AUTOMATA → LTL.

$L(3) = \top$

$$L(2) = (q \wedge \neg p) \wedge X L(3) \vee (p \wedge q) \wedge X L(3) \vee (\neg p \wedge \neg q) \wedge X L(2) \vee (p \wedge \neg q) \wedge X L(2)$$

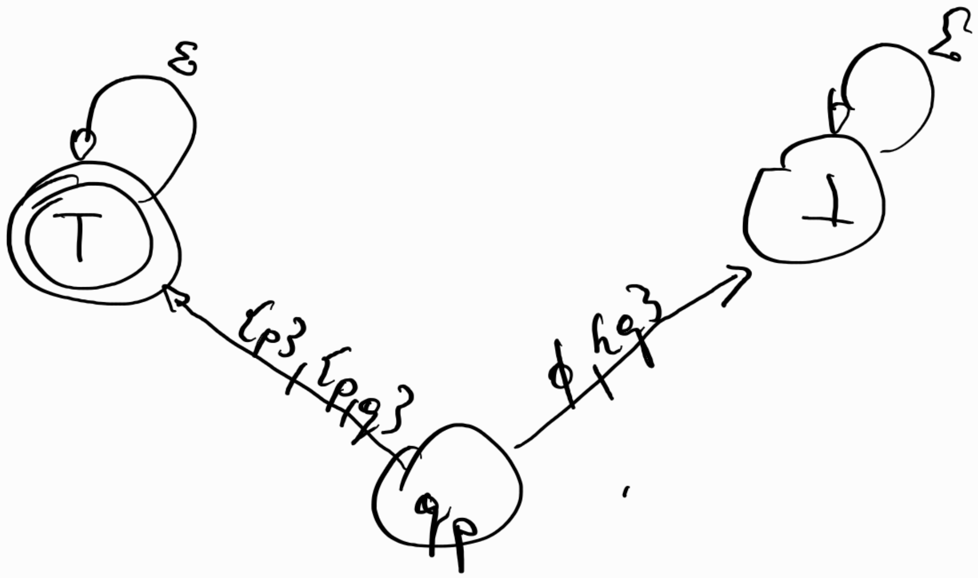
$$= q \wedge X L(3) \vee \neg q \wedge X L(2)$$

$$= q \wedge X \top \vee \neg q \wedge X L(2)$$

$$= q \vee \neg q \wedge X L(2)$$

$L(2) := \neg q \, U \, q$

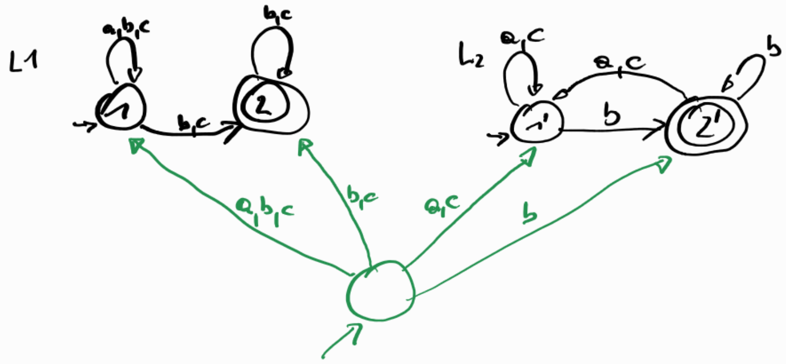
$P = \{p, q\}$



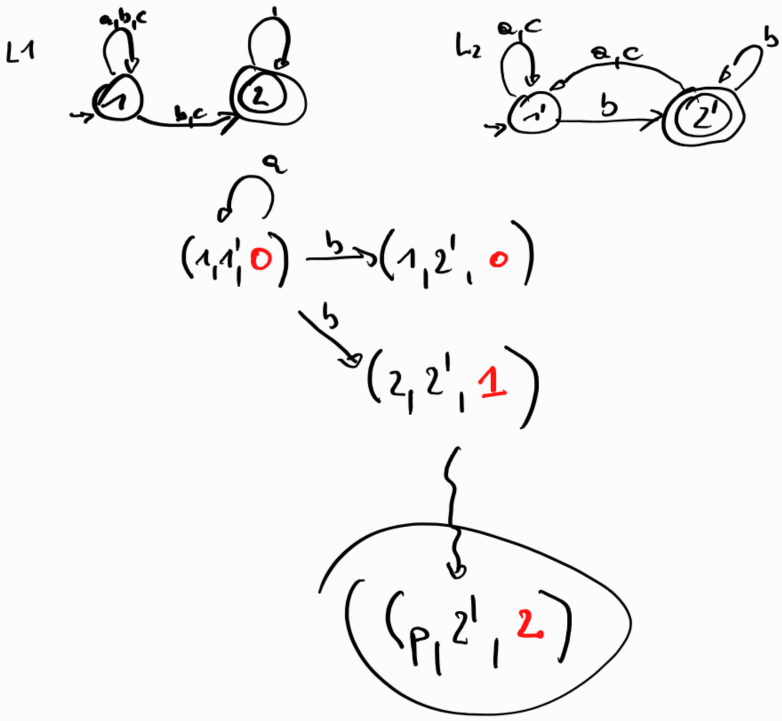
Exercise 1

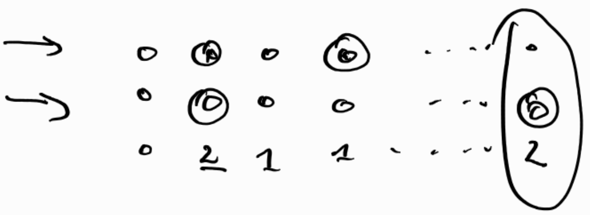
$$w \in L(A) :\Leftrightarrow \quad |w|_a \neq \infty \ \vee \ |w|_b = \infty$$

$$= w \in \Big(\underbrace{\{ w \mid |w|_a \neq \infty \}}_{L_1} \cup \underbrace{\{ w \mid |w|_b = \infty \}}_{L_2} \Big)$$



$L_1 \cap L_2$:





EXERCISE 2

PROOF BY CONTRADICTION.
IDEA $\neg A \Leftrightarrow A \rightarrow \text{FALSE}$

FOR A PROOF BY CONTRADICTION,
SUPPOSE $\mathcal{A} = (Q, \{a,b\}, q_I, \delta, F)$ ACCEPTS $L = \{w \in \{a,b\}^\omega \mid |w|_a \neq \infty\}$

- THUS $q_I \xrightarrow{ab^{i_0}} q_0$ FOR SOME $q_0 \in F$
- ALSO $q_I \xrightarrow{ab^{i_0}ab^{i_1}} q_1$ FOR SOME $q_1 \in F$. SINCE $\mathcal{A}$ IS DETERMINISTIC,

  EVEN $q_I \xrightarrow{ab^{i_0}} q_0 \xrightarrow{ab^{i_1}} q_1$.
- SIMILAR, $q_I \xrightarrow{ab^{i_0}} q_0 \xrightarrow{ab^{i_1}} q_1 \xrightarrow{ab^{i_2}} q_2$ FOR SOME $q_2 \in F$

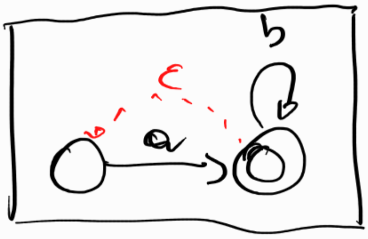
CONTINUING THIS REASONING, THERE NEEDS TO EXIST AN INFINITE RUN

$$q_I \xrightarrow{ab^{i_0}} q_0 \xrightarrow{ab^{i_1}} q_1 \xrightarrow{ab^{i_2}} \cdots$$

WHERE $q_i \in F$ FOR ALL $i \in \mathbb{N}$.

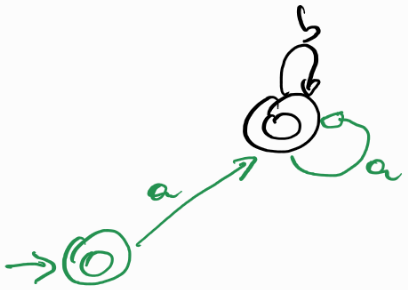
THUS $ab^{i_0}ab^{i_1}ab^{i_2}\ldots \in L(\mathcal{A})$. THIS CONTRADICTS $L(\mathcal{A}) = L$, HENCE $\mathcal{A}$ CANNOT EXIST.

EXERCISE 31



$\mathcal{A}$

$L = ab^*$



$\mathcal{A}^*$

$L^* = (ab^*)^*$