Advanced Logic

http://www-sop.inria.fr/members/Martin.Avanzini/teaching/2021/AL/

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Summer Semester 2021

Last Lecture

- a language $L \subseteq \Sigma^\omega$ is ω-regular if $L = \bigcup_{0 \le i \le n} U_i \cdot V_i^\omega$ for regular languages U_i, V_i $(0 \le i \le n)$
- a Büchi Automaton is structurally similar to an NFA, but recognizes words $w \in \Sigma^\infty$ that visit final states infinitely often

Theorem

For recognisable $U \in \Sigma^*$ and $V, W \in \Sigma^\omega$ the following are recognisable:

1. *union* $V \cup W$
2. *intersection* $V \cap W$
3. *left-concatenation* $U \cdot V$
4. ω*-iteration* U^ω
5. *complement* $\overline{V}$

Theorem

$L \in \omega REG(\Sigma)$ *if and only if* $L = \mathrm{L}(\mathcal{A})$ *for some NBA* $\mathcal{A}$

Theorem

For every MSO formula ϕ *there exists an NBA* $\mathcal{A}_\phi$ *s.t.* $\hat{\mathrm{L}}(\phi) = \mathrm{L}(\mathcal{A}_\phi)$.

Today's Lecture

1. Linear temporal logic (LTL)
2. LTL model checking

Linear temporal logic

Motivation

- linear temporal logic is a logic for reasoning about events in time
  - always not $(\phi \wedge \psi)$ — safety
  - always (Request implies eventually Grant) — liveness
  - always (Request implies (Request until Grant)) — liveness
- LTL shares algorithmic solutions with MSO

Formal Definition

- the set of LTL formulas over propositions $\mathcal{P} = \{p, q, \ldots\}$ is given by

$$\phi, \psi ::= p \;\big|\; \phi \vee \psi \;\big|\; \neg\phi \qquad \textit{(Propositional Formulas)}$$

$$\big|\; \mathsf{X}\,\phi \;\big|\; \phi\,\mathsf{U}\,\psi \qquad \textit{(Next and Until)}$$

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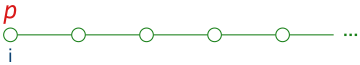
- LTL is a logic of temporal sequences, modeled as infinite words over $\Sigma \triangleq 2^{\mathcal{P}}$
- for a sentence ϕ and $w = P_0 P_1 P_2 \dots$, we define $w \models \phi$ as $w; 0 \models \phi$ where

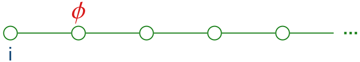
$$w; i \models p \quad :\Longleftrightarrow \quad p \in P_i$$



$$w; i \models \phi \vee \psi \quad :\Longleftrightarrow \quad w; i \models \phi \text{ or } w; i \models \psi$$

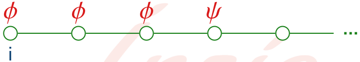
$$w; i \models \neg\phi \quad :\Longleftrightarrow \quad w; i \not\models \phi$$

$$w, i \models \mathsf{X}\,\phi \quad :\Longleftrightarrow \quad w; i+1 \models \phi$$



$$w; i \models \phi \,\mathsf{U}\, \psi \quad :\Longleftrightarrow \quad \text{exists } k \geq i \text{ s.t. } w; k \models \phi$$

$$\text{and } w; j \models \psi \text{ for all } i \leq j < k$$



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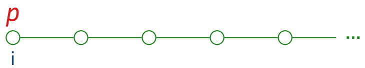

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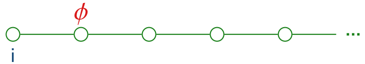

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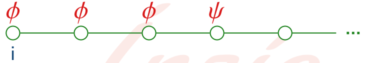


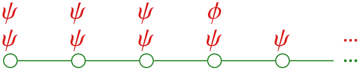
- a LTL formula ϕ defines the language $\mathsf{L}(\phi) \triangleq \{w \mid w \vDash \phi\}$

Derived Operators and Positive Normal Forms

finally: $\mathsf{F}\,\phi \quad :\Longleftrightarrow \quad \top \;\mathsf{U}\; \phi$

globally: $\mathsf{G}\,\phi \quad :\Longleftrightarrow \quad \neg(\mathsf{F}\,\neg\phi)$

release: $\phi \;\mathsf{R}\; \psi \quad :\Longleftrightarrow \quad \neg(\neg\phi \;\mathsf{U}\; \neg\psi)$



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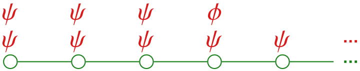

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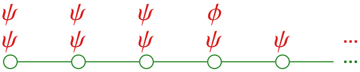

- ⋆ $\mathsf{F}\,\phi$, $\mathsf{G}\,\phi$ and $\mathsf{X}\,\phi$ are sometimes denoted by $\diamond\phi$, $\square\phi$ and $\circ\phi$, respectively
- ⋆ a formula ϕ is in positive normal form (PNF) if it is derived from the following grammar:

$$\phi,\psi ::= p \;\mid\; \neg p \;\mid\; \phi\wedge\psi \;\mid\; \phi\vee\psi \;\mid\; \mathsf{X}\,\phi \;\mid\; \phi\;\mathsf{U}\;\psi \;\mid\; \phi\;\mathsf{R}\;\psi$$

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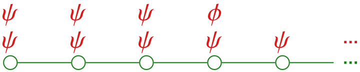

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Lemma

Every formula ϕ can be turned into an equivalent formula ψ in PNF with $|\psi| \leq 2|\phi|$

Safety Properties in LTL

Safety = something bad never happens = $\mathsf{G}\,\neg\phi_{\mathsf{bad}}$

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Example

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- c ...A train is crossing
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$$G\,(\neg b \wedge \neg l \rightarrow \neg a \wedge \neg c) \equiv G \neg (\neg b \wedge \neg l \wedge (a \vee c))$$

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$$\mathsf{G}\,(c \wedge \mathsf{X}\,\neg c \rightarrow \mathsf{X}\,\mathsf{F}\,\neg b)$$

Characterising LTL

- ⋆ LTL can be "expressed" within MSO $\equiv$ Büchi Automata
- ⋆ MSO and Büchi Automata are strictly more expressive

LTL recognisability $<$ ω-regular

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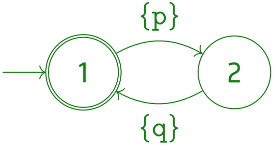
Example

the Büchi Automaton $\mathcal{A}$ over $\mathcal{P} = \{p, q\}$ given by



is not loop-free $\Rightarrow$ $L(\mathcal{A})$ not expressible in LTL

(Very Weak) Alternating Büchi Automata

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- ⋆ execution on words $w \in \Sigma^\omega$ are now infinite tree T_w
- ⋆ an execution is accepting in the sense of Büchi: every path visits F infinitely often
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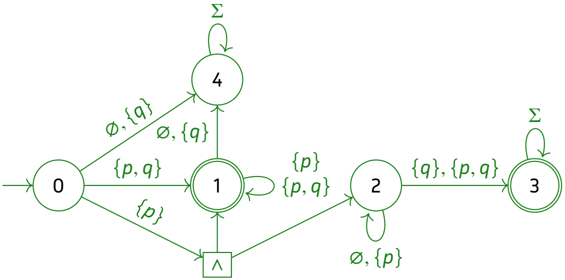

$\mathrm{G}\, p \wedge \mathrm{F}\, q$

LTL and Automata

Theorem

Let L be a language over $\Sigma = 2^{\mathcal{P}}$. The following are equivalent:

- ⋆ *L is LTL definable.*
- ⋆ *L is recognizable by VWABA.*

From Automata to LTL

fix a VWABA $\mathcal{A} = (\{q_0, \ldots, q_n\}, 2^{\mathcal{P}}, q_0, \delta, F)$ where wlog. $q_0 > q_1 > \cdots > q_n$

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- the construction differs whether the state is final, we thus consider two cases

From Automata to LTL (II)

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- note that $\mathrm{L}_{\mathcal{A}}(q_i)$ satisfies

$$\mathrm{L}_{\mathcal{A}}(q_i) \equiv \bigvee_{P \subseteq \mathcal{P}} \chi_P \wedge \mathrm{X}\,(\delta(q_i, P)[q_i/\mathrm{L}_{\mathcal{A}}(q_i), q_{i+1}/\mathrm{L}_{\mathcal{A}}(q_{i+1}) \ldots, q_n/\mathrm{L}_{\mathcal{A}}(q_i)])$$

- if $q_i \notin F$ then we rewrite $\mathrm{L}_{\mathcal{A}}(q_i)$ as $\psi \vee (\rho \wedge \mathrm{X}\,\mathrm{L}_{\mathcal{A}}(q_i))$ and set

$$\phi_i \triangleq \rho \;\mathrm{U}\; \psi$$

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the ABA $\mathcal{A}_\phi$ for a PNF formula ϕ is given by $(Q, 2^{\mathcal{P}}, \phi, \delta, F)$ where

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- ⋆ $Q \triangleq \{\top, \bot\} \cup \{q_\psi \mid \psi \text{ occurs as sub-formula in } \phi\}$
- ⋆ the transition function $\delta : Q \times 2^{\mathcal{P}} \to \mathbb{B}^+(Q)$ is given by

$$\delta(\top, P) \triangleq \top \quad \delta(\bot, P) \triangleq \bot \quad \delta(q_p, P) \triangleq \begin{cases} \top & \text{if } p \in P \\ \bot & \text{if } p \notin P \end{cases} \quad \delta(q_{\neg p}, P) \triangleq \begin{cases} \bot & \text{if } p \in P \\ \top & \text{if } p \notin P \end{cases}$$

$$\delta(q_{\psi_1 \wedge \psi_2}, P) \triangleq \delta(q_{\psi_1}, P) \wedge \delta(q_{\psi_2}, P) \qquad \delta(q_{\psi_1 \vee \psi_2}, P) \triangleq \delta(q_{\psi_1}, P) \vee \delta(q_{\psi_2}, P)$$

$$\delta(q_{\mathsf{X}\,\psi}, P) \triangleq q_\psi$$

$$\delta(q_{\psi_1 \mathsf{U} \psi_2}, P) \triangleq \delta(q_{\psi_2}, P) \vee (\delta(q_{\psi_1}, P) \wedge q_{\psi_1 \mathsf{U} \psi_2})$$

$$\delta(q_{\psi_1 \mathsf{R} \psi_2}, P) \triangleq \delta(q_{\psi_2}, P) \wedge (\delta(q_{\psi_1}, P) \vee q_{\psi_1 \mathsf{R} \psi_2})$$

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Notes

- ⋆ $\mathcal{A}_\phi$ is linear in size in $|\phi|$
- ⋆ using the construction for AFAs, this ABA can be transformed to an NBA of size $O(2^{|\phi|})$

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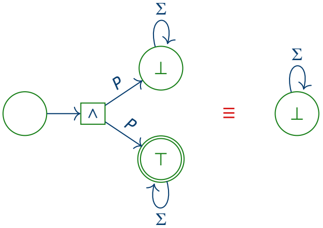

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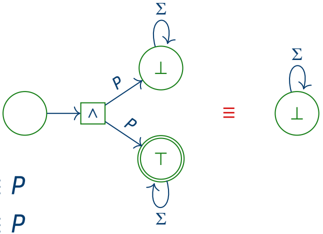

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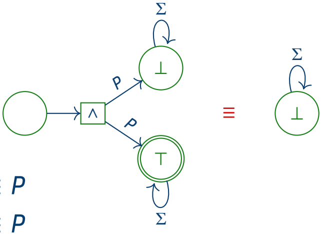

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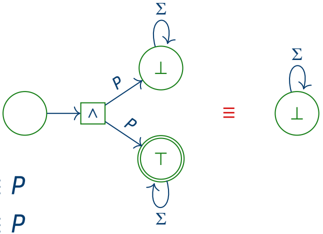

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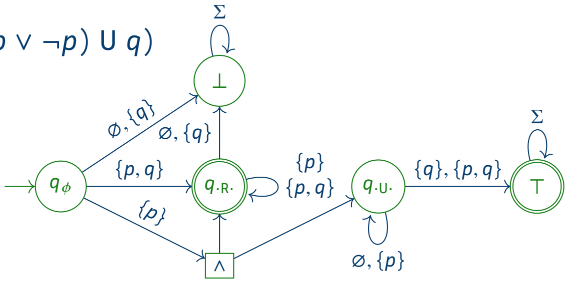

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Model Checking

Transition Systems (TSs)

- ★ transition systems capture evolution of state based programs etc.
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- ⋆ $\mathrm{L}(\mathcal{S}) \triangleq \{w \mid w \text{ is a run in } \mathcal{S}\}$ is the set of all runs

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- ⋆ emptyness of $\mathcal{S} \otimes \mathcal{A}_{\neg\phi}$ is decidable in time linear in $|\mathcal{S} \otimes \mathcal{A}_{\neg\phi}| \in O(|S|^2) \cdot 2^{O(|\phi|)}$

LTL Model Checking In Practice

Explicit Model Checking: each automaton node is an individual state

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Symbolic Model Checking: each automaton node represents a set of state, symbolically

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Main Challenge

- ⋆ while real problems have a finite number of states, we deal with an astronmoical number of cases
- ⋆ industrial-strength tools such as the ones above generate $\mathcal{S} \otimes \mathcal{A}_{\neg\phi}$ on-the-fly and implement several techniques to combat state-space explosion
  - partial order reduction: detects when an ordering of interleavings is irrelevant. E.g., the $n!$ transitions of n concurrently executing processes is reduced to 1 representative transition, when ordering irrelevant for property under investigation
  - Bounded Model Checking: check that ϕ is violated in $\leq k$ steps
  - ...

Thanks!