Parameter determination for surface acoustic wave filters*

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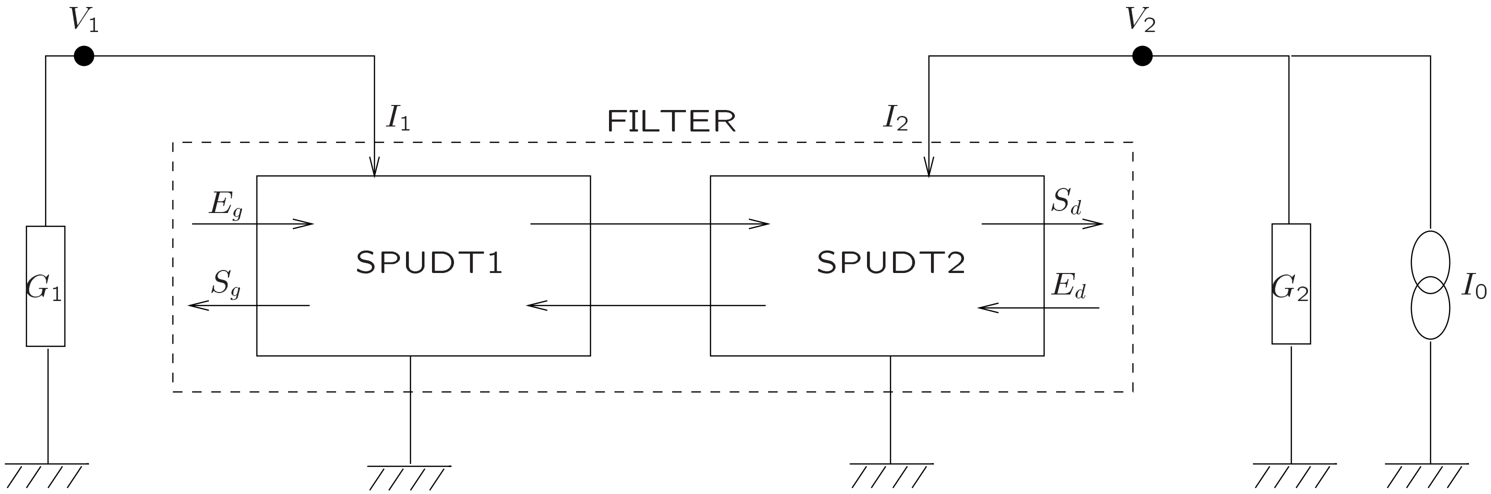
SPUDT Filters Design*

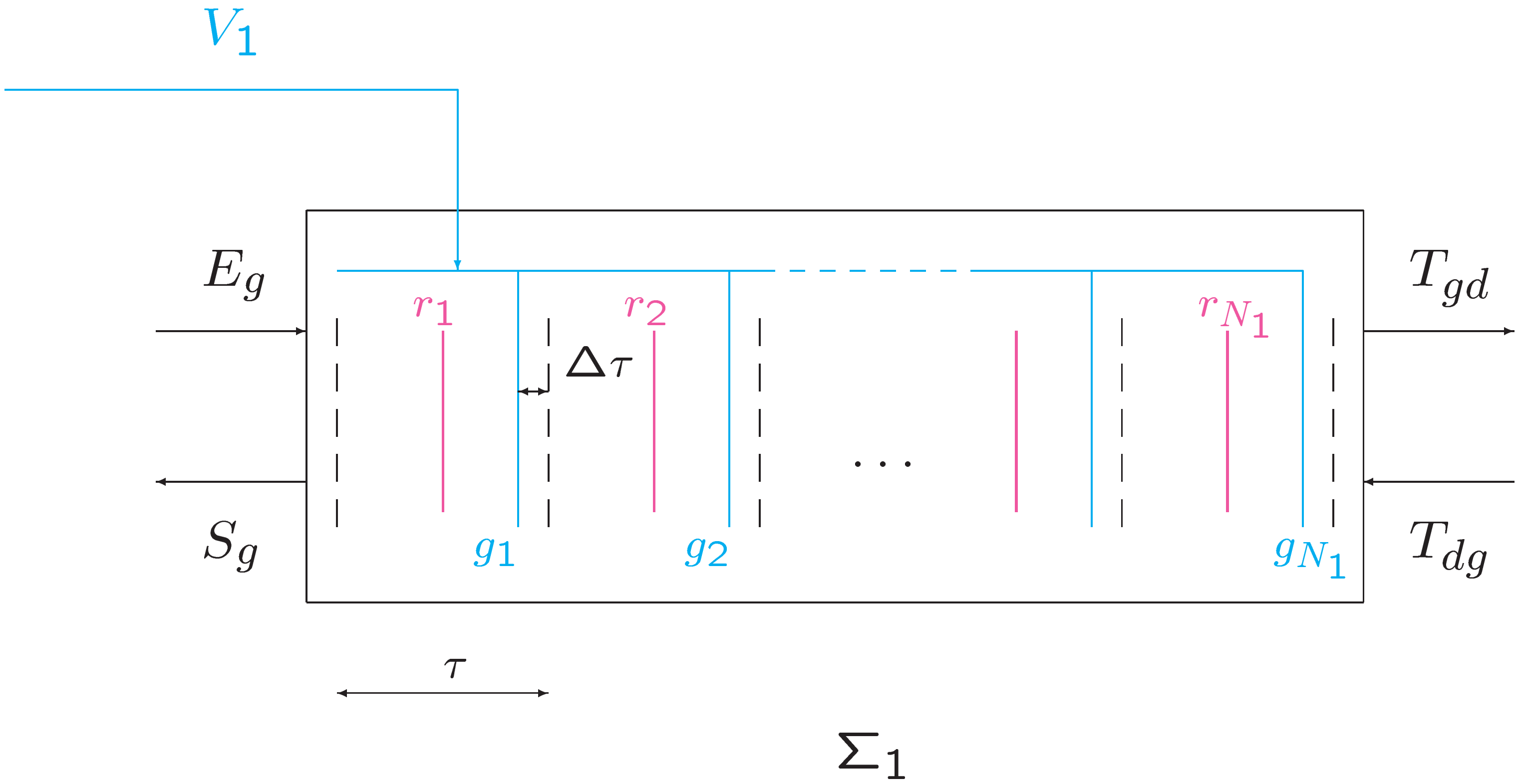


The filter is specified in the frequency domain in terms of amplitude and phase of the electrical transfer function (power transmission):

$$S_{12} = 2\sqrt{G_1 G_2}\,\frac{V_1}{I_0}.$$

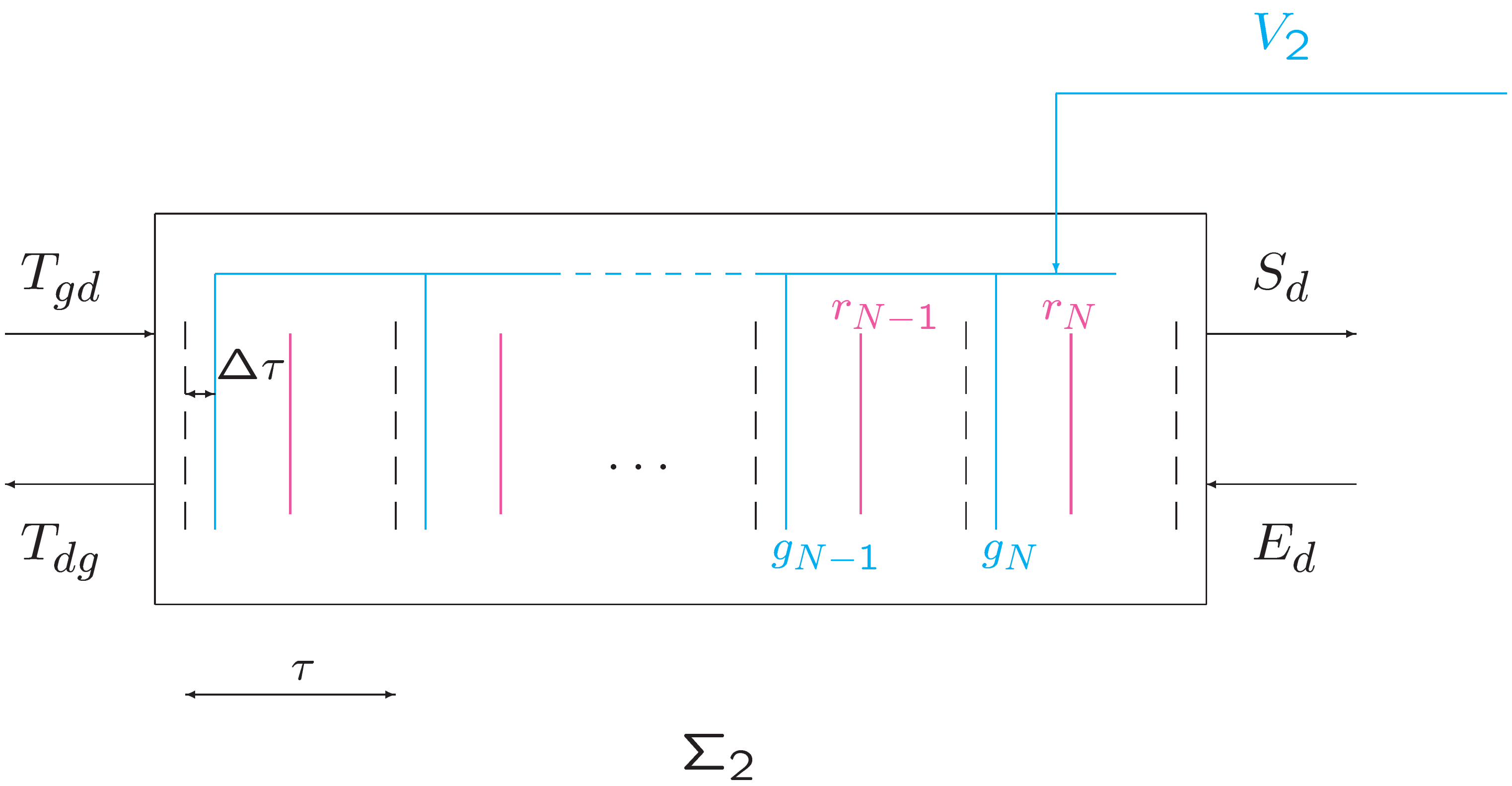
*J. M. Hodé, J. Desbois, P. Dufilié, M. Solal, P. Ventura (1995)

The Left Transducer



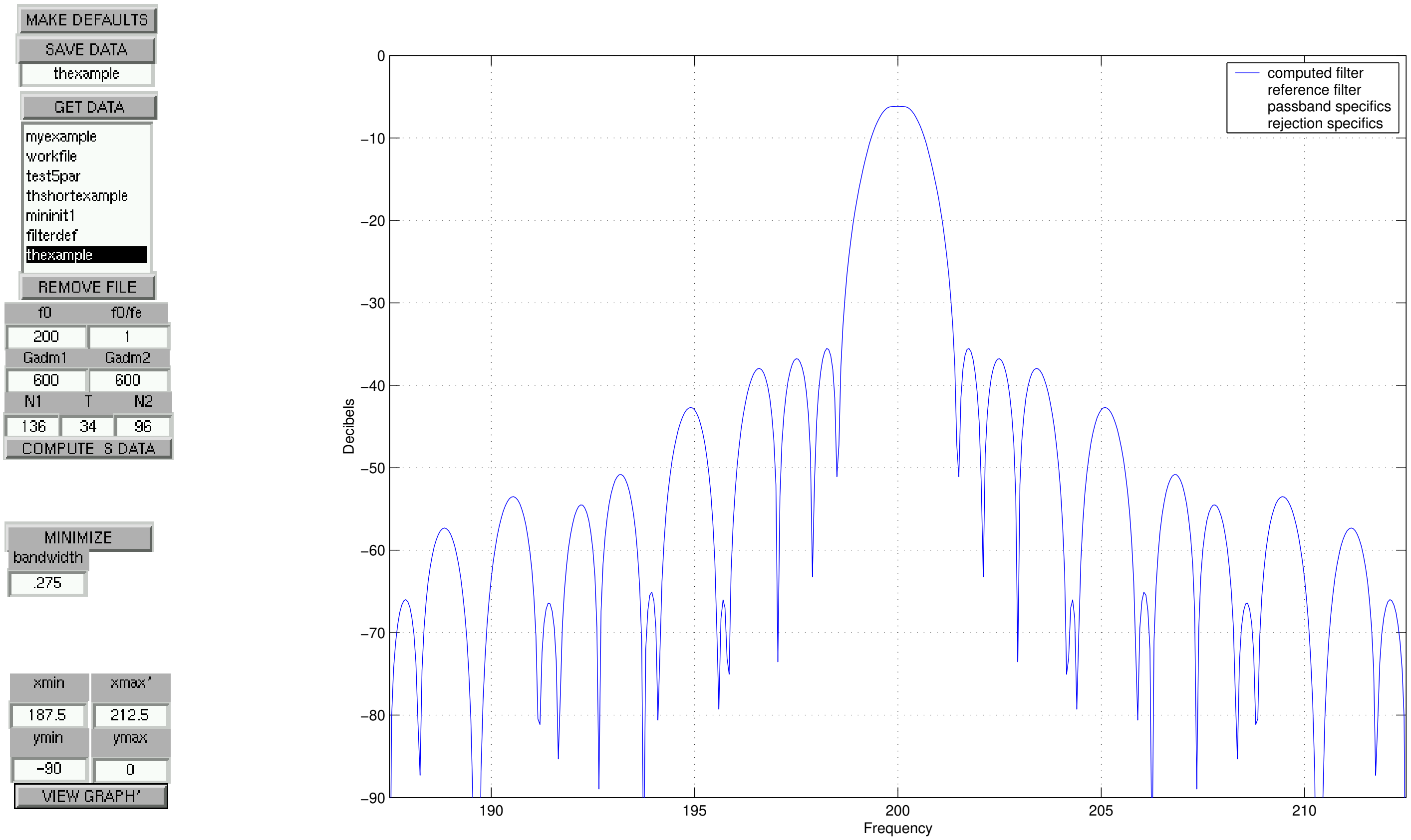
$\Delta\tau = \frac{1}{8\,f_0}$: unidirectionality for $f \approx f_0$

The Right Transducer



$$z = e^{-j2\pi f\tau}, \quad \delta = e^{-j2\pi f\Delta\tau}, \quad j^2 = -1$$

An example of filter



Mixed Matrix Representation

$$\left[\begin{array}{c} S \\ \hline I \end{array}\right] = \left(\begin{array}{c|c} M & \alpha \\ \hline \beta & Y \end{array}\right) \left[\begin{array}{c} E \\ \hline V \end{array}\right]$$

M, α, β and Y, 2×2 complex matrices of z, δ, r_i, g_i

By reciprocity and energy conservation:

M and Y symmetric,
$\beta = -\alpha^T$

$$\begin{aligned} MM^* &= Id \\ \alpha^*\alpha &= 2\,\mathrm{Re}\,Y \\ \alpha &= M\beta^* \end{aligned}$$

the electrical transfer function is the entry S_{12} of

$$S = (Y + Id)^{-1}(Id - Y)$$

Our approach

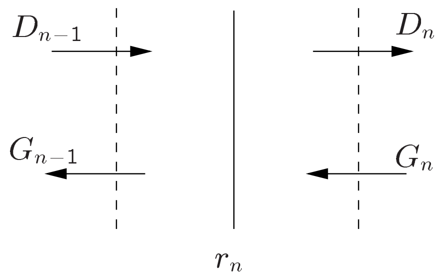
Plug this problem into the framework of mathematical system theory: matrices of analytic functions and their representations

State it as an approximation problem in an appropriate normed space

Use the skills developed within our team and already applied to an identification problem for an hyperfrequency filter*

*problem provided by the French space agency CNES

Diffraction and Chain Matrices



Diffraction matrix of the n-th cell:

$$\begin{bmatrix} G_{n-1} \\ D_n \end{bmatrix} = \begin{pmatrix} -j\, r_n\, z & t_n\, z \\ t_n\, z & -j\, r_n\, z \end{pmatrix} \begin{bmatrix} D_{n-1} \\ G_n \end{bmatrix}$$

Chain matrix:

$$\begin{bmatrix} D_n \\ G_n \end{bmatrix} = \frac{1}{t_n} \begin{pmatrix} z & -j\, r_n \\ j\, r_n & \frac{1}{z} \end{pmatrix} \begin{bmatrix} D_{n-1} \\ G_{n-1} \end{bmatrix}$$

$$z = e^{-j2\pi f \tau} \qquad t_n = \sqrt{1 - r_n^2}$$

Diffraction matrices and Schur polynomials

ϕ_n, ψ_n Schur polynomials of degree n; ϕ_n stable.

$$\begin{cases} \phi_{n+1}(z) &= z\,\phi_n(z) + r_{n+1}\,\psi_n(z) \\ \psi_{n+1}(z) &= r_{n+1}\,z\,\phi_n(z) + \psi_n(z) \end{cases} \qquad \phi_1(z) = z;\ \psi_1(z) = r_1 z.$$

The matrix of the cells 1 to n

$$M_{1,n} = \frac{1}{\tilde{\phi}_n(z^2)} \begin{bmatrix} -j\,z^{-1}\,\psi_n(z^2) & P_n\,z^n \\ P_n\,z^n & -j\,z\,\tilde{\psi}_n(z^2) \end{bmatrix}$$

$$\tilde{\phi}_n(z) = z^n\ \phi_n(1/z),\ \tilde{\psi}_n(z) = z^n\ \psi_n(1/z),\ P_n = t_1 \dots t_n,$$

is a rational inner function of z of degree $2n$.

Mathematical context

Assume δ constant $(\delta = j\bar{\delta})$

α, Y, and S are rational matrices of degree $2N-2$ of the complex variable z, analytic inside the unit disk.

$\alpha = M\beta^*$ Douglas-Shapiro-Shields factorization

$\alpha \in H(M) = H^2 \ominus M\, H^2$,

$$\alpha \;=\; \left[\sum_{n\in\Sigma_1} g_n\ \xi_n\ ;\ \sum_{n\in\Sigma_2} g_n\ \xi_n \right] \qquad \{\xi_n\}\ \text{O.B. of } H(M)$$

$\alpha^*\alpha = 2\mathrm{Re}\, Y = Y + Y^*$ spectral factorization

Y real positive, S Schur

$$(r,g) \;\to\; \alpha \;\to\; Y \;\to\; S = (Y + Id)^{-1}(Y - Id) \;\to\; S_{12}$$

A realization of α

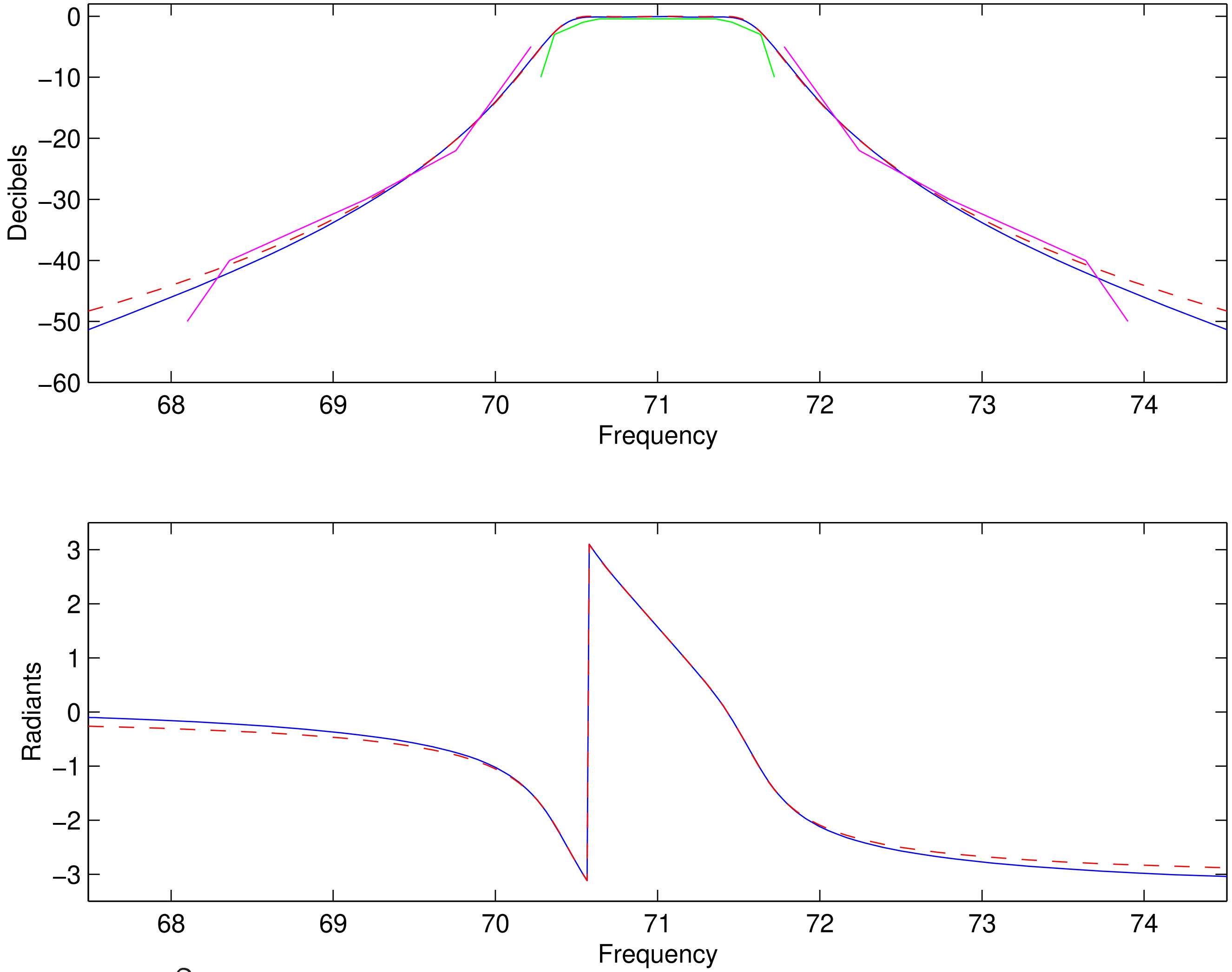
$$N_1 = N_2 = 2$$

$$A = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & jr_1 & 0 & 0 \\ t_2 & 0 & 0 & 0 & jr_2 & 0 \\ 0 & t_3 & 0 & 0 & 0 & jr_3 \\ \hline jr_2 & 0 & 0 & 0 & t_2 & 0 \\ 0 & jr_3 & 0 & 0 & 0 & t_3 \\ 0 & 0 & jr_4 & 0 & 0 & 0 \end{array}\right] \quad B = \left[\begin{array}{cc} t_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & t_4 \end{array}\right]$$

$$A^*A + B^*B = Id$$

$$C = \left[\begin{array}{ccc|ccc} g_1 & g_2 & 0 & j\,g_1 & j\,g_2 & 0 \\ 0 & j\,g_3 & j\,g_4 & 0 & g_3 & g_4 \end{array}\right] \delta$$

$\rightarrow$ realizations of Y, S



L^2-norm minimization to a Chebyschev filter.

$N = 7$, rmax$= 0.82$, gmax$= 0.89$.

Conclusion

$(r,g) \;\rightarrow\; \alpha \;\rightarrow\; Y \;\rightarrow\; S = (Y+Id)^{-1}(Y-Id) \;\rightarrow\; S_{12}$

Open questions:

S_{12}: $4N-3$ coefficients / $2N$ parameters (r,g)

caracterize S_{12} ? reference filter ?

Further results:

$\alpha = M\ \beta^* = -M\ \overline{\alpha}$ $\quad\rightarrow\quad$ zeros

$S_{12} \;\rightarrow\; S$

$\rightarrow$ work with Y

Mixed Matrix Representation

$$\left[\begin{array}{c} S \\ \hline I \end{array}\right] = \left(\begin{array}{c|c} M & \alpha \\ \hline \beta & Y \end{array}\right) \left[\begin{array}{c} E \\ \hline V \end{array}\right]$$

M, α, β and Y, 2×2 complex matrices of z, δ, r_i, g_i

$S = \begin{bmatrix} S_g \\ S_d \end{bmatrix}$ outgoing waves, $I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ currents

$E = \begin{bmatrix} E_g \\ E_d \end{bmatrix}$ incoming waves, $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ voltages

M pure acoustic part,

α electro-acoustic part, Y admittance

Global Diffraction Matrix

acoustic waves $\leftrightarrow$ currents and voltages

$$\left[\frac{Sa}{Se}\right] = \left(\begin{array}{c|c} M - \alpha(Y+Id)^{-1}\beta & \sqrt{2}\ \alpha(Y+Id)^{-1} \\ \hline -\sqrt{2}\ (Y+Id)^{-1}\beta & (Y+Id)^{-1}(Id-Y) \end{array}\right)\left[\frac{Ea}{Ee}\right]$$

reciprocity $\Leftrightarrow$ symmetric matrix

non lossy system $\Leftrightarrow$ unitary matrix

M and Y symmetric,
$\beta = -\alpha^T$

$$\begin{aligned} MM^* &= Id \\ \alpha^*\alpha &= 2\ \mathrm{Re}\, Y \\ \alpha &= M\beta^* \end{aligned}$$

the electrical transfer function is the entry S_{12} of

$$S = (Y+Id)^{-1}(Id-Y)$$

The Structure of α

$$\alpha \;=\; -j \left[\sum_{n\in\Sigma_1} g_n \left(\delta\, v_n + \bar{\delta}\, w_n\right) \;;\; \sum_{n\in\Sigma_2} g_{n+1} \left(\bar{\delta}\, v_n + \delta\, w_n\right) \right]$$

$$[v_n \;\; w_n] = \begin{pmatrix} P_n\, z^n & 0 \\ 0 & \frac{P_N\, z^N}{P_n\, z^n} \end{pmatrix} \frac{\begin{pmatrix} \tilde{\phi}^R_{N-n}(z^2) & -jz\,\tilde{\psi}^R_{N-n}(z^2) \\ -jz\tilde{\psi}_n(z^2) & \tilde{\phi}_n(z^2) \end{pmatrix}}{\tilde{\phi}_N(z^2)}$$

$$\begin{cases} M^{-1} v_n &= \; w_n^* \\ M^{-1} w_n &= \; v_n^* \end{cases}$$

The Structure of α

$$\alpha \;=\; -\,\delta\, \frac{\begin{pmatrix} j \displaystyle\sum_{n=1}^{N_1} g_n \; P_n\, z^n \left[\widetilde{\phi}^R_{N-n} - z\widetilde{\psi}^R_{N-n}\right] & \displaystyle\sum_{n=N_1+T}^{N-1} g_{n+1}\; P_n\, z^n \left[\widetilde{\phi}^R_{N-n} + z\widetilde{\psi}^R_{N-n}\right] \\ \displaystyle\sum_{n=1}^{N_1} g_n\; \frac{P_N\, z^N}{P_n\, z^n} \left[\widetilde{\phi}_n + z\widetilde{\psi}_n\right] & j \displaystyle\sum_{n=N_1+T}^{N-1} g_{n+1}\; \frac{P_N\, z^N}{P_n\, z^n} \left[\widetilde{\phi}_n - z\widetilde{\psi}_n\right] \end{pmatrix}}{\widetilde{\phi}_N}$$

Realizations of Y and S

$$Y = \left[\begin{array}{c|c} A & A\,C^* \\ \hline C & \frac{1}{2}CC^* \end{array}\right]$$

$$S = \left[\begin{array}{c|c} A(I - C^*(\frac{1}{2}CC^* + I)^{-1}C) & \sqrt{2}\,AC^*(\frac{1}{2}CC^* + I)^{-1} \\ \hline \sqrt{2}\,(\frac{1}{2}CC^* + I)^{-1}C & (I - \frac{1}{2}CC^*)(\frac{1}{2}CC^* + I)^{-1} \end{array}\right]$$