

Infinite Regular Languages Exercises

Deadline: 09/05 09:00

Exercise 1

Give an NBA \mathcal{A} over $\Sigma = \{a, b, c\}$ s.t.

$$L(\mathcal{A}) = \{w \in \Sigma^\omega \mid |w|_a = \infty \Rightarrow |w|_b = \infty\}$$

Exercise 2

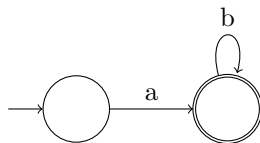
Show that $L = \{w \in \Sigma^\omega \mid |w|_a \neq \infty\}$ is not recognised by any DBA, for $\Sigma = \{a, b\}$.

Hint: For a proof by contradiction, suppose DBA \mathcal{A} recognises L and consider runs on the family of words $w_0 = ab^\omega$, $w_1 = ab^{i_0}ab^\omega$, $w_2 = ab^{i_0}ab^{i_1}ab^\omega$, \dots , for a carefully chosen i 's. Reason then that $w = ab^{i_0}ab^{i_1}ab^{i_2}a \dots \in L(\mathcal{A})$.

Exercise 3

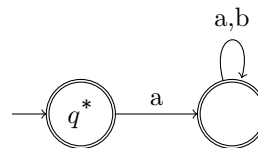
Show that for every NFA \mathcal{A} with $\epsilon \notin L(\mathcal{A})$, there exists a NBA \mathcal{B} s.t. $L(\mathcal{B}) = L(\mathcal{A})^\omega$.

Hint: The NFA construction for $L(\mathcal{A})^$ does not work, see the following counter example. You may assume that the initial state of \mathcal{A} does not have incoming edges (why?). For the construction of \mathcal{B} , use a fresh final state.*



NFA \mathcal{A}

$$L(\mathcal{A}) = ab^*$$



NBA \mathcal{B}

$$L(\mathcal{B}) = a(a+b)^\omega \neq L(\mathcal{A})^\omega$$