

# WMSO Exercises

M1 Master Info – 2021

## Exercise 1

*Explain, in your own words, WSMO.*

Show that the following properties are expressible in WSMO:

1.  $x = y$  (without using the equality symbol);
2.  $x = y + 1$ ; and
3. more generally  $y = x + k$  for a fixed constant integer  $k \geq 0$ .

## Exercise 2

*How do we define the language  $L(\phi)$  of a WMSO formula  $\phi$ .*

For each language  $L$  below, give a formula  $\phi_L$  of WMSO such that  $L = L(\phi_L)$ .  
When possible, take a formula in FO.

1.  $\{a_0 \dots a_n \mid a_{3i} = a \text{ for all } i < \frac{n}{3}\}$ , in other words language of words containing an  $a$  on all positions that are multiples of 3.
2. Generalize to the language  $L_k = \{a_0 \dots a_n \mid a_{ki} = a \text{ for all } i < \frac{n}{k}\}$ .
3.  $a^*b^*$ .
4.  $(ac^*bc^*)^*$ .

## Exercise 3

Determine the language  $L(\phi)$  for the following WMSO formulas  $\phi$ :

1.  $\forall x.P_a(x) \rightarrow \forall y.x < y \rightarrow P_b(y)$
2.  $\exists x.P_b(x) \wedge \forall y.y < x \rightarrow P_a(y)$
3.  $\exists X, Y. \forall z.X(z) \vee Y(z) \wedge \forall x, y.X(x) \wedge Y(y) \rightarrow x < y \wedge P_a(x) \wedge P_b(y)$
4.  $\forall x.(\exists z.x < z) \rightarrow \exists y.x < y \wedge (P_a(x) \leftrightarrow \neg P_a(y))$

#### Exercise 4

*Outline the construction of an NFA from  $\phi$  that recognizes the same language. Why is this useful?*

Give an NFA which recognizes  $\hat{L}(\phi)$  for  $\phi = X(x) \rightarrow \exists y.y < x \wedge Y(y)$ . Use the variable order  $X, Y, x, y$  in the construction.