Advanced Logic

http://www-sop.inria.fr/members/Martin.Avanzini/teaching/2023/AL/

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Last Lecture

- ⋆ a language $L \subseteq \Sigma^\omega$ is ω-regular if $L = \bigcup_{0 \leq i \leq n} U_i \cdot V_i^\omega$ for regular languages U_i, V_i $(0 \leq i \leq n)$
- ⋆ a Büchi Automaton is structurally similar to an NFA, but recognizes words $w \in \Sigma^\infty$ that visit final states infinitely often

Theorem

For recognisable $U \in \Sigma^*$ and $V, W \in \Sigma^\omega$ the following are recognisable:

1. *union* $V \cup W$
2. *intersection* $V \cap W$
3. *left-concatenation* $U \cdot V$
4. ω*-iteration* U^ω
5. *complement* $\overline{V}$

Theorem

$L \in \omega REG(\Sigma)$ *if and only if* $L = \mathrm{L}(\mathcal{A})$ *for some NBA* $\mathcal{A}$

Theorem

For every MSO formula ϕ *there exists an NBA* $\mathcal{A}_\phi$ *s.t.* $\hat{\mathrm{L}}(\phi) = \mathrm{L}(\mathcal{A}_\phi)$.

Today's Lecture

1. Linear temporal logic (LTL)
2. LTL model checking

Linear temporal logic

Motivation

- linear temporal logic is a logic for reasoning about events in time
  - always not $(\phi \wedge \psi)$ safety
  - always (Request implies eventually Grant) liveness
  - always (Request implies (Request until Grant)) liveness
- LTL shares algorithmic solutions with MSO

Formal Definition

- the set of LTL formulas over propositions $\mathcal{P} = \{p, q, \ldots\}$ is given by

$$\phi, \psi ::= p \;\big|\; \phi \vee \psi \;\big|\; \neg\phi \qquad \textit{(Propositional Formulas)}$$

$$\big|\; \mathsf{X}\,\phi \;\big|\; \phi\,\mathsf{U}\,\psi \qquad \textit{(Next and Until)}$$

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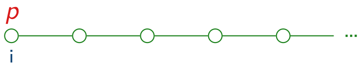
- ⋆ for a sentence ϕ and $w = P_0 P_1 P_2 \ldots$, we define $w \vDash \phi$ as $w; 0 \vDash \phi$ where

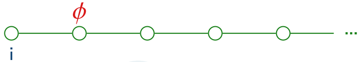
$$w; i \vDash p \quad :\Longleftrightarrow \quad p \in P_i$$



$$w; i \vDash \phi \vee \psi \quad :\Longleftrightarrow \quad w; i \vDash \phi \text{ or } w; i \vDash \psi$$

$$w; i \vDash \neg\phi \quad :\Longleftrightarrow \quad w; i \nvDash \phi$$

$$w, i \vDash \mathsf{X}\,\phi \quad :\Longleftrightarrow \quad w; i + 1 \vDash \phi$$



$$w; i \vDash \phi \,\mathsf{U}\, \psi \quad :\Longleftrightarrow \quad \text{exists } k \geq i \text{ s.t. } w; k \vDash \phi \text{ and } w; j \vDash \psi \text{ for all } i \leq j < k$$

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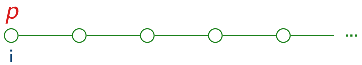

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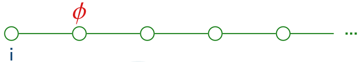

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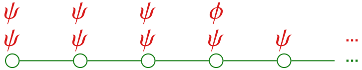
- ⋆ a LTL formula ϕ defines the language $\mathsf{L}(\phi) \triangleq \{w \mid w \vDash \phi\}$

Derived Operators and Positive Normal Forms

finally: $\mathsf{F}\,\phi \quad :\Longleftrightarrow \quad \top\ \mathsf{U}\ \phi$

globally: $\mathsf{G}\,\phi \quad :\Longleftrightarrow \quad \neg(\mathsf{F}\,\neg\phi)$

release: $\phi\ \mathsf{R}\ \psi \quad :\Longleftrightarrow \quad \neg(\neg\phi\ \mathsf{U}\ \neg\psi)$



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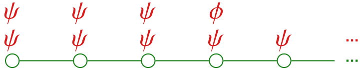

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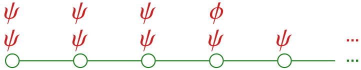

- ⋆ $\mathsf{F}\,\phi$, $\mathsf{G}\,\phi$ and $\mathsf{X}\,\phi$ are sometimes denoted by $\Diamond\phi$, $\Box\phi$ and $\circ\phi$, respectively
- ⋆ a formula ϕ is in positive normal form (PNF) if it is derived from the following grammar:

$$\phi,\psi ::= p \;\Big|\; \neg p \;\Big|\; \phi \wedge \psi \;\Big|\; \phi \vee \psi \;\Big|\; \mathsf{X}\,\phi \;\Big|\; \phi\;\mathsf{U}\;\psi \;\Big|\; \phi\;\mathsf{R}\;\psi$$

  - negation only in front of literals

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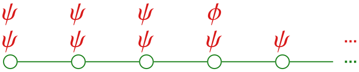

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  – negation only in front of literals

Lemma

Every formula ϕ can be turned into an equivalent formula ψ in PNF with $|\psi| \leq 2|\phi|$

Safety Properties in LTL

Safety = something bad never happens = $\mathsf{G}\,\neg\phi_{\text{bad}}$

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Example

- ⋆ a ...A train is approaching
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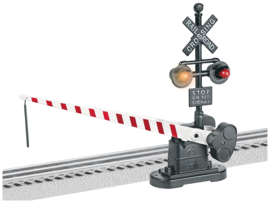

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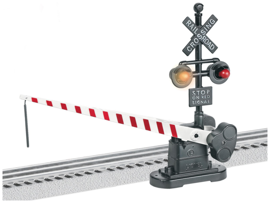

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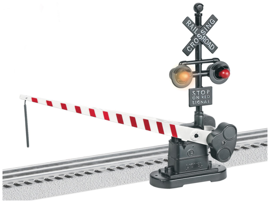

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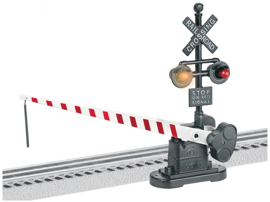

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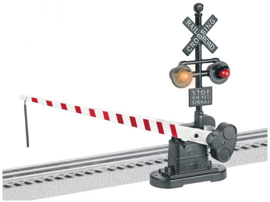

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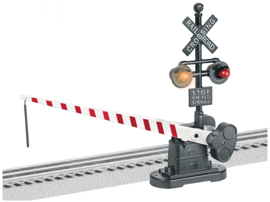

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$$G\,(\neg b \wedge \neg l \rightarrow \neg a \wedge \neg c) \equiv G\,\neg(\neg b \wedge \neg l \wedge (a \vee c))$$

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$$\mathsf{G}\,(c \wedge \mathsf{X}\,\neg c \rightarrow \mathsf{X}\,\mathsf{F}\,\neg b)$$

Characterising LTL

- ⋆ LTL can be "expressed" within MSO ≡ Büchi Automata
- ⋆ MSO and Büchi Automata are strictly more expressive

$$\text{LTL recognisability} < \omega\text{-regular}$$

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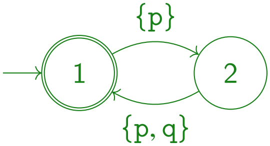
Example
the Büchi Automaton $\mathcal{A}$ over $\mathcal{P} = \{p, q\}$ given by



is not loop-free (and cannot be turned into equivalent loop-free one)

$\Longrightarrow$ $L(\mathcal{A})$ not expressible in LTL

(Very Weak) Alternating Büchi Automata

- ⋆ an alternating Büchi Automaton (ABA) is a tuple $\mathcal{A} = (Q, \Sigma, q_I, \delta, F)$ identical to an AFA
- ⋆ execution on words $w \in \Sigma^\omega$ are now infinite tree T_w
- ⋆ an execution is accepting in the sense of Büchi: every path visits F infinitely often
- ⋆ $\mathrm{L}(\mathcal{A}) \triangleq \{w \in \Sigma^\omega \mid \text{there exist an accepting execution } T_w \text{ for } w\}$

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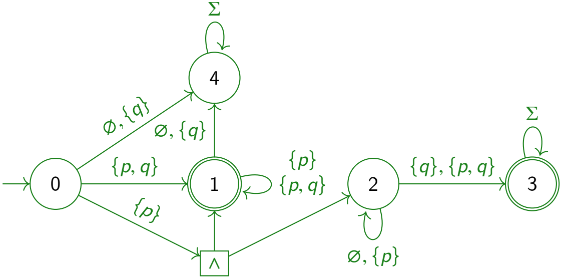

$\mathsf{G}\, p \wedge \mathsf{F}\, q$

LTL and Automata

Theorem

Let L be a language over $\Sigma = 2^{\mathcal{P}}$. The following are equivalent:

- ⋆ L *is LTL definable.*
- ⋆ L *is recognizable by VWABA.*

From Automata to LTL

fix a VWABA $\mathcal{A} = (\{q_0, \ldots, q_n\}, 2^{\mathcal{P}}, q_0, \delta, F)$ where wlog. $q_0 > q_1 > \cdots > q_n$

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- ★ the construction differs whether the state is final, we thus consider two cases

From Automata to LTL (II)

fix a VWABA $\mathcal{A} = (\{q_0, \ldots, q_n\}, 2^{\mathcal{P}}, q_0, \delta, F)$ where wlog. $q_0 > q_1 > \cdots > q_n$

- informally, ϕ_i should satisfy

$$\phi_i \equiv \bigvee_{P \subseteq \mathcal{P}} \chi_P \wedge \mathsf{X}\,(\delta(q_i, P)[q_i/\phi_i, q_{i+1}/\phi_{i+1} \ldots, q_n/\phi_n])$$

- to get rid of the "recursive definition", we distinguish two cases:
  - if $q_i \notin F$ then we rewrite the right-hand side as $\psi \vee (\rho \wedge \mathsf{X}\,\phi_i)$ and set

$$\phi_i \triangleq \rho \;\mathsf{U}\; \psi$$

  - if $q_i \in F$ then we rewrite the right-hand side as $\psi \wedge (\rho \vee \mathsf{X}\,\phi_i)$ and set

$$\phi_i \triangleq \mathsf{G}\,\psi \vee (\psi \;\mathsf{U}\; (\rho \wedge \psi))$$

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Notes

- ⋆ $\mathcal{A}_\phi$ is linear in size in $|\phi|$
- ⋆ using the construction for AFAs, this ABA can be transformed to an NBA of size $O(2^{|\phi|})$

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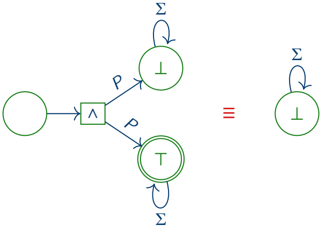

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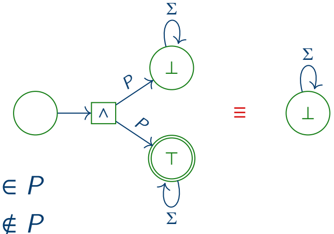

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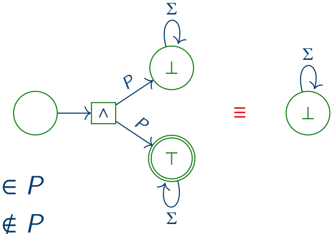

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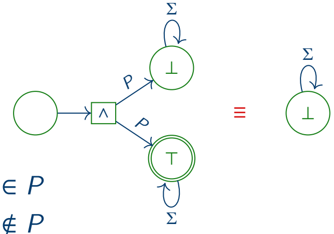

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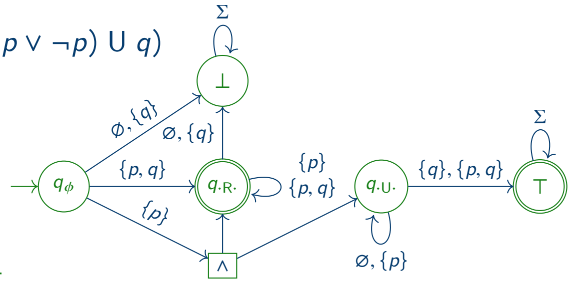

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Model Checking

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- ⋆ they can be seen as finite representations of potentially infinitely many program runs

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- $\mathrm{L}(\mathcal{S}) \triangleq \{w \mid w \text{ is a run in } \mathcal{S}\}$ is the set of all runs

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- emptyness of $\mathcal{S} \otimes \mathcal{A}_{\neg\phi}$ is decidable in time linear in $|\mathcal{S} \otimes \mathcal{A}_{\neg\phi}| \in O(|S|^2) \cdot 2^{O(|\phi|)}$

LTL Model Checking In Practice

Explicit Model Checking: each automaton node is an individual state

- ⋆ SPIN model checker: `http://spinroot.com/`

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- ⋆ while real problems have a finite number of states, we deal with an astronmoical number of cases
- ⋆ industrial-strength tools such as the ones above generate $\mathcal{S} \otimes \mathcal{A}_{\neg\phi}$ on-the-fly and implement several techniques to combat state-space explosion
  - partial order reduction: detects when an ordering of interleavings is irrelevant. E.g., the $n!$ transitions of n concurrently executing processes is reduced to 1 representative transition, when ordering irrelevant for property under investigation
  - Bounded Model Checking: check that ϕ is violated in $\leq k$ steps
  - ...

Thanks!