Finite Automata Exercises

M1 Master Info – 2021

Exercise 1

Explain what a language is. When is a language regular? What does it mean for a language to be recognized by an NFA.

Consider the regular language $(\mathtt{a} \cup \mathtt{b})^*\mathtt{a}(\mathtt{ab})^*$

1. construct an NFA recognizing this language;
2. translate this NFA to an equivalent DFA; and
3. minimise the resulting DFA.

Exercise 2

What does Kleene's Theorem tell us?

Let L and M be two languages recognized by NFAs, say $\mathcal{A}$ and $\mathcal{B}$, respectively. Show that the following languages are recognizable as well.

1. concatenation $L \cdot M$;
2. union $L \cup M$; and
3. intersection $L \cap M$.

Hint: Construct corresponding new automatons based on $\mathcal{A}$ and $\mathcal{B}$. For the last point, use a product construction, where states of the new NFA are pairs of states of the NFAs $\mathcal{A}$ and $\mathcal{B}$.

Exercise 3

What is a decision problem, and what does it mean for such a problem to be in PTIME*?*

Let $\mathcal{A}$ and $\mathcal{B}$ be two DFAs. Reason that the following decision problems are in PTIME:

1. the inclusion problem;
2. the equivalence problem.

How would you reason about these two problems when $\mathcal{A}$ and $\mathcal{B}$ are NFAs?
Hint: Exploit the closure properties of recognizable languages and the known complexity bounds for the emptyness/universality problem