Tradeoffs in Green Cellular Networks

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ABSTRACT
The growing awareness to negative impact of wireless technology on our environment has lead to designing green networks in which energy saving plays an important role. We consider energy saving by switching off a fraction of the base stations. This saving comes at some cost: the coverage is reduced, and moreover, the uplink transmission power of mobiles may increase. This may imply exposure of the human body to stronger electromagnetic fields. We quantify this through the deactivation of base stations under the assumptions that the random location of base stations and mobiles form Poisson processes. While simple calculation yield explicit expression under the light traffic assumption (i.e., negligible interference), stochastic differential equations are used when the interference is non negligible, for the case of exponential attenuation. We observe that when the mobiles have no power constraints, unlike in the case of negligible interference, switching off base stations reduces uplink power.

1. INTRODUCTION
What is green networking? The following definition is given in http://searchnetworking.techtarget.com/ "Green networking is the practice of selecting energy-efficient networking technologies and products, and minimizing resource use whenever possible".

Why is green networking relevant? The volume of traffic is expected to increase dramatically in the coming future and the energy consumed for mobile networks is around 2% of total carbon emission [2]. Moreover, more than 50% of the energy consumption is directly attributed to base station equipment and 30% more to mobiles switching and core transmission equipment [7].

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A growing awareness to the dangers related to large scale energy consumption and drafting of many international agreements as well as legislation have reduced energy consumption in several sector. There is also a growing willingness to reduce energy consumption in wireless networks.

We study another aspect of what we consider as green networking, that of minimizing the average uplink transmitted power, as the latter is proportional to the amount of energy that our body is exposed in communications by wireless terminals. Standards on the maximum amount of permitted radiation to the human exist (see [8]) due to the awareness that the radiation can cause health problems [3]. The energy saving obtained by switching off base stations can results in larger uplink energy and poor coverage. In this paper we quantify the tradeoff between these aspects of green networking: total energy saving and uplink energy transmission.

Several works focus on the base station deployment in order to reduce power while taking into account the Capital Expenditure (CapEX) and Operational Expenditure (OpEx) [6, 5]. Other literature deals with improving the energy efficiency in order to accomplish the same task with less energy. Several solutions aiming at reducing power from base station may be divided into different types as following

- Increasing the number of cells in order to reduce the cell size leading to a reduction in the average transmitted power. This approach is more efficient for indoor network [11, 10]
- Femtocells and indoor distributed antenna systems using MIMO channel: This architecture is used to reduce co-channel interference introduced by frequency reuse among the femto cells and maintain high spectral efficiency [1].
- Cooperation at the base stations level: In [9], the authors show how the degree of redundancy of a network may reduce the power. The authors propose an approach based on cooperation between base stations in order to minimize the active number of base stations while satisfying the minimum required quality of service and minimum coverage.

In this paper we study a sleep mode where a fraction of base stations can be shut down when possible to save energy. Our goal is to provide some insight on the cost of switching off base stations on the uplink energy. We chose to illustrate this tradeoff by an example that involves particular assumptions on the distribution of the base stations and on the interferences: the location of the base stations is assumed to form a homogeneous Poisson point process (Sec 2), and the radio interference is assumed to be negligible. The latter is a feature of operating at a light traffic, which is usually the one in which it is proposed to switch off base stations. Under those assumptions we obtain explicit expressions for the impact of switching off base stations on the total expected power consumption, on the coverage, and on the amount of radiation to the human's body. In section 3, we consider another example which does account for the interference. We derive expression for the expected interference using stochastic difference equation method. We calculate again the expected uplink power and study the impact of switching off base stations. Section 4 contains the concluding remarks and discussions.

2. MODEL & PERFORMANCE ANALYSIS

We consider an OFDMA cellular network. We shall focus on one (arbitrary) resource unit (a given time slot at a given frequency band). Assume that mobiles are Poisson distributed with parameter β and base stations are Poisson distributed with parameter λ. These two processes are assumed to be independent. We assume that each mobile connects to its closest base station. Let the random variable L denote the distance between a mobile and its nearest base station. We first assume that the mobiles are quite sparse or traffic is very light so that we can neglect the interferences. We later provide a more complex model that takes into account interference.

We assume that in order to save power of the base stations at times with low load, one keeps only a fraction $0 < q < 1$ of base stations turned on and the remaining are turned off. The stations that are switched off are chosen at random. Indeed, we assume that the duration of a call is much shorter than the duration of the period during which the base station is turned off, so it is not beneficial to use mechanisms that take into account the current state of the network in order to decide which station should go to sleep. We note that the distance between a mobile to the closest base station is greater than l if and only if within an d-dimensional ball of radius l, there is no base station. The probability of the latter is $\exp(-\lambda V(l))$ where $V(l)$ is the volume of a d-dimensional ball with radius l. In particular, if we consider the problem on a line, i.e., $(d = 1)$,then $V(l) = 2l$. For $d = 2$ it is $V(l) = \pi l^2$.

Let p be the transmission power of any mobile. We assume that there is a limit p_m on the transmitted power p. We call it green limit threshold; its size is determined by health considerations: it is the limit amount of power absorbed by the brain that is allowed. We assume that $p = p(l)$ is controlled such that a target SNR, η, is reached at the closest base station given at a distance l. $p(l)$ is the smallest transmitted power that guarantees the required SNR.

We consider here attenuation due to path loss only so that

$$\frac{p(l)l^{-a}}{\sigma^2} = \eta, \tag{1}$$

where σ^2 denotes noise variance and a denotes the path loss exponent. If $p(l)$ exceeds p_m we assume that there is an outage. When dealing with the line, we assume throughout that $a > 1$, and dealing with the plane, we assume throughout that $a > 2$. Inverting equation (1), we obtain $p(l) = \sigma^2\eta l^a$ The distance at which p_m is reached is denoted by l_m and is given by

$$l_m = \left(\frac{p_m}{\sigma^2\eta}\right)^{\frac{1}{a}} \tag{2}$$

We consider the following frameworks to react to outage:

- **(i) No transmission (NT):** there is no transmission when a mobile is not covered, or

$$p_{nt}(l) = \begin{cases} \sigma^2\eta l^a & \text{if } l \le l_m \\ 0 & \text{otherwise} \end{cases}$$

- **(ii) Always transmit (AT):** Transmission occurs at the maximum power when $l > l_m$ resulting in bad quality of service. Thus

$$p_{at}(l) = \begin{cases} \sigma^2\eta l^a & \text{if } l \le l_m \\ p_m & \text{otherwise} \end{cases}$$

  This is equivalent to $p_{at}(l) = \min(\sigma^2\eta l^a, p_m)$.

2.1 Uplink power and coverage probability

Let $\Delta(\lambda, l_m)$ denote the expected uplink power, i.e.,

$$\Delta(\lambda, l_m) := \mathbf{E}[p(L)] = \int_{B(l_m)} p(s)dP(s),$$

where $B(l)$ is the ball of radius l at the origin. We compute the expected power transmitted by a mobile in the following proposition.

PROPOSITION 1. *In the case of no transmission at outage, the expected power that a mobile node transmits, on a line, is given by*

$$\Delta_{nt}(\lambda, l_m) = 2\lambda \int_0^{l_m} \sigma^2\eta l^a \exp\{-2\lambda l\}dl$$

$$= \frac{\sigma^2\eta 2^{-\frac{a}{2}} l_m^a (\lambda l_m)^{-\frac{a}{2}} \exp\{-\lambda l_m\} WM\left(\frac{a}{2}, \frac{a}{2} + \frac{1}{2}, 2\lambda l_m\right)}{a+1},$$

where $WM(\cdot,\cdot,\cdot)$ denotes the WhittakerM function. On the two dimensional plane, it is given by

$$\Delta_{nt}(\lambda, l_m)$$

$$= \frac{\sigma^2\eta\pi^{-\frac{a}{4}} l_m^a (\lambda l_m^2)^{-\frac{a}{4}} \exp\{-\frac{\pi\lambda l_m^2}{2}\} WM\left(\frac{a}{4}, \frac{a}{4} + \frac{1}{2}, \pi\lambda l_m^2\right)}{\frac{a}{2}+1}$$

In the case of always transmit, the expected transmitted power is given by

$$\begin{aligned}\mathbf{E}[p(L)] &= \Delta_{nt}(\lambda, l_m) + P(L > l_m)p_m \\ &= \Delta_{nt}(\lambda, l_m) + p_m \exp\Big(- \lambda V(l_m)\Big).\end{aligned}$$

The proof of the above proposition and that of the next corollary is direct, except for the expressions for $\Delta(\lambda, l_m)$ for which we thank Maple.

Assume that there is no bound on power transmitted by mobiles p, and denote the expected power in this regime as $\overline{\Delta}(\lambda)$. Then we have the following corollary.

Corollary 1. *As $p_m \to \infty$, we have for the line:*

$$\overline{\Delta}(\lambda) := \lim_{l\to\infty} \Delta(\lambda, l) = \sigma^2\eta(2\lambda)^{-a}\Gamma(a+1)$$

For the plane we get:

$$\overline{\Delta}(\lambda) = \sigma^2\eta(\pi\lambda)^{\frac{-a}{2}}\Gamma\Big(\frac{a}{2}+1\Big)$$

A mobile is connected to a base station if it is within a distance of l_m from any base station, otherwise it will not be covered. The following proposition gives the expression for coverage probability a given mobile is covered.

Proposition 2. *The coverage probability at the target SNR is given by*

$$c(\lambda, l_m) = 1 - \Pr\{L > l_m\} = 1 - \exp(-\lambda V(l_m))$$

in both regimes.

2.2 Effect of Base station deactivation

The aim of the network operator is to minimize the total power spend in the system. We consider a scenario in which operator tries to achieve this goal by turning off those base station that are not loaded heavily. For example, turning off those base stations when a number of mobiles served by them is small. However, it is not possible for the network operator to know a priori which base stations are lightly loaded. So, the operator can decide to switch them off randomly. Recall that q denotes the probability that a given base station turned on. By the thinning property of Poisson point process it is clear that the resulting point process is still Poisson with intensity $q\lambda$. Then the expected power transmitted by any mobile is given by $\Delta(q\lambda, l_m)$.

The figure 1 shows the variations of $\Delta(q\lambda, l_m)$ in q for the 'no transmit'(NT) scenario in a plane. Note that when a large fraction of base stations are turned off, i.e., $q << 1$, the probability that a mobile connected to a base station is small and most of the mobiles do not transmit any power in the NT case. This leads to decrease in the expected power near origin in the above plot. However, the coverage is very poor in this region. This is also shown in figure 1.

The term $\Delta(\lambda, l_m)$ averages (with respect to the distance to the base station) over all potential calls, including those that are in outage conditions. We shall be more interested in measures that characterizes successful calls. We thus define for the "no transmission" regime: $J_{nt}(\lambda) = \frac{\Delta_{nt}(\lambda, l_m)}{c_{nt}(\lambda, l_m)}$. For the case of "always transmit" we have $J_{at}(\lambda) = \Delta_{at}(\lambda, l_m)$. The variation of $J_{nt}(q\lambda)$ as function in q is also depicted in the figure 1.

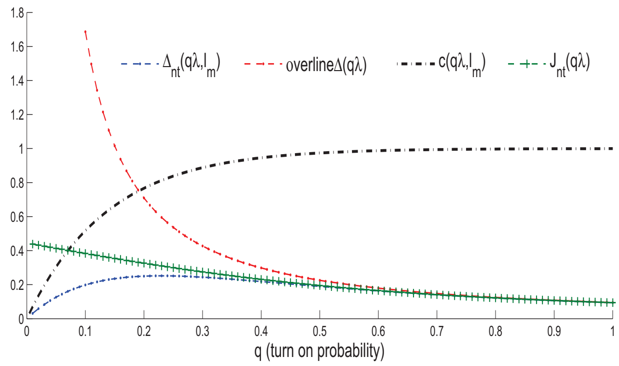

Figure 1: Expected Uplink Power, Coverage and Successful calls: $\sigma^2 = 0.01, \eta = 35, \lambda = 1, a = 2.5, p_m = 1$

2.3 Exponential Attenuation

In this subsection we consider the absorbing channel model instead of path loss model, i.e., the power received at a distance of D from an antenna is given by $\exp(-\xi D)$ times the transmitted power, where ξ is the attenuation factor. (e.g. very humid air: the attenuation is thus exponential in distance). Again we assume that each mobile connects to a nearest base station and transmits power just enough to meet the target SNR η, i.e., a mobile at a distance l from base station transmit power $p(l)$ such that

$$\frac{p(l)\exp\{-l\xi\}}{\sigma^2} = \eta.$$

If p_m is the maximum power that mobile can transmit, then inverting the above equation the corresponding maximum distance is $l_m = (1/\xi)\log\big(p_m/(\sigma^2\eta)\big)$. Repeating the calculation for expected power we obtain following expressions. On the line it is given by

$$\begin{aligned}\Delta(\lambda, l_m) : &= \mathbb{E}[p(L)] = 2\lambda\int_0^{l_m} \sigma^2\eta\exp\{\xi l\}\exp\{-2\lambda l\}dl \\ &= \frac{2\lambda\sigma^2\eta}{\xi - 2\lambda}\Big\{\exp\{(\xi-2\lambda)l_{max}\} - 1\Big\},\end{aligned}$$

and on the plane

$$\begin{aligned}\Delta(\lambda, l_m) &= 2\pi\lambda\int_0^{l_m} \sigma^2\eta l\exp\{\xi l\}\exp\{-\lambda\pi l^2\}dl \\ &= \sigma^2\eta\Big\{1 - e^{(\xi l_m - \lambda\pi l_m^2)} + \frac{\xi\exp\{\xi^2/(4\lambda\pi)\}}{2\sqrt{\lambda}}\cdot \\ &\quad \Big(erf\left\{\frac{\xi}{2\sqrt{\lambda\pi}}\right\} + erf\left\{\frac{2\pi\lambda l_m - \xi}{2\sqrt{\lambda\pi}}\right\}\Big)\Big\}.\end{aligned}$$

The coverage probability remains unchanged.

The behavior of expected transmitted power when a fraction of base stations are switched off is same as in the case of path loss model. In the next section we take the effect of interference into consideration. However, we consider only the absorbing model (exponential attenuation) for analytic tractability.

3. ACCOUNTING FOR THE INTERFERENCE

In this section we take into account the interference in a simple linear model. We make the following assumptions:

- The mobiles and base stations are scattered on a line at locations given by a Poisson process with parameter β and λ respectively, and are independent
- We focus on one resource (in time/frequency)
- Power control: Consider the absorbing channel as in subsection 2.3, but assume that there are no power limitations. Each mobile transmits at a power that guarantees a target SINR of η. Thus for an interference I and a noise variance σ^2 at a mobile, the transmission power should be
$$p(y) = (\sigma^2 + I)\eta \exp(\xi y) \tag{3}$$
where y is the distance between mobile and its base station.
- Each BS has a directional antenna. Assume all antennas transmit towards the east. (For example, in order to communicate with vehicles that go in that direction.) It is then natural to consider also directional receiving antennas at the base stations: they would receive signals sent from the west.
- We assume that if some resource is used at a given cell then the resource is reserved so that within some radius R of the base station, no other call is accepted with this resource. (We allow for $R = 0$ in which case there is no resource reservation). Such reservation is useful in pico-cells as it facilitates fast switching between neighboring pico-cells.
- A base station is restricted to receive one call at a time on a given resource (frequency or time). Therefore if a base station is at the same time the closest to a mobile at x and to a mobile at y, then we have to decide which of them will be chosen to be first to transmit.

Blocking Rate. Let d_n be the location of some mobile that transmits. Let $d_n + y_n$ be the location of the base station that receives the transmission. We assume that this is the base station which is the closest to the mobile on its east. Because of the memoryless property of the exponential random variables, y_n are i.i.d. exponential distributed random variables with parameter λ. Since only one source can transmit to the base station, all other calls whose location is between d_n and $d_n + y_n + R$ are blocked and are thus assumed not to transmit. The next mobile that can transmit is then the one located at $d_{n+1} = d_n + y_n + R + x_n$ where x_n are i.i.d. exponentially distributed random variables with parameter β and independent of y_n. We conclude that the expected distance between two consecutive transmitting mobiles is $E[y_n]+E[x_n]+R = 1/\lambda+1/\beta+R$. Hence the density of mobiles that transmit (and that are not blocked) is

$$\gamma = \frac{1}{\frac{1}{\lambda} + \frac{1}{\beta} + R} \tag{4}$$

which is the harmonic mean of λ and β (when $R = 0$). The blocking rate is then $\beta - \gamma$.

The interference. The interference I_n of a mobile $n \in \mathbb{Z}$, where $\mathbb{Z}$ denotes the set of integers, is the sum of powers received at its base station located at $d_n + y_n$ from mobiles transmitting from d_i over all $i < n$. Note that it satisfies the recursion:

$$I_n = \Big(I_{n-1} + p_{n-1}(\exp(-\xi y_{n-1}))\Big) \exp(-\xi(x_{n-1} + y_n + R)).$$

Recall that we use power control so that the SINR of a mobile will equal a target value η. Substituting equation (3) we get for any n,

$$\begin{aligned} I_n &= \Big(I_{n-1} + \eta(\sigma^2 + I_{n-1})\Big) \exp(-\xi(x_{n-1} + y_n + R)) \\ &= A_{n-1} I_{n-1} + B_{n-1} \end{aligned} \tag{5}$$

where

$$A_{n-1} = (1+\eta)\exp(-\xi(x_{n-1} + y_n + R)), \text{ and} \tag{6}$$

$$B_{n-1} = \eta\sigma^2 \exp(-\xi(x_{n-1} + y_n + R)). \tag{7}$$

We note that the two component random vectors (A_n, B_n) are i.i.d. and that

$$E[A] = (1+\eta)\exp(-\xi R)\frac{\beta\lambda}{(\xi+\beta)(\xi+\lambda)} \tag{8}$$

$$E[B] = (\eta\sigma^2)\exp(-\xi R)\frac{\beta\lambda}{(\xi+\beta)(\xi+\lambda)} \tag{9}$$

THEOREM 1. *The stationary solution of (5) satisfies the following iteration*

$$I_n = \sum_{j=0}^{n-1}\Big(\prod_{i=n-j}^{n-1} A_i\Big) B_{n-j-i} + \Big(\prod_{i=0}^{n-1} A_i\Big) I_0 \tag{10}$$

If the following condition

$$1 + \eta < \exp(\xi R)\frac{(\xi+\beta)(\xi+\lambda)}{\beta\lambda}. \tag{11}$$

is satisfied, then,

$$I_n^* = \sum_{j=0}^{\infty}\Big(\prod_{i=n-j}^{n-1} A_i\Big) B_{n-j-1} \quad \textit{for all} \quad n \in \mathbb{Z} \tag{12}$$

is the finite stationary solution of (5).

PROOF. (A_n, B_n) are a sequence of i.i.d, non negative and finite random variables. Under the assumption (11), we obtain

$$\log E[A] = \log(1+\eta)\exp(-\xi R)\frac{\beta\lambda}{(\xi+\beta)(\xi+\lambda)} < 0.$$

Further Jenson's inequality yields $E[\log A] \le \log E[A] < 0$. Also $E[\log B] < \infty$. Hence we have verified that condition (6) in [4][Thm 2A] holds and the result follows. $\square$

Corollary 2. *Under assumption (11), we have in stationary regime*

$$E[I^*] = \frac{E[B]}{(1-E[A])} = \frac{\eta\sigma^2}{\exp(\xi R)(\frac{(\xi+\beta)(\xi+\lambda)}{\beta\lambda}) - (1+\eta)} \quad (13)$$

For the case where there is no resource reservation, i.e., $R = 0$, the condition (11) becomes

$$\eta < \frac{\xi^2}{\lambda\beta} + \frac{\xi}{\lambda} + \frac{\xi}{\beta} \quad (14)$$

and the expected of interference is obtained as follows

$$E[I^*] = \frac{\sigma^2\eta}{\frac{\xi^2}{\lambda\beta} + \frac{\xi}{\lambda} + \frac{\xi}{\beta} - \eta} \quad (15)$$

Next we obtain the expression for expected power as stated in the following corollary.

Corollary 3. *The expected power is given by*

$$\begin{aligned} E[p_n] &= \eta\Big(\frac{\sigma^2\lambda}{\lambda-\xi} + E[I_{n-1}](1+\eta)\frac{\beta}{\beta+\xi}\exp(-\xi R) \\ &\quad +\eta\frac{\sigma^2\beta}{\xi+\beta}\exp(-\xi R)\Big) \end{aligned}$$

where $E[I_{n-1}]$ is given by (13).

Proof. For any given I_n and y_n from eq. (3) we have

$$\begin{aligned} p_n &= \eta(\sigma^2 + I_n)\exp\{\xi y_n\} \\ &= \eta(\sigma^2 + A_{n-1}I_{n-1} + B_{n-1})\exp\{\xi y_n\} \quad (16) \\ &= \eta(\sigma^2\exp\{\xi y_n\} + (1+\eta)I_{n-1}\exp\{-\xi(x_{n-1}+R)\} \\ &\quad +\sigma^2\eta\exp\{-\xi(x_{n-1}+R)\}) \quad (17) \end{aligned}$$

where equality (16) follows from (5), and equality (17) follows from (6). Note that I_{n-1} does not depend on x_{n-1}. Taking expectation on both sides in (17) and recalling that y_n and x_n are exponentially distributed with parameter λ and β respectively, we get the desired result. □

With the above expression we can study the effect of switching off a fraction of base stations. The following figure 2 shows the expected power as a function of turn on probability q, for different values of β. The interesting point to note is that the expected power is increasing when more and more base stations are turned on while in the case of no interference, as in figure 1 it is decreasing. However, it can be easily see from equation 4 that blocking rate improves (i.e., lesser mobiles are blocked) as q increases.

4. CONCLUSION AND DISCUSSION

We studied tradeoffs arising when turning off base stations. We presented two simple scenarios that allow us to quantify the tradeoffs. In both examples we derived several performance measures related to the network and investigated their dependence on the fraction of base stations that remains operational (not turned off). Our main contribution was to consider the cost of the energy saving obtained by switching off base stations on the uplink. This cost, seldom studied, is relevant to green networking as it is known that the uplink power is the main source of electro-magnetic energy to which humans are exposed.

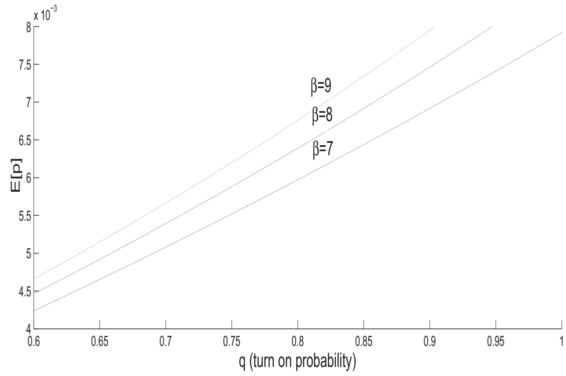

Figure 2: Expected power as function of q: with $\lambda = 3, \eta = 0.2315, \xi = 1\sigma = 0.01, R = 0$

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