Iterative learning control based on quasi-Newton methods

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Abstract

In this paper we propose an iterative learning control scheme based on the quasi-Newton method. The iterative learning control is designed to improve the performance of the systems working cyclically. We consider the general type of systems described by continuously differentiable operator acting in Banach spaces. The sufficient conditions for the convergence of quasi-Newton iterative learning algorithm are provided. In the second part of the paper we apply this general approach to the motion control of robotic manipulators. We also recommend to use the conventional feedback control in addition to the learning control. Finally some simulation results are presented.

Key words: iterative learning control; quasi-Newton method; robotic manipulators.

1 Introduction

In this paper we study a general iterative learning control scheme based on the quasi-Newton methods. The iterative learning control is employed to improve the performance of systems working in the cyclical mode such as robotic manipulators, hard-disk drivers, chemical processing. The most primitive way to control such systems is to calculate the desired control by some method, to memorize it and then to apply it during each cycle. However there are several disadvantages of this approach. Firstly the structure and parameters of the system is not known precisely and hence the desired control input is calculated with some error. Secondly the calculation of the desired control can be quite complicated. And thirdly the parameters of the system can be changed significantly after a large number of cycles. The alternative scheme of iterative learning control helps to overcome the mentioned above difficulties. It utilizes the periodicity property of the system. Namely, the controller improves the accuracy of the operations at the current cycle by using the information about the error and the input from the previous cycles. This basic idea was first formulated as a strict mathematical algorithm by Katkovnik and Pervozvanski [7] for the discrete-time systems and by Arimoto *et al* [2] for the continuous-time systems. The algorithm of Arimoto is the particular case of our general approach. Nowadays there is a large number of papers devoted to the subject of iterative learning control. Therefore we refer to the surveys and tutorials [4, 8, 9, 10, 12, 13, 14] in which an interested reader can find the extensive bibliography on this topic. Especially we would like to mention the survey of Moore [10] which comprises the recent developments. The reminder of this paper is organized as follows: In the next section we discuss the abstract approach to the iterative learning control. This approach is based on the quasi-Newton method for the solution of operator equations in Banach spaces [6]. Then in the third section we apply the general scheme to the motion control of robotic manipulators. The sufficient conditions for the convergence of the learning algorithm are given. These conditions demonstrate the advantage of using the conventional feedback control in addition to the learning control. Finally we provide some simulation results, which confirm the fast convergence of the proposed algorithm.

2 Quasi-Newton learning control scheme

Consider the most general setting of learning control scheme. Let the system be described by the operator $F : U \to Y$, where U and Y are Banach spaces. Moreover suppose that the system works cyclically, that is during each cycle we have

$$y_k = F(u_k), \quad k = 0, 1, ...,$$

where $u_k \in U$ and $y_k \in Y$ are the control and the observable output of the system at k-th cycle, respectively. Note that here we do not specify the structure of the abstract spaces U and Y. In real applications it could be either numbers or vectors or functions. For example, in the application that we provide in the next section the output and the control of the system are vector-valued functions over the finite time interval $[0, T]$. Let y_d be a desired behaviour of the system. Our aim is to find the control u_d which is a solution of the operator equation

$$y_d = F(u). \tag{1}$$

Recall that we are dealing with the cyclically working system. This allows us to use the information obtained from the previous cycles to improve the performance of the system at the next cycle. The latter prompts us to use some iterative numerical methods for the solution of the operator equation (1) on line. Namely, let us consider the iterative learning control scheme [13, 14, 15] defined by

$$u_{k+1} = u_k - L(u_k)z_k, \quad k = 0, 1, ..., \tag{2}$$

where $z_k = y_k - y_d$ is the error at the k-th cycle and $L(u_k)$ is a linear operator. We will refer to the operator $L(u_k)$ as the *learning operator*. Note that many well-known iterative methods for the solution of operator equations are also described by the formula (2). In particular, if $L(u_k) = [F'(u_k)]^{-1}$, then the learning procedure (2) becomes the classical Newton method. However, we would like to remind that in this paper we are mainly interested in the case, where the operator F is not given precisely. Indeed there are always some uncertainties in the system description of any real plant. Therefore it is possible to take only some approximation of $[F'(u_k)]^{-1}$ as the learning operator. This idea corresponds to the quasi-Newton method [3, 6]. Despite the numerous publications on this topic (see e.g., [3, 6, 11]), we decided to present our own convergence conditions for the quasi-Newton methods in the Banach spaces. Probably our results are mainly influenced by [3].

Theorem 1 *Let $F : U \to Y$ be continuously differentiable in $S(u_d, r) = \{u \in U \mid ||u - u_d|| < r\}$, where u_d is a solution of the operator equation (1) and $r > 0$. Let the operator $L(u)$ be well defined in $S(u_d, r)$ and suppose that the following conditions hold.*

1) $||L(u)|| \leq \beta, \quad u \in S(u_d, r)$,
2) $||F'(u_1) - F'(u_2)|| \leq \gamma ||u_1 - u_2||, u_1, u_2 \in S(u_d, r)$,
3) $||L(u)F'(u) - I|| \leq \delta < 1, \quad u \in S(u_d, r)$.

Then there exists $\varepsilon > 0$ such that for all $u_0 \in S(u_d, \varepsilon)$ the sequence $\{u_k\}_{k=o}^{\infty}$ generated by (2) converges to u_d. The rate of convergence is linear and can be estimated by

$$\alpha = \delta + \frac{1}{2}\beta\gamma\varepsilon.$$

PROOF: Let us show by induction on k that $||u_{k+1} - u_d|| \leq \alpha ||u_k - u_d||$ with $\alpha < 1$. Consider firstly the case $k = 0$.

$$\begin{aligned} ||u_1 - u_d|| &\leq ||u_0 - u_d - L(u_0)z_k|| \\ &\leq ||u_0 - u_d - L(u_0)(F(u_0) - F(u_d))|| \\ &\leq ||L(u_0)[F(u_d) - F(u_0) \\ &\quad -L^{-1}(u_0)(u_d - u_0)]|| \end{aligned} \tag{3}$$

The expression in the square brackets of (3) can be rewritten as follows:

$$F(u_d) - F(u_0) - L^{-1}(u_0)(u_d - u_0) =$$

$$= \int_0^1 F'(u_0 + t(u_d - u_0))(u_d - u_0)dt - L^{-1}(u_0)(u_d - u_0)$$

$$= \int_0^1 [F'(u_0 + t(u_d - u_0)) - F'(u_0) + F'(u_0) - L^{-1}(u_0)]$$

$$\times (u_d - u_0)dt$$

Substituting the above expression back into inequality (3), we get

$$\begin{aligned} ||u_1 - u_d|| &\leq ||L(u_0) \int_0^1 [F'(u_0 + t(u_d - u_0)) - F'(u_0) \\ &\quad +F'(u_0) - L^{-1}(u_0)](u_d - u_0)dt|| \\ &\leq ||L(u_0) \int_0^1 [F'(u_0 + t(u_d - u_0)) \\ &\quad -F'(u_0)](u_d - u_0)dt|| + ||L(u_0)(F'(u_0) \\ &\quad -L^{-1}(u_0))(u_d - u_0) \int_0^1 dt|| \\ &\leq ||L(u_0)|| \int_0^1 ||F'(u_0 + t(u_d - u_0)) \\ &\quad -F'(u_0)|| ||u_d - u_0||dt \\ &\quad +||L(u_0)F'(u_0) - I|| ||u_d - u_0|| \\ &\leq ||L(u_0)|| \int_0^1 \gamma ||t(u_d - u_0)|| ||u_d - u_0||dt \\ &\quad +\delta ||u_d - u_0|| \\ &\leq \beta \frac{\gamma}{2} ||u_d - u_0||^2 + \delta ||u_d - u_0|| \\ &\leq (\delta + \frac{1}{2}\beta\gamma\varepsilon) ||u_d - u_0|| \end{aligned}$$

All conditions 1)–3) were used in the above calculations.

In order that $u_1 \in S(u_d, r)$ and $\alpha := \delta + \frac{1}{2}\beta\gamma\varepsilon < 1$, we define

$$\varepsilon = \min\{r, \frac{2(1 - \delta)}{\beta\gamma}\}.$$

The parameter ε is positive, since $\delta < 1$ according to Condition 3. Thus,

$$||u_d - u_1|| \leq \alpha ||u_d - u_0||$$

with $\alpha < 1$. To prove the induction step statement $||u_d - u_{k+1}|| \leq \alpha ||u_d - u_k||$, one can use exactly the same consideration. This inequality demonstrates the linear rate of convergence of the iterative procedure (2). □

Remark 1 *We would like to note that Condition 1 can be changed to less restrictive one $||L(u_0)|| \le \beta$ by some modification of the proof.*

3 Application to robotic manipulators

In this section we apply the general results of Theorem 1 to the motion control of robotic manipulators working in the cyclical mode. A multi-link robotic manipulator can be described by the following model

$$A(q)\ddot{q} + b(q,\dot{q}) = u, \qquad (4)$$
$$q(t)|_{t=0} = q(0),\ \dot{q}(t)|_{t=0} = \dot{q}(0),$$

where $q \in \mathbb{R}^n$ is a vector of generalized coordinates, $u \in \mathbb{R}^n$ is a vector of torques applied to the joints, $A(q) \in \mathbb{R}^{n\times n}$ is an inertia matrix and $b(q,\dot{q}) \in \mathbb{R}^n$ is a vector consisting of Coriolis, centrifugal and gravity forces. Note that here we consider the rigid model of robotic manipulators. The robotic manipulators with flexible links demand a special treatment. For instance, the learning control algorithm that is robust in respect to the singular perturbations, which occur in flexible manipulators, was proposed in [15].

Suppose that the task of the robotic manipulator is to track repeatedly the trajectory $q_d(t), \dot{q}_d(t), t \in [0,T]$, where T is the duration of the cycle. After each cycle the manipulator is reset at the initial position $q_d(0), \dot{q}_d(0)$, that is not changed from cycle to cycle. At each cycle k one applies the torques $u_k(t), t \in [0,T]$ and observes the output of the system $y_k(t) := [q_k(t), \dot{q}_k(t), \ddot{q}_k(t)], t \in [0,T]$. The objective of the iterative learning control is to decrease successively the error $z_k(t) = q_k(t) - q_d(t), t \in [0,T], k = 0,1,...$, by using the observations of inputs and outputs from the previous cycles.

Let us now apply the general scheme presented in the previous section to this particular case. First we have to define the system operator F and Banach spaces U and Y. Let F be an operator representing the solution of differential equation (4) if the input $u(t), t \in [0,T]$ is given. Regarding to the Banach spaces U and Y, suppose that $U = C[0,T]$ is the space of bounded vector-valued functions over $[0,T]$ and $Y = C^2[0,T]$ is the space of twice differentiable vector-valued functions with bounded second derivatives over $[0,T]$. The next norms are used in these spaces

$$||u||_C \stackrel{def}{=} \sup_{[0,T]} ||u(t)||_2,$$

$$||q||_{C^2} \stackrel{def}{=} \sup_{[0,T]} ||q(t)||_2 + \sup_{[0,T]} ||\dot{q}(t)||_2 + \sup_{[0,T]} ||\ddot{q}(t)||_2,$$

where $||\cdot||_2$ is an Euclidean norm for vectors in $\mathbb{R}^n$. Suppose that the differential equation (4) has a property such that the operator F is continuously differentiable in the region $S(u_d, r)$ with $r > 0$. Let us also introduce the corresponding region $\Delta = F(S(u_d,r)) \subset Y$. This region represents the admissible changes of the generalized coordinates $q(t)$.

Next we consider the linearization of the differential equation (4) near the trajectory of the k-th cycle.

$$[A(q_k) + \frac{\partial A}{\partial q}(q_k)\Delta q][\ddot{q}_k + \Delta q] + b(q_k,\dot{q}_k) + \frac{\partial b}{\partial q}(q_k,\dot{q}_k)\Delta q$$
$$+ \frac{\partial b}{\partial \dot{q}}(q_k,\dot{q}_k)\Delta\dot{q} = u_k + \Delta u + o(||\Delta q||_{C^2})$$

Taking into account only the linear terms and the dynamics of the system (4), we obtain the linear approximation

$$A(q_k)\Delta\ddot{q} + \frac{\partial b}{\partial \dot{q}}(q_k,\dot{q}_k)\Delta\dot{q} + \frac{\partial b}{\partial q}(q_k,\dot{q}_k)\Delta q$$
$$+ \frac{\partial A}{\partial q}(q_k)\Delta q\ddot{q}_k = \Delta u,$$
$$\Delta q(t)|_{t=0} = 0,\ \Delta\dot{q}(t)|_{t=0} = 0.$$

Note that the solution of the above linear time-variant system can be written in operator terms as $\Delta q = F'(q_k(u_k))\Delta u$. Now for this linear model choose Δu in such a way that $q_{k+1}(t) = q_k(t) + \Delta q(t) = q_d(t)$ or equivalently $\Delta q(t) = q_d(t) - q_k(t) = -z_k(t)$ for $t \in [0,T]$. Since we do not know $A(q)$ and $b(q,\dot{q})$ precisely, we shall use their estimates $\hat{A}(q)$ and $\hat{b}(q,\dot{q})$, respectively. This results in the learning control procedure

$$u_{k+1} = u_k - \hat{A}(q_k)\ddot{z}_k - \frac{\partial \hat{b}}{\partial \dot{q}}(q_k,\dot{q}_k)\dot{z}_k \qquad (5)$$
$$- \frac{\partial \hat{b}}{\partial q}(q_k,\dot{q}_k)z_k - \frac{\partial \hat{A}}{\partial q}(q_k)z_k\ddot{q}_k, \quad k = 0,1,...$$

which can be initialize with $u_0 = \hat{A}(q_d)\ddot{q}_d + \hat{b}(q_d,\dot{q}_d)$. We can see that the derived learning procedure (5) is the particular case of the general scheme (2) with the learning operator

$$L(q_k(u_k))[\cdot] = \hat{A}(q_k)\frac{d^2}{dt^2}[\cdot] - \frac{\partial \hat{b}}{\partial \dot{q}}(q_k,\dot{q}_k)\frac{d}{dt}[\cdot] \qquad (6)$$
$$- \frac{\partial \hat{b}}{\partial q}(q_k,\dot{q}_k)[\cdot] - \frac{\partial \hat{A}}{\partial q}(q_k)[\cdot]\ddot{q}_k.$$

Therefore we can reformulate Theorem 1 in terms of the learning operator (6) for robotic manipulators.

Theorem 2 *Let $F : u \in C[0,T] \mapsto q \in C^2[0,T]$ be an operator associated with the differential equation (4). Let F be continuously differentiable in $S(u_d,r), r > 0$, where $u_d(t)$ is the desired control corresponding to the desired output $q_d(t), t \in [0,T]$. Suppose that the following conditions hold.*

1) $\sup_{t\in[0,T]} \{||\hat{A}(q(t))||, ||\frac{\partial \hat{b}}{\partial \dot{q}}(q(t),\dot{q}(t))||, ||\frac{\partial \hat{b}}{\partial q}(q_k,\dot{q}_k) + \frac{\partial \hat{A}}{\partial q}(q(t))\ddot{q}(t)||\} \leq \beta, \quad q(t) \in \Delta,$

2) *Let $u_1, u_2 \in S(u_d, r)$ be control inputs with corresponding outputs $q_1 = F(u_1) \in \Delta$ and $q_2 = F(u_2) \in \Delta$. Then for any $v \in U$ such that $||v||_C = \sup_{[0,T]} ||v(t)||_2 \leq 1$ the solutions x_1, x_2 of the linear time-variant systems*

$$\begin{aligned} & A(q_1)\ddot{x}_1 + \frac{\partial b}{\partial \dot{q}}(q_1,\dot{q}_1)\dot{x}_1 \\ & + \frac{\partial b}{\partial q}(q_1,\dot{q}_1)x_1 + \frac{\partial A}{\partial q}(q_1)x_1\ddot{q}_1 = v, \\ & x_1(0) = 0,\ \dot{x}_1(0) = 0, \end{aligned}$$

and

$$\begin{aligned} & A(q_2)\ddot{x}_2 + \frac{\partial b}{\partial \dot{q}}(q_2,\dot{q}_2)\dot{x}_2 \\ & + \frac{\partial b}{\partial q}(q_2,\dot{q}_2)x_2 + \frac{\partial A}{\partial q}(q_2)x_2\ddot{q}_2 = v, \\ & x_2(0) = 0,\ \dot{x}_2(0) = 0, \end{aligned}$$

obey the inequality

$$||x_1 - x_2||_{C^2} \leq \gamma ||u_1 - u_2||_C.$$

3) *For any $u \in S(u_d, r)$ and v such that $\sup_{[0,T]} ||v(t)||_2 \leq 1$ let x be a solution of the differential equation*

$$\begin{aligned} & A(q(u))\ddot{x} + \frac{\partial b}{\partial \dot{q}}(q(u),\dot{q}(u))\dot{x} \\ & + \frac{\partial b}{\partial q}(q(u),\dot{q}(u))x + \frac{\partial A}{\partial q}(q(u))x\ddot{q}(u) = v, \\ & x(0) = 0,\ \dot{x}(0) = 0. \end{aligned}$$

There exists a constant $\delta < 1$ such that

$$\sup_{t\in[0,T]} ||\hat{A}(q(u))\ddot{x}(t) + \frac{\partial \hat{b}}{\partial \dot{q}}(q(u),\dot{q}(u))\dot{x}(t) +$$

$$\frac{\partial \hat{b}}{\partial q}(q(u),\dot{q}(u))x(t) + \frac{\partial \hat{A}}{\partial q}(q(u))x(t)\ddot{q}(u) - v(t)||_2 \leq \delta.$$

Then there exists $\varepsilon > 0$ such that for all $u_0 \in S(u_d, \varepsilon)$ the iterative learning procedure (5) converges. Namely,

$$||u_k - u_d||_C \to 0, \quad as \quad k \to \infty.$$

The estimated rate of convergence is given by

$$\alpha = \delta + \frac{1}{2}\beta\gamma\varepsilon.$$

To prove the above theorem one needs only to verify the conditions of Theorem 1. Theorem 2 has not only the theoretical importance, but also it provides us with some practical suggestions. For example, if the learning operator is chosen properly, that is $\delta < 1$, then there always exists the region of convergence with non-zero radius. However this region can be relatively small. Hence it is recommended to start the learning procedure with a good approximation of the desired control. The latter can be achieved by using the conventional feedback circuit in addition to the iterative learning control. The idea of such combination was also proposed in [1, 5, 13, 14]. This suggestion can be formalized as the next algorithm.

$$\begin{aligned} u_{k+1} &= u_k - K_v\dot{z}_{k+1} - K_p z_{k+1} \\ & \quad - \hat{A}(q_k)\ddot{z}_k - \frac{\partial \hat{b}}{\partial \dot{q}}(q_k,\dot{q}_k)\dot{z}_k \\ & \quad - \frac{\partial \hat{b}}{\partial q}(q_k,\dot{q}_k)z_k - \frac{\partial \hat{A}}{\partial q}(q_k)z_k\ddot{q}_k, \quad k = 1,2,... \end{aligned} \tag{7}$$

initializing with $u_0 = -K_v\dot{z}_0 - K_p z_0 + \hat{A}(q_d)\ddot{q}_d + \hat{b}(q_d,\dot{q}_d)$. By appropriate choice of the feedback coefficients K_v and K_p one can ensure that $u_0 \in S(u_d, \varepsilon)$. Moreover the feedback circuit stabilizes the system and decreases the error level inside each cycle. Note that the computation of the time-variant coefficients $\hat{A}(q_k(t))$, $\frac{\partial \hat{b}}{\partial \dot{q}}(q_k(t),\dot{q}_k(t))$,... at each cycle k could be difficult. Therefore we propose to compute these coefficients only for the initial trajectory $q_0(t), t \in [0,T]$ or for the desired trajectory $q_d(t), t \in [0,T]$ and then use them for all iterations. This idea corresponds to the modified Newton method [3, 6]. The algorithm can be written as

$$\begin{aligned} u_{k+1} &= u_k - K_v\dot{z}_{k+1} - K_p z_{k+1} \\ & \quad - \hat{A}(q_k)\ddot{z}_k - \frac{\partial \hat{b}}{\partial \dot{q}}(q_d,\dot{q}_d)\dot{z}_k \\ & \quad - \frac{\partial \hat{b}}{\partial q}(q_d,\dot{q}_d)z_k - \frac{\partial \hat{A}}{\partial q}(q_d)z_k\ddot{q}_d, \quad k = 1,2,... \end{aligned} \tag{8}$$

with $u_0 = -K_v\dot{z}_0 - K_p z_0 + \hat{A}(q_d)\ddot{q}_d + \hat{b}(q_d,\dot{q}_d)$.

All proposed algorithms (5),(7),(8) were tested on the model of the robotic manipulator with two degrees of freedom. In particular, we have

$$A(q) = \begin{bmatrix} m & 0 \\ 0 & m(l^2/3 + (q_1 - l)^2) \end{bmatrix};$$

$$b(q,\dot{q}) = \begin{bmatrix} -m(q_1 - l)\dot{q}_2^2 \\ 2m(q_1 - l)\dot{q}_1\dot{q}_2 \end{bmatrix},$$

where q_1 and q_2 describe the transition and rotation of the robot arm respectively, m is the mass of the robot arm and l is the length of the robot arm. The matrices $\hat{A}(q)$ and $\hat{b}(q,\dot{q})$ have the same structure as the original matrices $A(q)$ and $b(q,\dot{q})$ but we use estimates $\hat{m}$ and $\hat{l}$ instead of m and l. The simulation shows the fast convergence of all algorithms. For example, using the basic algorithm (5) with estimates $\hat{m}$ and $\hat{l}$ such that $|m-\hat{m}|/m = 25\%$ and $|l-\hat{l}|/l = 25\%$, we obtain $||q_5 - q_d||_{C^2}/||q_d||_{C^2} \leq 1\%$. The simulation also shows that one can use the estimates $\hat{m}$ and $\hat{l}$ with greater deviation from the original parameters in the feedback algorithms (7) and (8). This is one more argument in favour of the combination of the classical feedback scheme with the iterative learning control.

4 Conclusions

In the first part of the paper we consider general systems described by operator acting in Banach spaces. The systems are assumed to be working in the cyclical mode. For such kind of systems we propose the iterative learning control based on quasi-Newton methods. We give the sufficient conditions for its convergence. The rate of convergence is linear and depends on our knowledge of the linear part of the system. Afterwards we apply this general scheme to the robotic manipulators. Again the convergence conditions are given. These conditions along with recommendations of other authors [1, 5, 13, 14] encourage us to use feedback circuit in addition to the learning control. Also we propose to calculate the time-variant coefficients of the learning control algorithm only once, for example for the desired trajectory, and then use them for all iterations. Finally we carry out the model simulation to confirm our theoretical results.

Acknowledgment

The author would like to exress his deep gratitude to Prof. A. A. Pervozvanski. The most of the work was done under his wise supervision.

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