

# Exercises

## Presburger, MONA, and $L^*$

Deadline: 2/05 09:00

Mona website: <https://www.brics.dk/mona/>

### Exercise 1

1. Define in MONA a WMSO predicate `leq(var2 X, var2 Y)` that is true if and only if the numbers  $x, y$  coded by  $X, Y, Z$  in binary (little endian) are such that  $x \leq y$ . For instance, `leq(pconst(2), pconst(3))` is valid.
2. Define in MONA a WMSO predicate `add(var2 X, var2 Y, var2 Z)` that is true if and only if the numbers  $x, y, z$  coded by  $X, Y, Z$  in binary (little endian) are such that  $x + y = z$ . For instance, `add(pconst(2), pconst(3), pconst(5))` is valid.
3. Consider the following formula  $\psi$  of the Presburger arithmetic

$$\exists x \ x + x = y \wedge x \leq 3 \leq y$$

Translate this formula into a WMSO formula  $\varphi$  such that all models of  $\varphi$  are the binary representations of the values of  $y$  that satisfy  $\psi$  (for instance,  $y = 4$  is such a value, so  $\varphi$  should accept 001). Run MONA and check your answer (you may need to look at the automaton to see *all* possible values for  $y$ ).

### Exercise 2

We consider the notions of observation table, closed table, and consistent table used in the  $L^*$  algorithm. For each of the following assertions, say if it is correct or not. When it is not correct, try to give a counter-example.

1. if a table is consistent before a row is promoted, it remains consistent
2. if a table is closed before a column is inserted, it remains closed
3. the table completion is deterministic, in the sense that the first conjecture sent to the teacher does not depend on which rows have been promoted, nor which columns have been inserted, in whatever order

4. the  $L^*$  algorithm is deterministic, in the sense that the sequence of conjectures sent to the teacher does not depend on the counter-examples found by the teacher