

WMSO Exercises

Deadline: 11/04 09:00

Exercise 1

Explain, in your own words, WMSO.

Show that the following properties are expressible in WMSO:

1. $x = y$ (without using the equality symbol);
2. $x = y + 1$; and
3. more generally $y = x + k$ for a fixed constant integer $k \geq 0$.

Exercise 2

How do we define the language $L(\phi)$ of a WMSO formula ϕ .

For each language L below, give a formula ϕ_L of WMSO such that $L = L(\phi_L)$, assuming $\Sigma = \{\mathbf{a}, \mathbf{b}\}$. When possible, take a formula in FO.

1. $\mathbf{a}^* \mathbf{b}^*$.
2. $\{a_0 \dots a_n \mid a_{3i} = a \text{ for all } i < \frac{n}{3}\}$, in other words language of words containing an a on all positions that are multiples of 3.
3. Generalize to the language $L_k = \{a_0 \dots a_n \mid a_{ki} = a \text{ for all } i < \frac{n}{k}\}$.

Exercise 3

Determine the language $L(\phi)$ over $\Sigma = \{\mathbf{a}, \mathbf{b}\}$ for the following WMSO formulas ϕ :

1. $\forall x. P_{\mathbf{a}}(x) \rightarrow \forall y. x < y \rightarrow P_{\mathbf{b}}(y)$.
2. $\exists x. P_{\mathbf{b}}(x) \wedge \forall y. y < x \rightarrow P_{\mathbf{a}}(y)$.
3. $\exists l. \exists X, Y. (\forall z. z \leq l \rightarrow (X(z) \vee Y(z))) \wedge (\forall x, y. X(x) \wedge Y(y) \rightarrow x < y \wedge P_{\mathbf{a}}(x) \wedge P_{\mathbf{b}}(y))$.
4. $\forall x. (\exists z. x < z) \rightarrow \exists y. x < y \wedge (P_{\mathbf{a}}(x) \leftrightarrow \neg P_{\mathbf{a}}(y))$

Exercise 4

Outline the construction of an NFA from ϕ that recognizes the same language. Why is this useful?

Give an NFA which recognizes $\hat{L}(\phi)$ for $\phi = X(x) \rightarrow \exists y.y < x \wedge Y(y)$. Use the variable order X, Y, x, y in the construction.