Exercise Hilbert polynomial of a set of points in $\mathbb{P}^n$. ($R = k[x_0, \ldots, x_n]$)

1) $p \in \mathbb{P}^n$ a single point. Up to change of coord
$I(p) = (x_1, \ldots, x_n)$ and $R/_{I(p)} \simeq k[x_0]$ so $HP(R/_{I(p)}, i) = 1$

2) $p_1 \neq p_2 \in \mathbb{P}^n$. We have an exact sequence

$$0 \to I(p_1) \cap I(p_2) \to I(p_1) \oplus I(p_2) \xrightarrow{\phi} I(p_1) + I(p_2) \to 0$$
$$(f, g) \to f - g$$
$$h \to (h, h)$$

3) $I(p_1) + I(p_2) = (x_0, \ldots, x_n)$ because one can assume
$I(p_1) = (x_1, \ldots, x_n)$ and $I(p_2) = (x_0, x_1, \ldots, x_{n-1})$
It follows that $HP\left(R/_{I(p_1)+I(p_2)}, i\right) = 0$

4) We deduce that $HP\left(R/_{I(p_1) \cap I(p_2)}, i\right)$
is $\parallel$

$\dim R_i - \dim (I(p_1) \cap I(p_2))_i = \dim R_i - \dim (I(p_1))_i - \dim (I(p_2))_i + \underbrace{\dim (I(p_1)+I(p_2))_i}_{\dim R_i}$

$= \dim R_i - \dim I(p_1)_i + \dim R_i - \dim (I(p_2))_i$

$= HP\left(R/_{I(p_1)}, i\right) + HP\left(R/_{I(p_2)}, i\right) = 2$.

5) $J = I(p_1 \cup p_2)$ and $I(p_3)$: same proof... $HP\left(R/_{\cap I(p_j)}, i\right) = d$.