LTL Exercises

Deadline: 16/05 09:00

Exercise 1

Consider $\Sigma = 2^{\mathtt{p},\mathtt{q}}$, and let $a = \varnothing$, $b = \{\mathtt{p}\}$ and $c = \{\mathtt{q}\}$. Define each of the following languages in terms of an LTL formula over propositions $\{\mathtt{p}, \mathtt{q}\}$.

1. $L_1 = a^* b^* c^* \Sigma^\omega$;
2. $L_2 = \{w \in \Sigma^\omega \mid |w|_a = \infty \Rightarrow |w|_b = \infty\}$;
3. $L_3 = (\Sigma^* a \Sigma^* b \Sigma^* c)^\omega$.

Exercise 2

Reason that the following equivalences hold, or give a counter example.

1. $\mathsf{G}\,\phi \land \mathsf{G}\,\psi \equiv \mathsf{G}\ * \phi \land \psi)$;
2. $\mathsf{F}\,\phi \land \mathsf{F}\,\psi \equiv \mathsf{F}\,(\phi \land \psi)$;
3. $\mathsf{G}\,\phi \rightarrow \mathsf{F}\,\psi \equiv \phi\ \mathsf{U}\ (\psi \lor \neg\phi)$;
4. $\mathsf{F}\,\mathsf{G}\,\phi \equiv \mathsf{G}\,\mathsf{F}\,\phi$;
5. $\mathsf{F}\,\mathsf{F}\,\phi \equiv \mathsf{F}\,\phi$;

Exercise 3

Prove that $\phi\ \mathsf{U}\ \psi \equiv \psi \lor \mathsf{X}\,(\phi\ \mathsf{U}\ \psi)$.
Assume $w, i \vDash \phi\ \mathsf{U}\ \psi$, and show that $w, i \vDash \psi \lor \mathsf{X}\,(\phi\ \mathsf{U}\ \psi)$; and vice verse.