Infinite Regular Languages Exercises

Deadline: 15/05 09:00

Exercise 1

Give an NBA $\mathcal{A}$ over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ s.t.

$$L(\mathcal{A}) = \{w \in \Sigma^\omega \mid |w|_\mathtt{a} = \infty \Rightarrow |w|_\mathtt{b} = \infty\}$$

Exercise 2

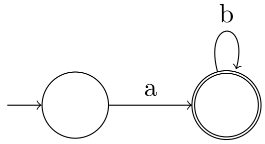
Show that $L = \{w \in \Sigma^\omega \mid |w|_\mathtt{a} \neq \infty\}$ is not recognised by any DBA, for $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

Hint: For a proof by contradiction, suppose DBA $\mathcal{A}$ recognises L and consider runs on the family of words $w_0 = \mathtt{ab}^\omega$, $w_1 = \mathtt{ab}^{i_0}\mathtt{ab}^\omega$, $w_2 = \mathtt{ab}^{i_0}\mathtt{ab}^{i_1}\mathtt{ab}^\omega$, ..., for a carefully chosen i's. Reason then that $w = \mathtt{ab}^{i_0}\mathtt{ab}^{i_1}\mathtt{ab}^{i_2}\mathtt{a}\cdots \in \mathsf{L}(\mathcal{A})$.

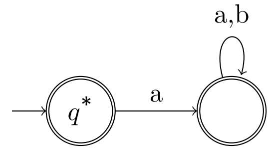
Exercise 3

Show that for every NFA $\mathcal{A}$ with $\epsilon \notin \mathsf{L}(\mathcal{A})$, there exists a NBA $\mathcal{B}$ s.t. $\mathsf{L}(\mathcal{B}) = \mathsf{L}(\mathcal{A})^\omega$.

Hint: The NFA construction for $\mathsf{L}(\mathcal{A})^*$ does not work, see the following counter example. You may assume that the initial state of $\mathcal{A}$ does not have incoming edges (why?). For the construction of $\mathcal{B}$, use a fresh final state.



NFA $\mathcal{A}$



NBA $\mathcal{B}$

$L(\mathcal{A}) = ab^*$

$L(\mathcal{B}) = a(a+b)^\omega \neq L(\mathcal{A})^\omega$