VEHICLE NETWORKS: ACHIEVING REGULAR FORMATION

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Abstract

In this paper we will consider a network of vehicles exchanging information among themselves with the intention of achieving a specified polygonal formation. A stochastic model for information transmission and reception is considered, allowing for the randomly breaking of the communication links among the vehicles. The network achieves the formation through decentralized feedback control, which is constructed from the available information. Several information flow laws are considered in order to improve the performance of the vehicle network.

1 Introduction

As we move into an era of autonomous vehicles, the control theory is increasingly used to design and analyze the performance of such decentralized systems. The cooperative use of unmanned vehicles requires some assurance of proper performance, especially when conditions are unfavorable. When centralized coordination is either disabled, impractical or tactically inadvisable, it is unclear whether individual vehicles will be able to properly use the information available to them, or how much information they may need to perform a desired task. One natural question is whether automated vehicles will be able to arrange themselves into a prespecified formation, assuming that one or all of the vehicles has incomplete information as to the whereabouts of other vehicles.

In recent papers [4, 3], the convergence of simple automated vehicles into formation using directed graphs to describe the vehicles' abilities to detect one another, has been explored. In [3] graphical conditions are developed, which allow us to predict when the dynamical systems describing these autonomous vehicle formations will be stable. Assuming an underlying network structure for communication method in which transmission of information amongst the vehicles facilitates a more efficient convergence to their predetermined relative positions has been demonstrated [4]. The formations considered are achieved when the vehicles in question assemble in proper position relative to a stationary central point which is not determined in advance, but will be arrived at through formation consensus.

In this paper we extend these ideas of vehicle formation to include several new features. We first provide a novel vehicle formation formulation, a definition which includes dynamic or moving formations in which the positions of vehicles relative to a fixed frame of reference is not required. This definition allows for convergence into stopped positions oriented about an undetermined final center ([4, 3]), as well as for convergence of moving formations of vehicles which maintain fixed positions relative to one another without coming to a halt (as is desirable in satellite and surveillance formations for example.)

Further, we consider systems in which communication and sensing amongst the vehicles may be faulty, intermittent, or otherwise randomly varying. We introduce a stochastic element to the underlying graph in which arcs representing lines of communication may randomly disappear. After devising a control system for individual vehicles facing the confusing conditions of intermittent sensory data, we also study the possibility of improved performance when vehicles transmit and receive information amongst themselves, again under the conditions of random loss.

2 Problem Formulation

In this work, we consider a network of N vehicles, among which there are some means of communication. The information exchange between vehicles is represented by directed graphs.

2.1 Spectral Graph Theory

We shall first give a brief overview of some spectral graph theory [2, 1] concepts that will be used in modeling the vehicle network.

Definition 2.1 *A directed graph G consists of a vertex set $V(G)$ and an edge set $A(G)$, where an arc or a directed edge is an ordered pair of distinct vertices $a = (u, v)$.The vertex u is called the tail of a and the vertex v is called the head of a.*

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Definition 2.2 *Let u and v be two vertices of a directed graph G. An $u-v$ walk of G is a finite, alternating sequence of vertices and arcs $u = u_0, a_1, u_1, a_2, \ldots, u_{n-1}, a_n, u_n = v$ beginning with u and ending with v, and such that $a_i = (u_{i-1}, u_i)$, for $i = 1, \ldots, n$.*

Definition 2.3 *Let $u, v \in V(G)$. Vertex v is said to be reachable from vertex u if the directed graph G contains an $u-v$ walk.*

Definition 2.4 *A directed graph G is strongly connected if, for every two distinct vertices of G, each vertex is reachable from the other.*

Definition 2.5 *The adjacency matrix of a graph G, denoted $A_d(G)$, is a square matrix of size $|V(G)| \times |V(G)|$ defined as follows:*

$$A_d(i,j) = \begin{cases} 1 & \text{for } (u_i, u_j) \in A(G) \\ 0 & \text{elsewhere.} \end{cases}$$

where $u_i, u_j \in V(G)$.

An equivalent characterization of a strongly connected graph can be given in terms of the adjacency matrix.

Definition 2.6 *A directed graph G is strongly connected if there exists an integer $m > 0$ such that the matrix $(I + A_d)^m$ has all entries strictly positive.*

Definition 2.7 *The number of arcs incident into a vertex u_i is the in-degree of u_i. We denote by D the diagonal matrix with the in-degree of vertex u_i as the (i,i)-th entry.*

Definition 2.8 *The Laplacian matrix of a graph G is defined as $L = I - D^{-1}A_d$.*

2.2 Vehicle Network Model

We assume that each vehicle is a node in a strongly connected graph, G, with the corresponding Laplacian L. The adjacency matrix of this graph represents the communication links among the N vehicles: if $(A_d)_{ij} = 1$ then vehicle i can sense information from vehicle j (we also say that "i can see j"). Note that the links need not be reversible, so i may receive information from j but j doesn't receive information from i. The individual dynamics of each vehicle will be modeled by a discrete-time linear system:

$$\begin{aligned} x^i_{k+1} &= A_1 x^i_k + B_1 u^i_k \\ z^i_k &= \frac{1}{|J_i|} \sum_{j \in J_i} ((x^i_k - h^i_0) - (x^j_k - h^j_0)) \qquad (\Sigma_1) \end{aligned}$$

where, for each time k, $x^i_k \in \mathbf{R}^n$ represents the state variable vector for vehicle i, $u^i_k \in \mathbf{R}^m$ represents the control vector and $z^i_k \in \mathbf{R}^n$ is the output. $J_i \subset [1,N] \setminus \{i\}$ represents the set of vehicles which vehicle i can sense. The output can be interpreted as the measurement of the positions of the neighbors $j \in J_i$ relative to the position of vehicle i. The vectors h^i_0 represent the desired relative positions of each vehicle with respect to the center of the formation.

We now consider the entire system of N vehicles, with state vector $x_k = ((x^1_k)^T, \ldots, (x^N_k)^T)^T \in \mathbf{R}^{nN}$, control vector $u_k = ((u^1_k)^T, \ldots, (u^N_k)^T)^T \in \mathbf{R}^{mN}$, output vector $z_k = ((z^1_k)^T, \ldots, (z^N_k)^T)^T \in \mathbf{R}^{nN}$ and offset vector $h_0 = ((h^1_0)^T, \ldots, (h^N_0)^T)^T \in \mathbf{R}^{nN}$. We let $A = I_N \otimes A_1$ represent the matrix A_1 repeated N times along the diagonal. Similarly, suitable dimensional adjustments are made for the B_1 and F_1 matrices below. Using this notation we rewrite the entire system dynamics as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ sz_k &= L_n(x_k - h_0) \qquad (\Sigma) \end{aligned}$$

where $A = I_N \otimes A_1$, $B = I_N \otimes B_1$ and $L_n = L \otimes I_n$ is the augmented Laplacian of the graph.

Definition 2.9 *N vehicles are said to be in a dynamic formation if at all times they are ordered as the vertices of a pre-specified 2-complex.*

We wish to investigate the problem of controlling this network of vehicles into a specified dynamic formation, where each vehicle will occupy a fixed position relative to the other vehicles (see Figure 1, for an example of a dynamic formation, and section 5 for more simulation results). Using the measurement signal z_k, we intend to construct a decentralized control law for the network of the form $u_k = Fz_k$, where $F = I_N \otimes F_1$ and $u^i_k = F_1 z^i_k$ regulates the system Σ_1 into its position in a dynamic formation.

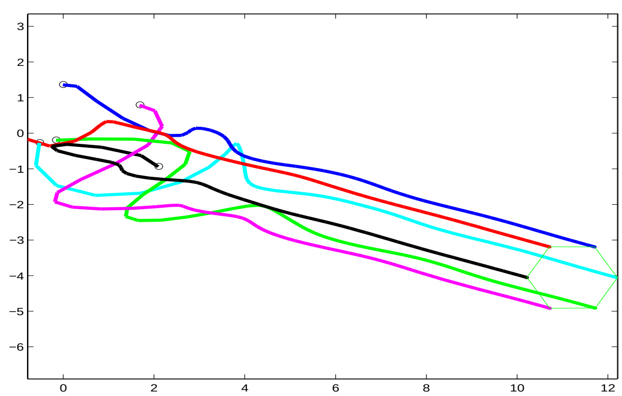

Figure 1: Achieving hexagonal formation, starting from random initial positions.

In section 3 we consider the communication among the vehicles to be in the form of sensory information (vehicle i

receives information from the vehicles it can see, according to the graph). In section 4, we consider another type of communication among the vehicles: both sensory and transmitted information (vehicle i also sends some kind of information to the vehicles that can see him). In both sections we also consider the possibility of random loss of information, how this affects the stability of the network formation, and how to improve the information flow laws for formation control. Finally, in section 5, a model is chosen for the vehicles' dynamics and several results of simulation tests are shown. Some remarks and conclusions end our paper.

3 Sensed Information and Stochastic loss

We will consider N vehicles to achieve "formation" if they are ordered on the vertices of a regular polygon. Notice that this formation can be dynamic, i.e. the positions of the vehicles may not be constant with respect to some global reference point. This is not the case in previous work presented in [4].

3.1 Stochastic loss of information

We first consider the case when sensed information is received with some probability p. At each time step, we associate to each arc (i,j) (with $(A_d)_{ij} \neq 0$), a number θ_k^{ij} which is zero if the sensed information that i would receive from j is lost — in this case, we say that the arc was broken or invisible at time k. In the same way as in [7], we define

$$\theta_k^{ij} = \begin{cases} 1, & \text{if } i \text{ sees } j \\ 0, & \text{if } i \text{ doesn't see } j. \end{cases} \qquad (1)$$

For each pair (i,j), θ_k^{ij} is a Bernoulli process with $\mathbf{P}(\theta_k^{ij} = 1) = p$. We assume that every arc (i,j) has the same probability p of not breaking at time k, and that the random variables θ_k^{ij}, $k = 1,2,\ldots$ are independent. We also assume that each vehicle has access to θ_k^{ij}, in other words, each vehicle knows when an arc is broken.

Whenever an arc is invisible, vehicle i does not receive the new position of its neighbor $j \in J_i$. To overcome this loss of information, we allow vehicle i to use the *last available information* received from vehicle j. So, as discussed in [7], a *sensory information vector* can be defined as follows:

$$w_k^{ij} = w_{k-1}^{ij} + \theta_k^{ij}(x_k^j - w_{k-1}^{ij}) \qquad (2)$$

which represents the information that i receives from j at time k. If the arc (i,j) was not broken, then i receives the new position information, x_k^j, but if the arc was broken, then i receives the latest position information available, w_{k-1}^{ij}. (Note that there is a different vector w_k^{ij} associated with each arc (i,j): if, for instance, $j_1, j_2 \in J_i$, then vehicle i may receive old information from neighbor j_1 and new information from another neighbor j_2.)

Vehicle i will now use w_k^{ij} to compute the control u_k^i. In the case of a broken arc, we improve the sensory information vector by *estimating the current position of vehicle* j instead of using previously received information. Using the old information w_{k-1}^{ij}, and assuming that j would follow its drift direction, the vector w_k^{ij} becomes

$$w_k^{ij} = A_1 w_{k-1}^{ij} + \theta_k^{ij}(x_k^j - A_1 w_{k-1}^{ij}). \qquad (3)$$

Then each vehicle, $i = 1,\ldots,N$, in the network evolves according to

$$\begin{aligned} x_{k+1}^i &= A_1 x_k^i + B_1 F_1 z_k^i \\ z_k^i &= \frac{1}{|J_i|}\sum_{j\in J_i}\left((x_k^i - h_0^i) - (w_k^{ij} - h_0^j)\right) \end{aligned} \qquad (4)$$

Using only the simplest mechanism for estimating the position of their intermittently disappearing neighbors, vehicles were able to collect themselves into position rather efficiently, as shown in Section 5.

4 Information Flow Laws and Stochastic Loss

We next consider that the vehicles that can sense each other may also be able to exchange information. We then introduce stochastic loss of data for a network of vehicles that can sense and communicate simultaneously. We again assume that vehicles know when data is received, or if the data was lost, i.e. they have access to θ_k. In particular, we suppose that whenever vehicle i can sense j, it also receives a relay of the positions of the vehicles that j sees.

As in [4] in the case when no information is lost we consider the following simple information law for each vehicle i:

$$p_{k+1}^i = z_k^i + \frac{1}{|J_i|}\sum_{j\in J_i} p_k^j$$

In other words, vehicle i senses information and also receives information at time k. The information it will have at time $k+1$ is the sum of the information sensed and the average of the information it received from its neighbors at the previous step. Globally, the information law for the system becomes:

$$p_k = \left(\sum_{l=0}^{k} G^l\right) L_n(x_k - h_0)$$

where $G = I - L_n$. The dynamics for the system are:

$$x_{k+1} = Ax_k + BFp_k.$$

In the case when vehicle i receives information from j with some probability p, we assume that every arc (i,j) has the same probability p of not breaking at time k (i.e. information can be transmitted), and that the random variables θ_k^{ij}, $k = 1,2,\ldots$ are independent.

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One question investigated by the authors was whether the incorporation of an information flow law as above into the model (3) would improve the convergence of the vehicles to the formation. Since in this random loss system there is "old" and not very accurate information on the vehicles' positions being passed around, it is conceivable that an information flow law would only increase the level of inaccuracy in the system and thus hinder, rather than improve, the convergence. Indeed the results from our simulations confirm this. To avoid increasing the level of inaccuracy we develop an alternative information flow law. We will see in the next section that this particular information flow law leads to some improvements. For each pair (i, j), such that $j \in J_i$, the information that vehicle i receives from j is given by:

$$\begin{aligned} p_k^{i,j} &= w_k^{i,j}, \quad \text{for } \theta_k^{i,j} = 1 \\ p_k^{i,j} &= \frac{\sum_{l \in J_j} \theta_k^{i,l} \theta_k^{l,j} w_k^{l,j}}{\sum_{j \in J_j} \theta_k^{i,l} \theta_k^{l,j}}, \quad \text{for } \theta_k^{i,j} = 0 \\ p_k^{i,j} &= 0, \quad \text{for } \sum_{j \in J_j} \theta_k^{i,l} \theta_k^{l,j} = 0. \end{aligned} \tag{5}$$

For $j = i$ we set $p_k^{i,i} = x_k^i$.

Note that this law does not involve any "old" information:

- if the arc (i, j) is not broken, then $\theta_k^{i,l} = 1$ and therefore $p_k^{i,j} = w_k^{i,j} \equiv x_k^j$;
- if the arc (i, j) is broken, then $\theta_k^{i,l} = 0$, but any other broken links ($\theta_k^{i,l} = 0$ or $\theta_k^{l,j} = 0$), which would bring in some old information, do not contribute to $p_k^{i,j}$.

The main idea behind this flow law is that, if the arc (i, j) is broken (so "i does not see j") at time k, then vehicle i may still receive vehicle j's new position information, in the following situation. Suppose there exists some other vehicle l, such that $l \in J_i$ and $j \in J_l$ and at time k both $\theta_k^{i,l} = 1$ and $\theta_k^{l,j} = 1$ (that is, both arcs (i,l) and (l, j) are not broken). Then, information flow law (5) states that vehicle i receives the new position information from j through its neighbor l. If several vehicles satisfy these two conditions, an average is computed.

The state transition for each vehicle, in this case, is given by

$$\begin{aligned} x_k^i &= A_1 x_k^i + B_1 F_1 z_k^i \\ z_k^i &= \frac{1}{|J_i|} \sum_{i \in J_i} (x_k^i - h_0^i) - (p_k^{i,j} - h_0^j). \end{aligned}$$

Information flow law (5) is expected to improve the communication in the network, especially in the case of high connectivity graphs.This is shown in Section 5.

5 Simulations

To test and explore our results, a linear discrete-time system was implemented and simulated for several different situations, as described below. A simple statistical analysis, based on Monte Carlo methods, is also presented, that provides a measure of our system's performance. We consider several networks and topologies for the various simulations.

5.1 Decentralized control

In this paper, we restrict our examples to deal with linear feedback control of the form

$$u_k = FL_n x_k - FL_n h_0, \tag{6}$$

where $h_0 \in \mathbf{R}^{nN}$ is a constant vector, containing information about the desired formation. In our examples, we set h_0 to be the positions at the vertices of a regular N-gon: The matrix $F \in \mathbf{R}^{mN \times nN}$ is determined so that individual vehicle will converge to its desired position in formation.

The general problem of finding a matrix K that stabilizes the linear system $x_{k+1} = Ax_k + Bu_k$ with output feedback $u_k = KLx_k$, has an optimal solution K provided by the LQ regulator method. Then the system $x_{k+1} = (A + BKL_n)x_k$ is stable and the solution K is optimal in the sense that some cost function of u_k and z_k is minimized. However, a matrix F obtained by the LQ method would generally result in a centralized control ([6].) For practical purposes, designing a *decentralized control*, is of greater interest for us and is one of the goals of this paper. With a decentralized control each vehicle in the network is able to independently construct its own control u_k^i from its sensed and communicated information. In our design each u_k^i picks up only the information that vehicle i receives form its neighbors, $j \in J_i$. Matrix F is *independent of L* (as opposed to what happens in the LQ regulator case). In all simulations described below we use this constant feedback matrix.

5.2 Simulation Results

As a first step towards understanding the role of an information flow law such as described in [4], we compare the evolution of our network of vehicles and its convergence to regular formation, in the two cases:

(i) each vehicle has access only to its sensed information;

(ii) each vehicle has access to its sensed information as well as to extra communicated information from its neighbors, in the form of an *information flow law*.

From Figure 2, it is clear that the network with information flow law added achieves the desired formation in a more efficient way. In this simulation there were no communication links broken.

Next we consider the effect that random losses of sensed information have on the vehicles in the network, and address

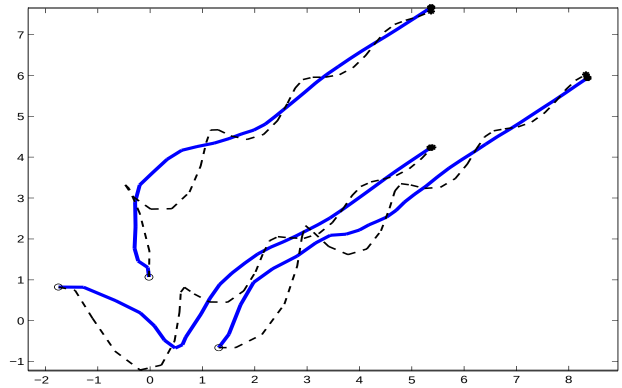

Figure 2: Triangle formation: no information flow (dashed line) and with information flow (solid line).

the question of how to control vehicles to the desired formation under these adverse conditions.

The estimation of the current position as in (3) (as opposed to the more basic equation (2)) is observed to introduce an improvement in the convergence of system (4) to the formation as shown in Figure 3. In this simulation there was no inter vehicle communication.

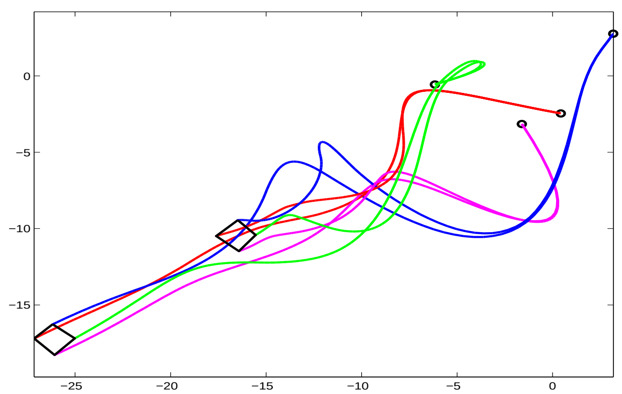

Figure 3: Square formation with $p = 0.4$ of information communication. Comparison between the two sensory information vectors: with estimation of the current position (left corner); with no estimation (center).

Once we allow inter vehicle communication the new information flow law (5) considerably improved the convergence for the vehicle network as shown in Figure 4.

5.3 Monte Carlo analysis

To evaluate the performance of our vehicles under random losses of information, and to try to provide some estimates of rapidity of convergence and deviation from the formation, we focused on the following two questions:

(i) How fast do the vehicles converge to the formation, depending on the probability p of no information loss?

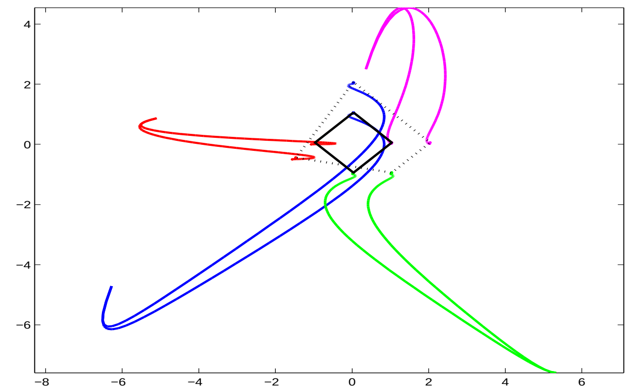

Figure 4: Square formation with random loss of information: no information flow law (dashed line) and with information flow law (solid line).

(ii) After a given fixed time has elapsed, what is the deviation of the formation from the desired formation, as a function of the probability p?

Using a Monte Carlo approach, for each probability $p \in \mathbf{P} := \{0.05s : s = 0, 1, \ldots, 20\}$, the system is simulated M times. For each of these simulations, the initial condition x_0 is a random vector, and the random variables θ_k^{ij} take the value 1 with probability p. For each of these simulations a certain quantity $f = f(p)$ (supposed to depend on the parameter p) is recorded

$$f(r, p), \quad r = 1, \ldots, M, \quad p \in \mathbf{P},$$

and then an estimate of the quantity $f = f(p)$ can be obtained by averaging

$$f(p) = \frac{1}{M} \sum_{r=1}^{M} f(r, p).$$

For question (i), it is necessary to establish the following measure: the network is said to be "within ε of the N-gon formation at time k" if the sides and the diagonals of the N-gon formed by the vehicles at time k do not differ from the ideal values for more than ε. Thus in case (i), the estimated quantity is $f(p) =$ "iteration k, when vehicles come within 0.05 of the square formation".

In Figure 5, for a network of four vehicles and $M = 10$, we can see the estimated number of iterations that are necessary for system (4) to be within $\varepsilon = 0.05$ of the square formation, as a function of p. As expected, for low p the probability of information loss is large $(1 - p)$, and the vehicles need quite a long time to reach the desired formation. An interesting observation is that, as p increases, there seems to be an abrupt transition and, for $p \geq 0.3$, the number of iterations necessary for the network to come to formation does not differ very much, and lies in the range $120 - 160$. In

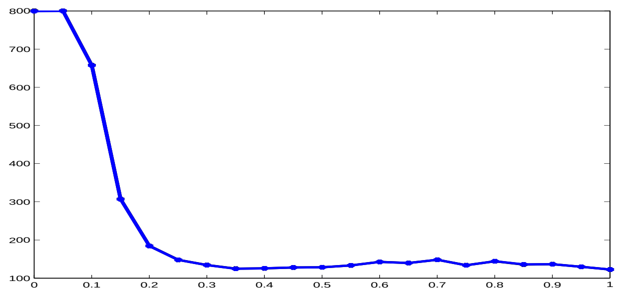

Figure 5: Estimated time to reach square formation, as a function of p.

Figure 6, the error between the exact formation and the position of the vehicles after a fixed number of iterations (40) is plotted against the probability p. As expected, this error decreases as the probability of having no broken arcs increases.

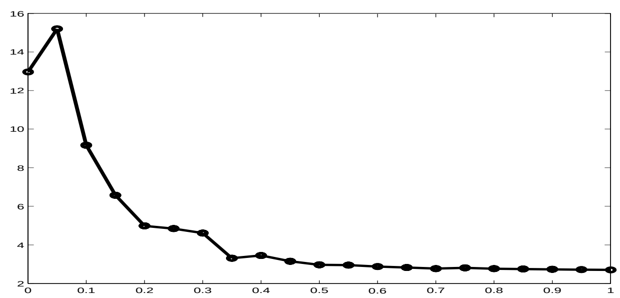

Figure 6: Deviation from formation, after 40 iterations, as a function of p.

6 Conclusion

Our initial goal of smoothly inserting stochastic loss of communication, both passive and active, into a vehicle network has been achieved. The stringency of the graph-theoretical sufficient conditions for system stability (proposed in [4]) made it unclear whether or not such vehicle networks could be expected to reach formation and stay in formation subject to more than a very modest stochastic disturbance of the graph.

We have found that although the static network model required strong connectivity to ensure stability, vehicles in a stochastic model were able to achieve and maintain formation even when the instantaneous graph was not even expected to be connected for the majority of time iterations! It seems from experiment that we need only require strong connectivity in the *underlying* network structure, that is, the collection of all arcs available over the entire course of the simulation. Statistical analysis shows that these vehicles were able to consistently converge into formation with even as much as two-thirds of all communication lost. Using only the simplest mechanism for estimating the position of their intermittently disappearing neighbors, they were able to collect themselves into position in a small number of time steps.

The static network information flow law proposed in [4] did not give satisfactory results when applied to our stochastic vehicle network model (due to its unwaveringly consistent circulation of information, obviously not suited for graphs which may not be connected for several instants).It is clear that a new information flow law based on makeshift single-layer approach has a much better performance.

7 Acknowledgments

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