Accessibility Games and Tree Automata Exercises

M1 Master Info – 2021

Exercise 1

Consider the BUTA $\mathcal{A} = (Q, \Sigma, \delta, F)$ where

1. Q = {NOM, DET, ADJ, VERB, GN, GV};
2. the following are letters of arity 0 in Σ: le, la, petit, chat, souris, mange, nargue; furthermore, Σ contains one symbol + of arity 2 and a symbol ++ of arity 3;
3. the transition function delta is given such that:
   - δ_{chat} = {NOM}, δ_{souris} = {NOM}, δ_{le} = {DET}, etc.;
   - δ_{+}(DET,NOM) = {GN};
   - δ_{++}(DET,ADJ,NOM) = {GN}, δ_{++}(GN,VERB,GN) = {GV}.
4. F = {GV}

Give an example of a tree recognised by this BUTA. Why are some "leaf-sentences" not correct in French? Can you repair the automaton? Can the so obtained language be recognised by a deterministic TDTA?

Exercise 2

Fix $\Sigma = \{0, 1, +, \times\}$ where $0, 1$ are constants and $+, \times$ are binary. Note that trees over this alphabet form arithmetical expressions. Which of the following languages are recognised by a BUTA? Which are recognised by a DTDTA?

1. The set of L_1 expressions that evaluate to an even integer.
2. The set L_2 of expressions parenthesised only on the left: E.g., $0 \in L_2$, $(1 + 0) \times 1 \in L_2$ but $(1 + 0) \times (1 + 1) \in L_2$.
3. The set L_3 of balanced binary trees.

Exercise 3

Show that we can associate an accessibility game with a TDTA, so that the player ◆ has a winning strategy iff the language of this TDTA is not empty.

Exercise 4

Show that the set of winning strategies for player ◆ of a *finite* accessibility game (seen as trees) are recognised by a TDTA.