Workshop on
Commutative Algebra, Syzygies and Singularities,
December 4-6, 2017, Nice, France.

This workshop will take place at the Laboratoire Jean-Alexandre Dieudonné, Université Côte d'Azur, as part of the activity of the GDRI ECO-Math (France-Hongrie-Roumanie).

1 Program of the workshop

Monday 4th:

- **9:30-10:15:** Alexandru Dimca: Open questions on free curves and free surfaces
- Coffee break
- **10:45-11:30:** Jean Vallès: Nearly free curves and arrangements: a vector bundle point of view
- **11:45-12:30:** Alessandro Oneto: A new question on planar polynomial interpolation and line arrangements.
- Lunch break and research in groups in the afternoon

Tuesday 5th:

- **9:00-9:45:** Dorin Popescu: Artin strong approximation for the systems of partial differential equations over the complex number field
- **9:45-10:30:** Claudia Andrei: The coordinate ring of a convex polyomino
- Coffee break
- **11:00-11:45:** Carlos Simpson: Resolution of singularities of a 4 dimensional compactified character variety
- **11:45-12:30:** André Galligo: Deformations of germs of curves with and without embedded components
- Lunch break and research in groups in the afternoon

Wednesday 6th:

- **9:30-10:15:** Marius Vladoiu: Hypergraph encodings of arbitrary toric ideals
- Coffee break
- **10:45-11:30:** Marc Chardin: Syzygies of rings of invariants
- **11:45-12:30:** Laurent Busé: Geometry of rational maps via syzygies.
- Lunch break and research in groups in the afternoon

The workshop activities will take place in room 1, except on Tuesday afternoon in room 2.

2 Abstracts of the talks

Claudia Andrei, University Bucharest, Romania: *The coordinate ring of a convex polyomino.*

The coordinate ring of a convex polyomino was introduced by Qureshi in [Ideals generated by 2-minors, collections of cells and stack polyominoes, Jour- nal of Algebra, 2012, 279-303]. In that paper, it was shown that for a convex polyomino P, the attached ideal IP is a Cohen-Macaulay prime ideal and also a classification of the Gorenstein stack polyominoes was given. We classify all convex polyomino whose coordinate rings are Gorenstein. For this, we use a rep- resentation of the coordinate ring of P as an edge ring of a suitable bipartite graph. In addition, we compute the Castelnuovo-Mumford regularity of the coordinate ring of any stack polyomino in terms of the smallest interval which contains its vertices.

Laurent Busé, Inria Sophia Antipolis, France: *Geometry of rational maps via syzygies.*

In this talk, I will describe how geometric properties of a given rational map between projective spaces can be extracted from the syzygies of its defining homogeneous polynomials. In particular, I will discuss the implicitization of a rational curve or surface in $\mathbb{P}^3$, the fibers of the graph of such a rational map and the computation of the distance.

Marc Chardin, University of Pierre et Marie Curie, France: *Syzygies of rings of invariants.*

I will present joint work with Peter Symonds in Manchester, establishing a conjecture of Derksen on degrees of syzygies of rings of invariants under the action of a finite group. The main ingredient is a result on Koszul homology that could be useful in other contexts (powers of graded ideals or the study of rational maps, for instance).

Alexandru Dimca, University of Nice, France: *Open questions on free curves and free surfaces.*

In this talk I will describe several related open questions on free plane curves and free surfaces in the 3-dimensional projective space. Line arrangements and Terao conjecture on one hand, and rational cuspidal curves on the other, will be discussed from a new point of view.

André Galligo, University of Nice, France: *Deformations of germs of curves with and without embedded components.*

Two recent articles provided generalizations of some results of my joint work with J. Briançon and J. M. Granger: "Déformations équisingulières des germes de courbes gauches réduites ", published in 1980, which described a classification of the natural notions of equisingularity, initially introduced for plane curve germs. These generalizations deal with 1-parameter deformations of complex curve germs with embedded components, but which are reduced surface germs. I will present the subject (topological and algebraic aspects), then question its relation with the construction of "Macaulayfication", i.e. CM-linking of a module.

Alessandro Oneto, Inria Sophia Antipolis, France: *A new question on planar polynomial interpolation and line arrangements.*

Polynomial interpolation problems have been largely studied in algebraic geometry and commutative algebra. The classical question is the following: how many independent conditions a general union of fat points in $\mathbf{P}^2$ give on the complete linear system of plane curves of given degree? A conjectural answer to this question is given by the famous SHGH Conjecture (due to B. Segre, B. Harbourne, A. Gimigliano and A. Hirschowitz) which has been proven to be true in some cases but, in general, is still open. In a recent paper,

D. Cook II, B. Harbourne, J. Migliore and U. Nagel, started to investigate a different question by looking at the conditions imposed by a general fat point to the incomplete linear system of curves of given degree passing through a given set of points X (not in general position). There are cases in which the expected number of conditions is not achieved and we have unexpected curves. In their work, they relate the existence of unexpected curves with properties of the line arrangement dual to the given set of points X. In particular, to the exponents, or splitting type, of the arrangement.

In this talk, after describing the problem and the relation between unexpected curves and line arrangements, I will present a joint project with M. Di Marca (U. of Genoa, Italy) and G. Malara (Pedagogical U. of Cracow, Poland). We classify supersolvable line arrangements whose dual configuration of points admits unexpected curves and we provide new families of line arrangements having this unexpected property.

Dorin Popescu, IMAR, Romania: *Artin strong approximation for the systems of partial differential equations over the complex number field.*

Let F be a system of polynomial equations in $z_1, \ldots, z_q$ and some of their differentials $\partial^{|j_1|} z_{i_1}/\partial x^{j_1}, \ldots, \partial^{|j_s|} z_{i_s}/\partial x^{j_s}$, $i_1, \ldots, i_s \in [q]$, and $j_1, \ldots, j_s \in \mathbf{N}^n$, with coefficients in $\mathbf{C}[[\mathbf{x}]]$. If $F = 0$ has approximate solutions up to any order then $F = 0$ has a solution with coefficients in $\mathbf{C}[[\mathbf{x}]]$.

Carlos Simpson, University of Nice, France: *From multi-V-filtration to V-filtration for a normal crossings divisor, and higher direct image of harmonic bundles.*

We would like to study the compactification of a 4-dimensional character variety, the moduli space of rank 2 local systems on $\mathbb{P}^1$ minus 5 points. I'll explain how a utilisation of SINGULAR has helped to understand what this compactification looks like. The overall goal is to compare it with the Hitchin fibration on the moduli space of Higgs bundles, and some things can be said in this direction.

Jean Vallès, University of Pau, France: *Nearly free curves and arrangements: a vector bundle point of view.*

Many papers are devoted to study logarithmic sheaves associated to reduced divi- sors, in particular logarithmic bundles associated to plane curves since forty years in differential and algebraic topology or geometry. An interesting family of these curves are the so-called free ones for which the associated logarithmic sheaf is the direct sum of two line bundles. When the curve is a finite set of distinct lines (i.e. a line arrangement), Terao conjectured thirty years ago that its freeness depends only on its combinatorics. A lot of efforts were done to prove it but at this time it is only proved up to 12 lines. If one wants to find a counter example to this conjecture a new family of curves arises naturally: the nearly free curves introduced by Dimca and Sticlaru. We prove here that the logarithmic bundle associated to a nearly free curve possesses a minimal non zero section that vanishes on one single point P , called jumping point, and that characterizes the bundle. Then we give a precise description of the behaviour of P . In particular we show, based on detailed examples, that the position of P relatively to its corresponding nearly free arrangement of lines is not a combinatorial invariant.

This is a work in collaboration with S. Marchesi.

Marius Vladoiu, University of Bucharest, Romania: *Hypergraph encodings of arbitrary toric ideals.*

We discuss a combinatorial classification of all toric ideals, via the bouquet structure, which allows us to answer (partially) a few open questions. We also show that hypergraphs exhibit a surprisingly general behavior: the toric ideal associated to any general matrix can be encoded by that of a 0/1 matrix, while preserving the essential combinatorics of the original ideal. Furthermore we provide a polarization-type operation for arbitrary positively graded toric ideals, which preserves all the combinatorial signatures and the homological properties of the original toric ideal.

---

3 Registered participants

- Claudia Andrei, University of Bucharest, Romania
- Laurent Busé, Inria Sophia Antipolis, France
- Marc Chardin, University Pierre and Marie Curie, Paris, France
- Yairon Cid Ruiz, University of Barcelona, Spain
- Mircea Cimpoeas, Institute of Mathematics of the Romanian Academy, Romania
- Clare D'Cruz, Chennai Mathematical Institute, India
- Alexandru Dimca, University of Nice, France
- Sorin Dumitrescu, University of Nice, France
- André Galligo, University of Nice, France
- Andreas Hoering, University of Nice, France
- Roser Homs Pons, University of Barcelona, Spain
- Denis Ibadula, University of Ovidius, Constanta, Romania
- Cristodor Ionescu, Institute of Mathematics of the Romanian Academy, Romania
- Navid Nemati, University Pierre and Marie Curie, Paris, France
- Alessandro Oneto, Inria Sophia Antipolis, France
- Christian Pauly, University of Nice, France
- Adam Parusinski, University of Nice, France
- Dorin Popescu, Institute of Mathematics of the Romanian Academy, Romania
- Carlos Simpson, University of Nice, France
- Hoa Quang Tran, University Pierre and Marie Curie, Paris, France
- Jean Vallès, University of Pau, France
- Marius Vladoiu, University of Bucharest, Romania Romania
- Andrei Zarojanu, Institute of Mathematics of the Romanian Academy, Romania

This workshop was organized with the help of

- Clara Salaun, University of Nice, France
- Sophie Honnorat, Inria Sophia Antipolis, France