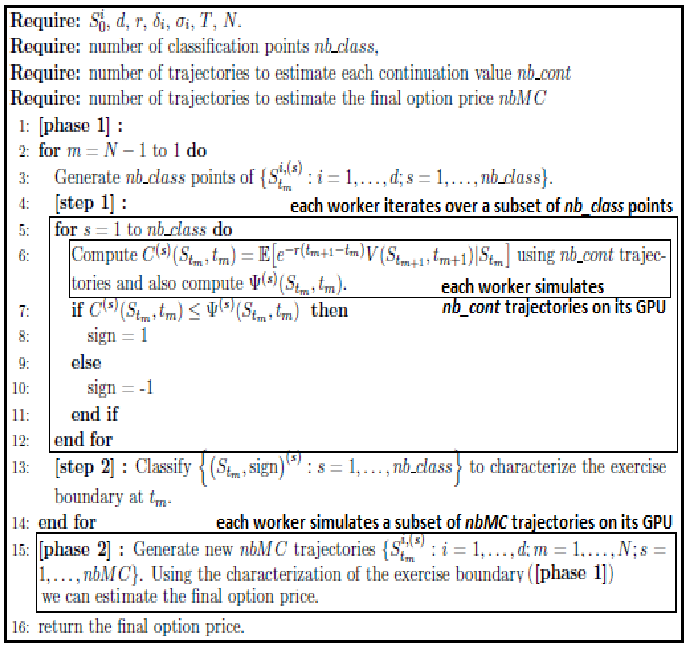
$$V(S_0,0) \approx \frac{1}{nbMC} \sum_{s=1}^{nbMC} e^{-rt} \Psi\left(f\left(S_t^{(S)}\right), t \in [0,T]\right)$$

As opposed to European contracts, American ones offer more flexibility for the exercise: it can be performed at any time until the maturity date, and this over all discrete times. This is reflected in the mathematical definition below

$$V(S_T,T) = \Psi(f(S_T),T)$$
$$V(S_{t_m},t_m) = \max\left(\Psi\left(f(S_{t_m}),t_m\right), \mathrm{E}\left[e^{-r(t_{m+1}-t_m)} V\left(S_{t_{m+1}},t_{m+1}\right) \mid S_{t_m}\right]\right)$$

The formula $\mathrm{E}[\,e^{-r(t_{m+1}-t_m)} V\left(S_{t_{m+1}},t_{m+1}\right) \mid S_{t_m}\,]$ defines the continuation value at time t_m, noted C in Figure 1, i.e. the forecasted option price at t_{m+1}. The option owner will keep it, if its forecasted price is over the benefit of immediately exercising it, i.e. the payoff.

Picazo method exposes an efficient way to define continuation or exercise regions, separated by a frontier named exercise boundary, by combining a machine learning technique with MC methods. The algorithm is shown in Figure 1. We note d the basket size, δi and σi respectively the dividends and volatilities of the $i = 1..d$ underlying assets, N the discrete time number.



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Require: $S_0^i$, $d$, $r$, $\delta_i$, $\sigma_i$, $T$, $N$.
Require: number of classification points $nb\_class$,
Require: number of trajectories to estimate each continuation value $nb\_cont$
Require: number of trajectories to estimate the final option price $nbMC$
 1: [phase 1] :
 2: for $m = N - 1$ to 1 do
 3:    Generate $nb\_class$ points of $\{S_{t_m}^{i,(s)} : i = 1, \ldots, d; s = 1, \ldots, nb\_class\}$.
 4:    [step 1] :                    each worker iterates over a subset of nb_class points
 5:    for $s = 1$ to $nb\_class$ do
 6:       Compute $C^{(s)}(S_{t_m}, t_m) = \mathbb{E}\left[e^{-r(t_{m+1}-t_m)} V(S_{t_{m+1}}, t_{m+1}) | S_{t_m}\right]$ using $nb\_cont$ trajec-
          tories and also compute $\Psi^{(s)}(S_{t_m}, t_m)$.          each worker simulates
 7:       if $C^{(s)}(S_{t_m}, t_m) \leq \Psi^{(s)}(S_{t_m}, t_m)$ then       nb_cont trajectories on its GPU
 8:          sign = 1
 9:       else
10:          sign = -1
11:       end if
12:    end for
13:    [step 2] : Classify $\left\{(S_{t_m}, \text{sign})^{(s)} : s = 1, \ldots, nb\_class\right\}$ to characterize the exercise
       boundary at $t_m$.
14: end for                   each worker simulates a subset of nbMC trajectories on its GPU
15: [phase 2] : Generate new $nbMC$ trajectories $\{S_{t_m}^{i,(s)} : i = 1, \ldots, d; m = 1, \ldots, N; s =$
    $1, \ldots, nbMC\}$. Using the characterization of the exercise boundary ([phase 1])
    we can estimate the final option price.
16: return the final option price.
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Figure 1. Picazo pricing method and the two parallelization levels (in rectangles)

The key pricing method strategy is to call a specific classifier per discrete time during the *nbMC* simulations of the final pricing phase [phase 2], to decide if current simulation must be stopped or not, i.e. if simulated prices reach or not an exercise region. To achieve this, we need during a previous phase [phase 1], to train each classifier [step 2] over *nb_class* training instances. Each training instance is composed of simulated underlying asset prices and a boolean, depending on if the option payoff is over or not an estimation of the continuation value. Each continuation value requires *nb_cont* MC simulations [step 1]. Consequently there are *nb_cont* MC simulations needed per training instance.

2.2 Distribution orchestration for coarse-grained parallelism

Our parallel version of the Picazo pricing algorithm introduces two degrees of parallelism as Figure 2 depicts. The first level follows a master-slave approach. We use the Java ProActive library [7] which offers an abstraction of distribution management by introducing the concept of Active Object. By this way, during the detection phase described in part I of Figure 2, whose role is to dynamically detect what are the available computing resources, we deploy as many active objects as cluster nodes and discover the number of residing CPU cores and GPUs per node. In our pricing strategy, more than workers, we require a merger to gather intermediate results. Finally during this initialization phase illustrated in part II, we allocate the merger active object on the node with the fewer GPUs and there will be as many workers active objects as GPUs. Running multiple workers to exploit GPUs on a single node will not significantly impact performance because workers jobs are GPU intensive.

Part III details the orchestration of the training instances computation for each classifier. To estimate a continuation value per training instance, a worker launches *nb_cont* MC simulations on its GPU. The merger recovers all training instances from workers to train a new classifier. This classifier will be used during MC simulations of final pricing phase, but also during MC simulations of continuation values. Therefore the merger broadcasts the trained classifier to all workers, at each time loop iteration. Once all classifiers are trained, each worker is distributed a subset of MC simulations to estimate the final price as part IV depicts.

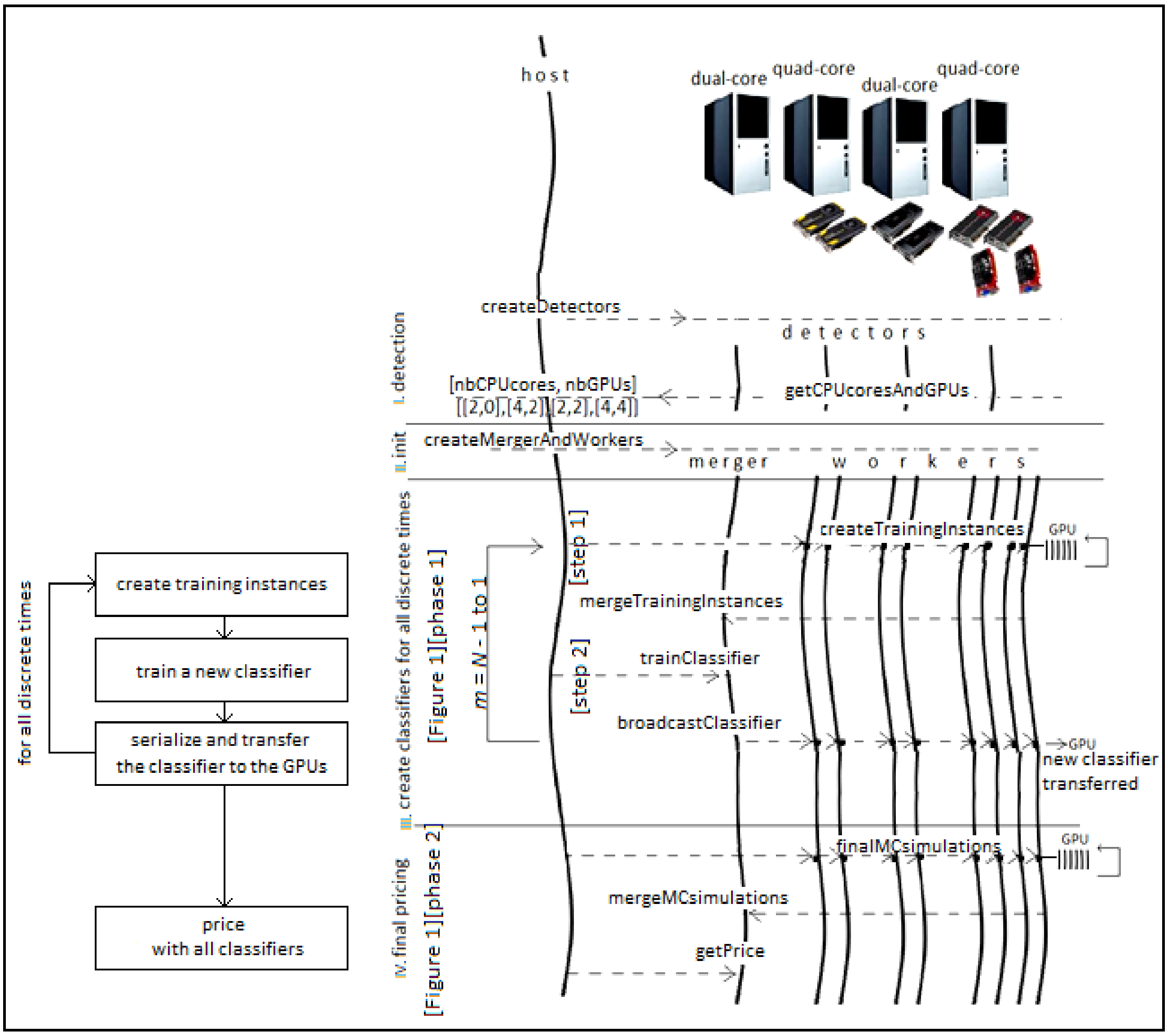

Figure 2. Parallelism orchestration of the Picazo pricing method

2.3 Kernel parameters calibration and load balancing

2.3.1 Dynamic kernel parameters calibration

Targeting GPU programming implies to cope with a wide variety of GPUs. To ensure high multiprocessor occupancies for each worker, we must calibrate kernel parameters, i.e. work-group size and global size. For this, we provide a Java class which imitates the CUDA occupancy spreadsheet. Before starting the first step of the pricing algorithm, each worker, in charge of one GPU device, computes theoretical multiprocessor occupancies for all possible work-group sizes: from the warp size up to the maximal work-group size allowed, increased by warp size. As required in the spreadsheet, some device specifications are required: each worker detects shared memory amount per multiprocessor, maximal work-group size, generates the program compilation log to parse used registers. Different kernel configurations can describe same multiprocessor occupancies, for instance 4 work-groups of 32 threads against 2 of 64. In such case, our program will keep the one offering more work-groups, to reduce waiting time between them (as each work-group would be given a smaller simulations number to perform). As intermediate calculus to deduce the multiprocessor occupancy, the theoretical active work-group number by multiprocessor is estimated, and will be reused to fix the total threads number to: work-group size multiplied by number of active work-group per multiprocessor multiplied by number of multiprocessors on the device. This strategy allows a fast estimation of kernel parameters for each of the detected GPUs to ensure a high multiprocessor occupancy without launching any preliminary fake pricing calculations.

2.3.2 GPU cluster load balancing

The estimated total number of threads per GPU is used as a criterion to load balance the calculus on the GPU cluster. Indeed, a high-end graphic card capable of launching twice as many threads than another will be given twice as many MC simulations. We set the subset of *nb_class* and *nbMC* for a given worker *W* as follows

$$nb\_class_W = \frac{totalGPUthreads_W}{\sum_{ALL\ WORKERS\ P} totalGPUthreads_P} \times nb\_class$$

[TABLE I] highlights our dynamic split strategy over a heterogeneous GPU-based cluster. Grid5000 [8] provides sufficient CPU and GPU resources to perform intensive tests. On Grid5000, each cluster node can directly interact with other cluster nodes, i.e. without having to traverse a cluster front-end node. Thus, virtually all Grid5000 nodes form a single heterogeneous cluster. Each node of the Grenoble Adonis cluster has 2 E5520 CPUs and 2 NVIDIA Tesla S1070. The Lille Chirloute cluster includes 3 Tesla M2050 and each node has 2 E5620 CPUs. Each node of the Lyon Sagittaire cluster holds 2 AMD Opteron 250. These sites are connected with 10Gbit/s optical fibers. We launch respectively 10 and 3 workers on the Adonis and Chirloute clusters. The merger is executed on a single node from the Sagittaire cluster. The dynamic kernel parameters calibration is activated on both tests, and estimates as best parameters <total threads number, work-group size>, for respectively Tesla S1070 and Tesla M2050, <5760, 64> and <7168, 64>. We disable the dynamic split in the first test by fixing the same subset of *nb_class* and *nbMC* for all GPUs. On the contrary, second column features better performances due to the use of the simple yet efficient load balancing strategy.

Table 1. Comparison of two algorithm phases execution times (in seconds) with Adaboost classifier. Geometric average American call option, d=40, K=100, N=50, T=1, r=3%, δ_i=5%, σ_i =40%, *nb_class*=5000, *nb_cont*=10^5, *nbMC*=2x10^6, 150 boosting iterations/decision stumps

	no dynamic calibrated split	dynamic calibrated split
Total duration of training instances computations Figure 1 [phase 1][step 1]	3502,2s	3320,4s
Final pricing time Figure 1 [phase 2]	3,7s	3,5s

2.4 Fine-grained parallelism with OpenCL

Each worker computes a subset of *nb_class* training instances and requires for each to estimate a continuation value through *nb_cont* MC simulations, c.f. Figure 1 line 6. MC simulations are launched through an OpenCL kernel function. There are as many parallel simulations on the GPU as threads iterating to provide the *nb_cont* simulations. Difficulty of pricing American option is the random length of simulations: a classifier can predict the exercise region is reached at any time before the maturity date. Consequently we cannot forecast the required random variables number and we use the GPU based Random Number Generator MWC64X [9] to generate at runtime only required variables. At each discrete time of a single simulation, a thread generates as many uniform random variables as underlying assets, performs the Box Muller transformation to retrieve the Gaussian values, simulates the underlying assets prices, call the specific classifier, and finally computes the actualized payoff, adds it to a variable allocated in a register, and start a new simulation.

This random stopping time leads to some threads finishing earlier their simulations than others. A "warp", for NVIDIA architecture or "wavefront" for AMD, is the smallest quantity of threads that are issued with a SIMT instruction. Because threads of the same warp cannot perform at the same time different instructions, some of them will block at the main loop condition if they perform short simulations (as dictated by the classifier call). These unwanted synchronizations lead to low occupancy of the multiprocessor. That's why we cannot simply iterate over the same fixed number of steps for all threads when computing the *nb_cont* simulations. Consequently each thread computes after *nbStepsBeforeReduction* time steps and through intermediate reductions (parallel sums), how many MC simulations have been achieved (see further details in [2]). This is repeated by each thread until at least the total number of MC simulations needed for getting a continuation value has been achieved.

We kept in mind all recommendations of the GPU device programming guide to avoid possible performance losses. In particular, (1) coalesced access allow threads to get asset prices from global memory in few instructions, (2) we employ constant cached memory to store read-only values such as volatilities or dividends, and (3) perform the intermediate parallel reductions in shared memory. Specific tests revealed that even a high number of reductions for summing do not impact global execution time.

Classifiers used during Monte Carlo simulations are previously created and trained on the CPU by the merger with the Weka library [10]. Since OpenCL does not allow advanced library call, each worker needs to work with a serialized version of the Weka Classifier object obtained at kernel launches. The two possible classifiers from Weka we experimented with, AdaBoost and SVM, part of Weka library were slightly modified to retrieve all private members of Weka object and only cope with basic structures in OpenCL. Then all of them are transferred to the global memory to imitate the Weka classify call on the GPU. At the end, we can afford to imitate the original Weka behavior with basic structures, and store as many classifiers as discrete times in arrays. During a kernel execution, threads work with position indexes to access in parallel different classifiers to predict the stopping times.

2.5 Experimental tests

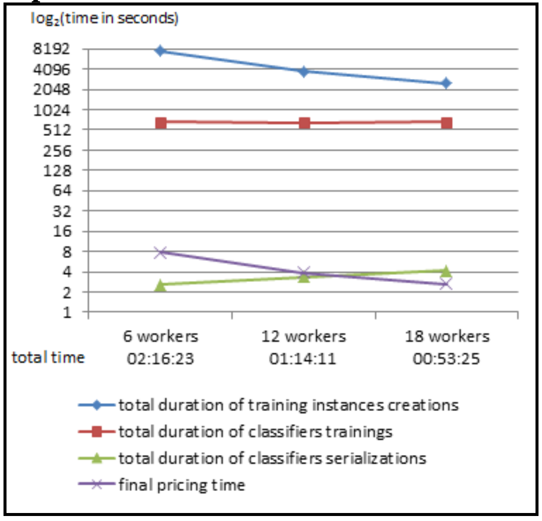

Figure 3. Comparison of algorithm phases execution times with AdaBoost classifiers over workers numbers. The pricing parameters are the same than previously (see TABLE 1)

Figure 3 depicts execution times on the Adonis cluster, of parts III and IV as illustrated in Figure 2. Parts I and II are not specified here due to their small execution times, and possibility to reuse the resulting active objects deployment for multiple program runs. The option price of a single run is around 0.64108 ± 0.0015, which is a reference price according to [3]. Times of training instances creations and final pricing phases include calculus and broadcast/merge operations from/to the merger. We fall below 1 hour when performing tests over 18 workers (i.e. GPUs). Tests reveal linear dependence of workers numbers with the computation part of each phase, but managing more workers complicates broadcast/merge operations and slowdowns their respective overall time. Because the merger sequentially trains each classifier through the Weka library and does not solicit workers, the implementation is not scalable. When increasing workers number, the training instances computation time decreases, and consequently tends to vanish in comparison to the constant time of the classifiers training. AdaBoost and SVM are based upon iterative algorithms during the learning phase, which are thus not parallelizable. The idea is to choose an alternate classification method, i.e. the Random Forests method, whose learning phase could be parallelized.

3. RANDOM FORESTS INTEGRATION FOR FAST CLASSIFIER TRAINING

We focus here on the integration of Random Forests in our pricing engine. Experimental tests will illustrate the scalability of our implementation, thanks to the parallelization of the learning phase.

3.1 Training Random Forests over cluster nodes

When distributing the Random Forests trainings, we decided to preserve the Weka behavior: the idea was to train in parallel small Random Forests with the same *buildClassifier()* call as it was for a single larger one. The Weka library was slightly modified so that the original Random Forest and the one obtained after merging all smaller forests built by workers provide strictly identical classification measures. By this way, we can train in parallel subsets of a Random Forest over cluster nodes (Figure 4). As complementary optimization, we decided to exploit the last Weka library version affording parallelization over CPU cores. For this, only one worker per node is in charge of a sub classifier to benefit of all CPU cores for the training.

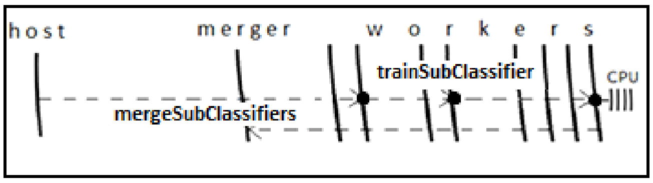

Figure 4. Two-level parallelization of the Random Forest training. Each subClassifier is trained over the detected CPU cores through the Weka library

We set the Weka parallelization degree of each node with the number of detected CPU cores. A simple load balancing mechanism affords each worker W to build a specific subset $nbTrees_W$ of the total number $nbTrees_{CLASSIFIER}$ of trees of a Random Forest, such as

$$nbTrees_W = \frac{nbCPUcores_W}{\sum_{ALL\ CPUs\ P} nbCPUcores_P} \times nbTrees_{CLASSIFIER}$$

For the following tests (Figure 5), we will disable this optimization, in order to highlight the benefit of the training distribution over cluster nodes. Once all workers have finished, the merger retrieves all sub classifiers, merges them and broadcasts the trained global Random Forest to all workers that will use them, as explained in the following subsection.

3.2 Parallel Random Forests classifications on GPU Units

As for AdaBoost or SVM, a Random Forest classifier per discrete time must be serialized by the worker, and transferred to the GPU global memory, in order to predict the exercise boundary at this time, during the simulations, c.f. Figure 1 line 6 and 15. The difficulty comes from the storage of the trees that are indeed incomplete. Only an experimental solution is provided by the JOCL team, to transfer tree structures to the device, so we had to imagine one solution that fits our needs. To cope with sparse tree storage, we work with compressed arrays representation. Once workers are broadcasted the merged global Random Forest, they parse all trees, retrieve and queue node information in specific arrays for the compression. Indeed considering all trees, there is an array for split values, another one for attribute indexes. We store indexes of tree roots in a dedicated array. Finally, we work with a left children indexes array and a right children indexes array, to imitate tree parsing when classifying instances in OpenCL. As for AdaBoost and SVM, we queue all the classifier representations in the same specific arrays to be accessed for each discrete time, complicating indexes management.

3.3 Experimental tests

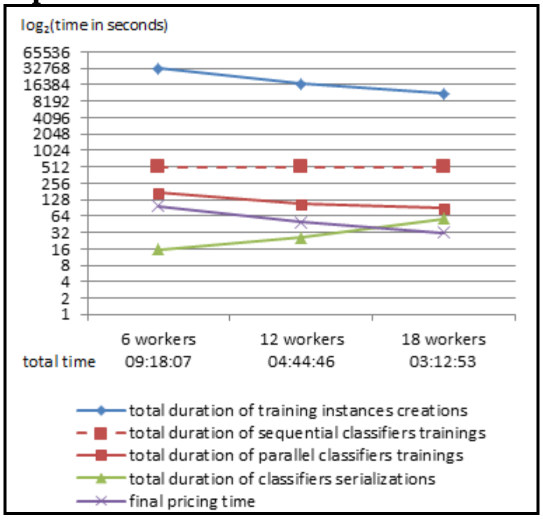

Figure 5. Comparison of algorithm phases execution times with Random Forest classifiers of 150 unlimited depth trees, over workers numbers. The pricing parameters are the same than previously and global execution times correspond to the situation where training of classifiers is distributed

The option price of a single run is around 0.63651 ± 0.0016 which is in line with the expected value. Tests are executed on the Adonis cluster, and working with such Random Forest parameters (150 trees) affords to reach the same order of confidence interval than AdaBoost tests presented in 2.5. The training instances creations (~3h13min with 18 GPUs) require more time than with AdaBoost (~53min with 18 GPUs) due to the cost of Random Forest classification. Indeed, to classify an instance, a GPU thread will take more time to parse the 150 unlimited depth trees, rather than the 150 one-level decision trees of the AdaBoost classifier. Conversely, we take advantage of the distributed CPUs during the classifiers trainings, to let the algorithm better scale.

4. RELATED WORK

Regarding the fine tuning for GPU configuration, Grauer and Cavazos present an auto-tuning implementation in [11] to produce the configuration that minimizes local memory accesses against registers and shared memory. Since they play with data partition sizes via changing the maximum occupancy, the strategy allows finest kernel parameters calibration but requires more calculus. Raphael Y. de Camargo [12] describes a load distribution algorithm for heterogeneous GPU cluster to reduce the total execution time of his neuronal network simulator. To estimate each quantity of data input assigned to each GPU, he formalizes the problem to a linear system of equations. Some variables in the system represent the execution time functions of each kernel on each GPU over input sizes. This requires each kernel to be executed a few times on each GPU, with different input sizes to get the interpolation function. This can spend a lot of time and become inconvenient in case of several types of GPUs, and compute-intensive kernels. Our dynamic strategy to calibrate kernel parameters, with no preliminary simulations, allows a fast comparison of the parallelism degrees of each GPU for a given kernel. Although it does not consider the program behavior with all implementation details such as branch divergences, non-coalesced memory accesses, our approach is not closed to a specific problem and is more generic.

In [13] is presented CudaRF, a CUDA-based implementation of Random Forests. During the training phase, each thread constructs a tree of the forest. It could be used within our ProActive-based distributed training phase so that huge Random Forests could benefit of a dual-level of parallelism offered at both worker and GPU sides. However, having a GPU thread handles one single tree of the forest during the classification phase, is not suited to our algorithm. We cannot afford to exploit at a specific time the entire device for a single instance, as our implementation exploits SIMT architecture to call simultaneously possibly different classifiers, depending on the discrete time reached by each thread.

5. CONCLUSION

Our works propose a multi GPU based implementation of Picazo method to price complex American options, allowing pricing time to fall below 1 hour with 18 GPUs (against almost 10 hours on a 64 cores cluster). To fully exploit the dual-level of parallelism of such architecture, we distribute the training instances computation over the cluster nodes and solicit the SIMT architecture of each detected device to parallelize all the Monte Carlo simulations of the algorithm. Our fast parallel strategy to estimate kernel parameters of devices can be adapted to a wide range of GPUs to target from any cluster: as such it allows us to easily involve heterogeneous GPUs for solving the pricing in parallel. The integration of Random Forests, tackles the sequential bottleneck effect due to the classifiers trainings by parallelizing them, but slowdown the training instances creations due the expensive classification. Working with more GPUs (100+) than in our experiments, would further decrease the computation operations but increase broadcast/merge operations, impacting the overall pricing time. Thus, to face this only remaining bottleneck effect, we could implement a broadcast ring, and merge operations could be parallelized along a tree of workers.

It would be exiting to take advantage of high end CPUs (Xeon Phi) if available on the cluster, to perform part of the Monte Carlo simulations. By relying on OpenCL in our pricing engine, it already abstracts the hardware architecture. The only point to consider in order to take advantage of such hybrid hardware environment is to extend our dynamic calibration and load balancing strategy. A natural exploitation of our work is to evaluate a portfolio of such complex assets, which is an ongoing task.

6. ACKNOWLEDGMENTS

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