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Cover Letter

Dear Editor(s),

Please note that this is the journal version of the paper presented in WiOpt 2010, Avignon, France.

This paper is one among the small set of selected WiOpt 2010 papers that has been chosen to form a special issue of
Elsevier's Performance Evaluation.

The details of the paper are as below:
Paper #1569279397 (Spatial queueing analysis for mobility in pico cell networks).

Thanks and best regards,
Authors

Spatial queueing for analysis, design and dimensioning of Picocell networks with mobile users[1]

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Abstract

In this work, we characterize the performance of Picocell networks in presence of moving users. We model various traffic types between base-stations and mobiles as different types of queues. We derive explicit expressions for expected waiting time, service time and drop/block probabilities for both fixed as well as random velocity of mobiles. We obtain (approximate) closed form expressions for optimal cell size when the velocity variations of the mobiles is small for both non-elastic as well as elastic traffic. We conclude from the study that, if the expected call duration is long enough, the optimal cell size depends mainly on the velocity profile of the mobiles, its mean and variance. It is independent of the traffic type or duration of the calls. Further, for any fixed power of transmission, there exists a maximum velocity beyond which successful communication is not possible. This maximum possible velocity increases with the power of transmission. Also, for any given power, the optimal cell size increases when either the mean or the variance of the mobile velocity increases.

1. Introduction

Since the time 3G technology was designed and deployed, various other technologies have appeared. The ambitious objectives in terms of quality of service offered by 3G technology turned to be quite expensive, which made the 3G technology vulnerable to cheaper competing technologies such as WIFI. Picocell technology has been recently proposed as an alternative that offers some basic connectivity and mobility support, and is yet sufficiently simple so as to be economically competitive ([15, 16]). To prevent a large number of handovers that would result from the small size of the cells ([8, 17]), it has been proposed to group together a number of Picocells in one virtual Macrocell and to restrict the effort of preventing losses due to the handover only to those handovers that occur between Picocells of the same virtual cell. In between the Picocells some fast switching mechanisms are proposed such as frequency following mechanism where the frequency used by a mobile follows it from one Picocell to the next. This requires reserving the same channel for a user in the entire Macrocell.

In this paper we consider a large Macrocell divided into a number of Picocells and study the impact of mobility on such systems, especially the effect of frequent handovers. We

[1]This article is a journal version of the paper [12] presented in WiOpt 2010, Avignon, France

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assume that the ongoing call is never dropped at the Picocell boundary, however base station switching (BSS) at any Picocell boundary requires some fixed amount of information (in terms of bytes) to be exchanged. There is however a possibility of calls being dropped at Macrocell boundaries. We further assume that the active users cross Macrocell boundaries at maximum once, i.e., the calls always end before reaching the second Macro boundary. The handovers at the Macrocell boundaries are modeled as independent Poisson process (this is a commonly made assumption, for example see [5, 6]).

This paper has several goals. First, to model the system so as to predict its performance measures. We are thus interested in developing tools using spatial queuing, that take into account not only the instantaneous geometry but also the way it varies in time. It should thus account for the impact of the speed of the users. We model the Macrocells by various types of queues and well known results from queuing theory (for example [1]) are used to obtain performance measures like expected waiting time, expected service time, drop or blocking probabilities, etc. We shall use these results for preliminary dimensioning purposes in planning the Picocell network catering to pedestrian and vehicular mobility, typical of urban and sub-urban areas. We derive closed form expressions of useful performance metrics considering free space path loss, handover constraints, traffic type etc. We also obtain closed form expressions for optimal cell size, optimal for various performance metrics, when all the users move at the same fixed velocity. To derive these performance measures, we would require the *moments* of the time taken by the system to serve [2] the customers, which in our case will equivalently be the time the Macrocell spends on a call. We derive the expressions for these service times, during which the information is exchanged between the moving user and the set of appropriate base stations (which it encounters during its journey), using variable rate of transmission. We further simplify the expressions for service time under the following assumption: in a typical Picocell network, a moving user would have traversed across a number of cells before the completion of call. In this system, arrivals occur in space and the service time depend on the position, movement of the user and the serving base station(s). The queues modeling these systems are referred to as spatial queues and have been used in interesting applications ([4, 9, 10, 14]). We make the following theoretical and or simulation based observations:
1) Maximum possible velocity: For any fixed power of transmission P, there exists a maximum user velocity $V_{lim}(P)$ and the users moving at speeds greater than V_{lim} can not successfully communicate with the BS;
2) Larger cells for larger velocities: Given P, the optimal cell size increases with an increase in the highest velocity that the system has to support. This is true as long as the highest velocity is less than $V_{lim}(P)$. However the system cannot cater for velocities above the limit $V_{lim}(P)$, even if one increases the cell size indefinitely;
3) Insensitivity to application: The optimal cell size remains the same for non-elastic (NES) as well as elastic (ES) calls for large file sizes, when the rest of the parameters remain same;
4) Two dimensional Manhattan Grid: The one dimensional results can be applied directly to a two dimensional regular street grid (for example [3, 7]).

Towards the end of this paper, we consider a system where call drops are possible at Picocell boundaries and obtain some initial results. To completely avoid call drops at Pico boundaries, one needs a centralized call admission control (a control based on the total number of calls in the entire Macrocell). This requirement can be relaxed if one can design a system

[2] Throughout we use the terms from queuing theory like arrivals, service time etc.

delivering required QoS, in spite of (negligible) call drops at Pico boundaries. We consider one such system (with decentralized call admission control) and obtain closed form expressions for the optimal cell size catering to non elastic traffic.

We describe our system in Section 2 while the service time is discussed in Subsection 3.1 and is optimized in 3.5. The NES, ES calls are modeled by appropriate queuing models and performance measures are derived in Subsections 3.7, 3.8 respectively. The two dimensional regular street grid is studied in Section 4 while the numerical examples are provided in Section 5. Some initial results on a system with possible call drops at Picocell boundaries and with a decentralized call admission control are in Section 6. Some lengthy derivations are provided in two appendices (Appendix M and P) placed at the end of the paper.

2. System Model

We consider a Macrocell, $[-D, D]$, divided into a number of Picocells of length L. Each Picocell has a base station (BS) located at the center and all these BS communicate to a central unit (CU), which controls the entire system. *We assume that there is no interference between any two transmissions.*

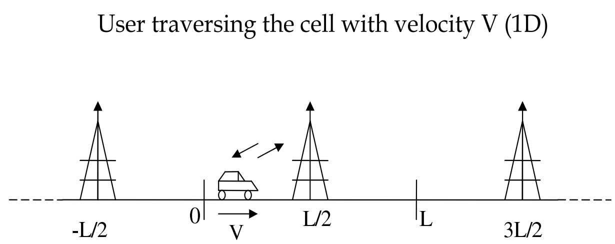

Figure 1: User moving with velocity V along a line

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Traffic Types: Define the waiting time as the duration between the arrival of the call and the instance its service starts. We consider two types of traffic: elastic (ES) and Non-elastic (NES). The non-elastic traffic is very sensitive to the waiting time (multimedia streaming, voice calls etc). A call of this type will be blocked if not picked up within a very small waiting time. The elastic traffic (data traffic) is less sensitive so it is never blocked. Expected waiting time is the appropriate performance measure for the ES calls while the probability of a call being blocked, P_B and the drop probability, i.e., the probability that an ongoing call is dropped before completion, P_D are important performance measures for NES calls. Systems are designed with more stringent requirements on P_D than P_B.
Arrivals: The two (ES, NES) arrivals are modeled by two independent Poisson arrivals with rates λ. Every arrival is associated with Marks (X, V, S): $X \in [-D, D]$ the location of arrival, S the file size requirement and V the user velocity, distributed respectively according to $P_{n,X}$, $P_{n,V}$ and $P_{n,S}$ with respective densities$f_{n,X}$, $f_{n,V}$ and $f_{n,S}$. Let $P_n := (P_{n,X}, P_{n,V}, P_{n,S})$. We *assume symmetry in both directions, i.e., that*

$$P_n((X, V, S) \in A) = P_n((X, V, S) \in -A)$$

for all Borel sets A. In this paper we thus *calculate and analyze without loss of generality (w.l.g.) for* $V \geq 0$.

Handovers: In major parts of this paper (except for Section 6), we assume that handovers are completely successful at Picocell boundaries. However, we do not assume the same for Macrocell boundaries, i.e., a crossover into a new Macrocell results in a successful handover only if the new Macrocell has free servers. We model each handover into a Macrocell as a Poisson arrival, stochastically independent of the new call arrivals (as done for example in [5, 6]). We further assume that there can be at maximum one handover, i.e., the calls get finished before reaching the second Macrocell boundary. This simplifies the analysis to a good extent and is quite a good assumption as the Macrocells are typically large in dimension. We consider generalization of this assumption in our future work. Lastly in Section 6 we obtain some initial results by relaxing the 'handovers at Pico boundaries are completely successful' assumption.

Radio Conditions: The BS communicates with the mobiles using a wireless link, at a rate that depends upon the distance between the two. Since our primary focus is on mobility we implicitly consider Picocells deployed outdoors, for example urban, suburban scenarios. Hence we can assume significant line of sight signal. Further, Picocells being small in size, it will be sufficient to consider only the direct path for communication. A user located at x communicates with BS of cell m using unit transmit power (when receiver noise variance is one) at rate[3] given by,

$$\bar{R}(x;m) := 1_{\left\{\left|x-\left(mL-\frac{L}{2}\right)\right|\le d_0\right\}} + 1_{\left\{\left|x-\left(mL-\frac{L}{2}\right)\right|>d_0\right\}} {d_0}^{\beta} \left|x-\left(mL-\frac{L}{2}\right)\right|^{-\beta}, \tag{1}$$

where $\beta \ge 1$ represents path loss factor and $d_0 > 0$ is a small distance up to which there is no propagation loss. The above model is valid for systems with low signal to noise ratios, where in the rates are directly given by the SNRs.

3. System Analysis

The users are moving continuously with a fixed but random velocity. The Macrocell can handle at maximum K parallel calls. Transmission always occurs at fixed power P. Since Picocells are small in size, the movement of the users results in frequent handovers. The number of handovers will be quite large that it would be complicated to design a reliable system without redundancy: We assume that every BS can also handle K parallel calls[4]. This ensures that, once a call is picked up it is not dropped at any Picocell boundary: when a user crosses over to a new BS, the new BS would at maximum be handling $K-1$ calls and hence will have at least one free server. However it is important to note that the maximum power used at any time in the system equals KP. We further assume that :

[3] The analysis will go through for any other rate functions, for example like $R(x;1) = (1 + (x - L/2)^2)^{-\beta/2}$ ([11]), $R(x;1) = log\left[1+(1+(x-L/2)^2)^{-\beta/2}\right]$ ([11]), $R(x;1) = log\left[1+\left(d_0^{\beta}|x-L/2|^{-\beta}\ 1_{\{|x-L/2|>d_0\}} + 1_{\{|x-L/2|\le d_0\}}\right)\right]$ etc. Some of the simplifications that we obtain in subsequent sections, may not be possible with these rate functions. However one can always conduct Monte Carlo simulations to obtain the required inferences.

[4] In practical systems, each BS will have M backup servers to manage handovers. This means each BS can handle M parallel calls. In general M need not be equal to K, however M has to be chosen large enough to ensure negligible call drops at Picocell boundaries, taking into consideration the large number of handover associated with Picocells. With this large enough M the system's behavior will be close to the system considered in this paper (the case with $M = K$).

1) Every BSS (base station switching at a Picocell boundary) requires fixed B_h bytes of information to be communicated (independent of the user's velocity), after which the user's service is resumed by the BS of the cell it just entered;

2) The user is served by the BS of the Picocell in which it is moving, as it is physically nearest to this BS.

In the following sections up to Section 6, all the results are derived under the assumption that, no call drops ever happen at Pico boundaries.

3.1. Time required for communicating S bytes (B_c)

Define by $B_c(S, X, V)$ the time required to communicate a packet of length S bytes to a user located at X (when the service starts) and moving with velocity V. If the user can communicate at a fixed rate r bytes/sec then the communication time would have been S/r. The maximum rate at which a user can communicate with the BS in cell m is given by (1). This position dependent rate varies: minimum when the user is at the cell edges and increases as the user moves towards the cell center. This poses a need to calculate the communication time considering the variable rates. The location of the user (under service) will change according to $X(t) = X + Vt$ (Figure 1). At time t, if the user is in cell m, i.e. if $X(t) \in [(m-1)L, mL]$, it communicates with the BS of m^{th} cell. Hence the user gets service at time varying rate given by

$$R(t; X, V) := P\bar{R}(X(t); m) \text{ if } t \in \left[\frac{(m-1)L - X}{V}, \frac{mL - X}{V}\right].$$

Without loss of generality we consider the users, whose communication started in the first Picocell, i.e., with $X \in [0, L]$. The communication time B_c required by the user, i.e., the time required to communicate S bytes satisfies :

$$S = \int_0^{B_c} R(t; X, V)dt. \tag{2}$$

Let,

$$g(l) := \int_0^{l/V} P\bar{R}(Vt; 1)dt = P\int_0^{l} \bar{R}(l'; 1)\frac{dl'}{V},$$

represent the number of bytes communicated while the mobile traverses interval $[0, l]$. Note that (throughout it is assumed that $L > 2d_0$: one can easily show that the optimal cell size has to be greater than $2d_0$),

$$\begin{aligned} g(L) &= \frac{Pd_0^\beta}{V}\int_0^{L/2-d_0}\left(\frac{L}{2} - l\right)^{-\beta} dl + \frac{Pd_0^\beta}{V}\int_{L/2+d_0}^{L}\left(l - \frac{L}{2}\right)^{-\beta} dl + \frac{2Pd_0}{V} \\ &= \begin{cases} \frac{2Pd_0}{V(\beta-1)}\left(\beta - \left(\frac{L}{2d_0}\right)^{1-\beta}\right) & \text{when} \quad \beta > 1 \\ \frac{2Pd_0}{V}\left(log(L/2) - log(d_0)\right) & \text{when} \quad \beta = 1. \end{cases} \end{aligned} \tag{3}$$

For any m, the number of bytes communicated as the user traverses through m^{th} Picocell (by change of variable $l = X + Vt - (m-1)L$),

$$\int_{\frac{(m-1)L-X}{V}}^{\frac{mL-X}{V}} R(t; X, V)dt = \int_{\frac{(m-1)L-X}{V}}^{\frac{mL-X}{V}} P\bar{R}(X + Vt; m)dt = \frac{P}{V}\int_0^L \bar{R}(l; 1)dl = g(L)$$

and thus is independent of m. Out of this number, B_h number of bytes are dedicated for BSS. Hence, irrespective of the cell which the user traverses, $g(L) - B_h$ number of bytes are transmitted during the user's journey via one Picocell. Thus the communication time can have three components : 1) Time taken in the originated cell: $(L - X)/V$, 2) Time taken to travel N full cells, where (with $\lfloor t \rfloor$ representing the largest integer in t)

$$N = N(S, X, V) := \left\lfloor \frac{(S - (g(L) - g(X)))}{g(L) - B_h} \right\rfloor$$

represents the number of cells traveled during the communication of S bytes and 3) Time taken in the cell in which the call terminates, i.e., time taken to communicate leftover bytes

$$S_l := S - (g(L) - g(X)) - N(g(L) - B_h).$$

Further a call can be handled only if the bytes that can be communicated while traversing through a cell $g(L)$, is greater than the number of bytes required for BSS B_h. From (2), the communication time $B_c(S, X, V)$ can be calculated as:

$$B_c(S, X, V) = \begin{cases} \frac{1}{V} \arg\inf_{l \in (X, L]} \{(g(l) - g(X)) \geq S\} & \text{if } S < (g(L) - g(X)) \\ \infty & \text{if } B_h > g(L) \\ \frac{L-X}{V} + N\frac{L}{V} + \frac{1}{V} \arg\inf_{l \in (0, L]} \{g(l) - B_h \geq S_l\} & \text{else.} \end{cases}$$

From (3), g is continuous and monotonically increasing function, so g^{-1} exists and thus:

Theorem 1. *Time to communicate S bytes with a user initially located at X and moving with velocity V is,*

$$B_c(S, X, V) \;=\; \begin{cases} \frac{g^{-1}(S+g(X);V)-X}{V} & \text{if } S < (g(L) - g(X)) \\ \infty & \text{if } \;\; B_h > g(L) \\ \frac{(L-X)+NL+g^{-1}(S_l+B_h;V)}{V} & \text{else, where} \end{cases}$$

$$g^{-1}(s; v)$$

$$= \begin{cases} \frac{L}{2}(1 - e^{-\frac{sv}{Pd_0}}) & \text{if } \beta = 1 \quad \frac{sv}{Pd_0} < log\left(\frac{L}{2d_0}\right) \\ \frac{L}{2} + \frac{2d_0^2 e^{-2}}{L} e^{\frac{sv}{Pd_0}} & \text{if } \beta = 1 \quad \frac{sv}{Pd_0} > log\left(\frac{L}{2d_0}\right) + 2 \\ \frac{L}{2} + \frac{sv}{P} - d_0 log\left(\frac{L}{2d_0}\right) - d_0 & \text{if } \beta = 1 \quad else \\ \frac{L}{2} - \left(\frac{sv(\beta-1)}{Pd_0^\beta} + \left(\frac{L}{2}\right)^{-\beta+1}\right)^{\beta-1} & \text{if } \beta > 1 \quad \frac{sv(\beta-1)}{Pd_0^\beta} < d_0^{-\beta+1} - \left(\frac{L}{2}\right)^{-\beta+1} \\ \frac{L}{2} + \left(\frac{2\beta d_0^{-\beta+1}}{\beta-1} - \frac{sv(\beta-1)}{Pd_0^\beta} - \left(\frac{L}{2}\right)^{-\beta+1}\right)^{\beta-1} & \text{if } \beta > 1 \quad \frac{sv(\beta-1)}{Pd_0^\beta} > \frac{(\beta+1)d_0^{-\beta+1}}{\beta-1} - \left(\frac{L}{2}\right)^{-\beta+1} \\ \frac{L}{2} + \frac{d_0^\beta}{\beta-1}\left(\frac{sv(\beta-1)}{Pd_0^\beta} + \left(\frac{L}{2}\right)^{-\beta+1} - d_0^{-\beta+1}\beta\right) & \text{if } \beta > 1 \quad else. \end{cases} \qquad \square$$

Approximation : In Picocell based systems, an user traverses a large number of Picocells while receiving service. Hence the communication time can be approximated by the product of number of cells, $S/(g(L) - B_h)$, and the time taken for traversing each cell L/V:

$$B_c(S, X, V) \;\approx\; \frac{S}{g(L) - B_h}\frac{L}{V} \quad \text{when } g(L) > B_h. \tag{4}$$

In Figure 2 we show that this approximation is very good. We plot the expected value of actual communication time (given in Theorem 1) and the expected value of the approximation (4), for two different velocity profiles. As expected the approximation is very good, in fact for all velocity profiles (one can hardly distinguish the two lines in the figure).

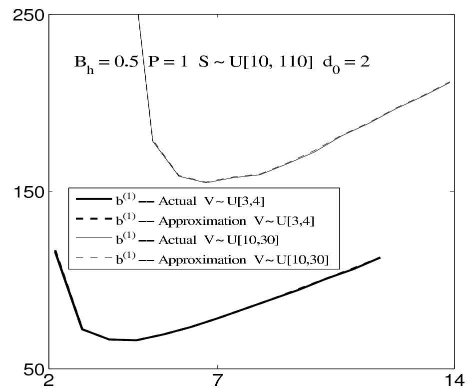

Figure 2: Accuracy of Communication time (B_c) approximation

Remark 1. *If a call is originated at position X and moves with velocity $V \neq 0$ then, in case of ES applications, the call will be picked up at a position X_s $(= X + VW$; W the waiting time) different from X. It is difficult to estimate X_s and the time taken to communicate S bytes, B_c, actually depends upon X_s but not on X. However with the above approximation, $B_c(S, X_s, V) = B_c(S, V)$, i.e., B_c does not depend upon the location of the user when its communication started.*

3.2. Maximum velocity handled by the system

Communication time, B_c, is finite only if the number of bytes transfered $g(L)$ per cell is strictly greater than the bytes required for BSS, B_h. The system can not handle velocities for which the communication times are infinite and hence we obtain (proof in Appendix **M.1**):

Theorem 2. *When $\beta = 1$, for any transmit power P, system can handle all velocity profiles. When $\beta > 1$, there exists a bound $V_{lim}(P) < \infty$ (increasing linearly with P) on the maximum velocity that can be handled by system, where*

$$V_{lim}(P) \quad := \quad \frac{2Pd_0\beta}{\beta - 1}. \qquad \square \tag{5}$$

Henceforth we consider only the velocity profiles that satisfy:

$$P_{n,V}(V < V_{max}) = 1, \text{ where } V_{max} < V_{lim}(P). \tag{6}$$

3.3. Service time : The time of the Macrocell spent for user's service

The user reaches the boundary of the Macrocell starting from a point X in time:

$$B_\partial(X, V) := \frac{D - X}{V}. \tag{7}$$

The Macrocell has to serve the user either until all its S bytes are transmitted (which takes time B_c) or till the user reaches the boundary. Thus, the overall service time requirement of the user in a Macrocell is $B_D := \min\{(D - X_s)/V, B_c(V, S)\}$, where X_s (defined in Remark 1) is the user position when its service starts. By Remark 1, B_c does not depend upon X_s. For NES applications $X_s = X$ the position of arrival. For ES applications, it is difficult to estimate

X_s, instead we approximate X_s with X, i.e., $B_D(X, V, S) \approx \min\{B_\partial(X, V), B_c(V, S)\}$. The error in this approximation is given (with W representing the waiting time) by:

$$E_{rr} = W 1_{\left\{B_c > \frac{D-X}{V}\right\}} + \left(B_c - \frac{D - X_s}{V}\right) 1_{\left\{\frac{D-X_s}{V} < B_c < \frac{D-X}{V}\right\}}$$

and so for any k,

$$E[E_{rr}^k] \le E\left[W^k 1_{\left\{B_c > \frac{D-X}{V} - W\right\}}\right].$$

The *error is small either whenever the number of servers is large (so waiting times are small) or when the Macrocell is large in size (which usually is the case).*

3.4. Macro Handovers

Handover are modeled as Poisson arrivals (as done in [5, 6]). In this subsection we derive the other characteristics of the handover calls.

Distribution of handover call marks (X, S): In general handover densities will be different from the new call densities $f_{n,X}$, $f_{n,S}$ and $f_{n,V}$. As the users move in either direction with equal probability, $P_{h,X}$ the position of handover arrival occurs either at $-D$ or at D with half probability. If handover occurs at $-D$ the corresponding velocity will be positive, which is the case we consider w.l.g. We assume $f_{n,S}$ *is exponential*, i.e., $f_{n,S}(s) = \mu e^{-\mu s}$ for some $\mu > 0$, in which case by memoryless property, $f_{h,S} = f_{n,S}$.

Rate of handovers: Let

$$\nu(v, L) := \frac{(\eta(L) - vB_h)}{L}. \tag{8}$$

Then from (4) and (7),

$$P_{ho} \quad := \quad Prob(B_\partial < B_c) \quad = \quad E_{n,X,V}\left[e^{-\mu\nu(V,L)(D-X)/V}\right] \tag{9}$$

gives the probability that a call is not completed in one Macrocell. This precisely represents that fraction of new arrivals which get converted into handover calls. So

$$\lambda_{ho} := \lambda P_{ho}$$

is the rate at which handovers occur.

Speed of handover arrival : A handover call arrives at $X = -D$ with velocity $v > 0$ only if a new call with velocity v is not completed before reaching the boundary. Here we use the assumption that handover occurs at maximum at one Macrocell boundary, i.e., an handover call is not converted to another handover. Let

$$P_{ho,v} \quad := \quad Prob(B_\partial < B_c | v) \quad = \quad E_{n,X}\left[e^{-\mu\nu(v,L)(D-X)/v}\right] \tag{10}$$

represent the conditional probability given $V = v$. Then the handover speed distribution,

$$f_{h,V}(v) = \frac{f_{n,V}(v) P_{ho,v}}{P_{ho}}.$$

3.5. Moments of Service time

Under assumption (6), the k^{th} moment of B_D exists (whenever the corresponding for S and V^{-1} exist) and equals, $b^{(k)} := E_{X,V,S}[(B_D(X,V,S))^k]$, where $E_{X,V,S}$ is the expectation w.r.t. the (new call and handover call) joint distribution,

$$P_{X,V,S} := \frac{\lambda(P_{n,X}, P_{n,V}, P_{n,S}) + \lambda_{ho}(P_{h,X,V}, P_{h,S})}{\lambda + \lambda_{ho}}.$$

In Appendix **M.2** we obtain (recall E_n is expectation w.r.t. P_n, the new call distribution):

$$b^{(k)} = E_n\left[\frac{B_D(x,v,s)^k + B_D(-D,v,s)^k P_{ho,v}}{1+P_{ho}}\right] \text{ and } \tag{11}$$

$$\frac{db^{(k)}}{dL} = E_n\left[\frac{d}{dL}\left(\frac{B_D(x,v,s)^k + B_D(-D,v,s)^k P_{ho,v}}{1+P_{ho}}\right)\right]. \tag{12}$$

3.6. Cell size optimizing the moments of the service time

The number of bytes that can be communicated in a cell increases with the increase in cell size: from (3) g is continuous and monotonically increasing w.r.t. L. For any given velocity there exists a minimum cell size (the smallest cell size at which one can transmit more than B_h bytes per cell), below which successful communication is not possible. When cell size is closer to this smallest one, the useful bytes transmitted per cell ($g(L)-B_h$) are very small and hence it takes more time to transmit S bytes, i.e., the communication time B_c will be large. As the cell size increases from this smallest size, the communication time B_c starts reducing. However after some point, due to path loss, the number of bytes per cell starts saturating and hence the gain in terms of useful bytes transmitted per cell will be small in comparison with the extra time taken to traverse each cell, resulting in increasing the communication time again. *Thus there exists an optimal cell size for every fixed velocity.* One can extrapolate similar things even for random velocity. We derive the optimal cell size $L^*_{b^{(1)}}$ and relate the same to the optimizer of more interesting performance measures for ES and NES calls in the subsequent sub-sections.

Define $L^*_\nu(v) := \arg\max_L \nu(v,L)$, the maximizer of the function ν given by (8). In Appendix **M.3** we show that, there exists an unique $L^*_\nu(v) > 0$ for every velocity v. In the Appendix **M.4** we further show that, for fixed velocities (i.e., when $V \equiv \bar{v}$), the derivatives of all the (existing) k^{th} moments of the service time vanish only at $L^*_\nu(\bar{v})$. Thus *for fixed velocities, all the (existing) moments of the service time have unique and common minima*:

$$L^*_{b^{(k)}} := \arg\min_L b^{(k)} = \arg\max_L \nu(\bar{v}, L) = L^*_\nu(\bar{v}) \quad \text{for all k.}$$

The common optimizer L^* (for example for $\beta > 1$) satisfies

$$\left.\frac{\partial \nu(\bar{v}, L)}{\partial L}\right|_{L=L^*} = 0 \quad \text{ or } \quad 2P\left(\frac{L^*}{d_0}\right)^{-\beta} L^* - \eta(L^*) + \bar{v}B_h = 0 \text{ and therefore}$$

Theorem 3. *For fixed velocity profile, i.e., $P_{n,V}(V=\bar{v})=1$ the optimal cell size for the expected service time is,*

$$L^*_{b^{(1)}} = L^*_\nu = \begin{cases} 2\left(\frac{2Pd_0^\beta \frac{\beta}{\beta-1}}{2Pd_0\frac{\beta}{\beta-1}-\bar{v}B_h}\right)^{\frac{1}{\beta-1}} & \text{when} \quad \beta>1 \\ 2d_0 e^{\frac{\bar{v}B_h}{2Pd_0}} & \text{when} \quad \beta=1. \end{cases}$$

Further, if the k^{th} moment of the service time exists then $L^*_{b^{(k)}} = L^*_{b^{(1)}}$. $\square$

For velocity profiles with small variance, the optimizers of all the moments of the service time will be equal approximately. Hence when $P_{n,V}$ has small variance with mean $\bar{v}$ then $L^*_{b^{(1)}}$ is close to $L^*_\nu(\bar{v})$. *From the above it is clear that $L^*_{b^{(1)}}$ increases when the mean $\bar{v}$ increases.*

Having obtained the service time, we now turn our attention to model various configurations of the Macrocell with appropriate queues to further obtain their performance measures.

3.7. ES Calls : Average Waiting time

Each Macrocell can handle at maximum K parallel calls. The CU of the Macrocell keeps a record of the users entered into the system and serves them in FIFO (first in first out) order via the BSs of the Picocells. When a new user initiates a call, it is immediately picked up if there are less than K active calls in the system. If not the user will have to wait. Its service will start at the time : 1) when one of the active K users finish their service and exit 2) if there are no other waiting users arrived before it. The BS nearest to the user, at the time of its service start, will initiate the call. Hence after, its call is served (by the Macrocell under consideration) as discussed in subsection 3.1 either till its service is over or till it reaches the Macrocell boundary. When it reaches the boundary the call will be transfered to the next Macrocell as a handover call and the handover call is treated by the new cell similar to that of a new call. Thus each Macrocell can be modeled by a $M/G/K$ queue with service time B_D and Poisson arrivals at rate $\lambda+\lambda_{ho}$. This queue has been analyzed to a good extent and the system is stable only if ([2])

$$\rho := \frac{(\lambda+\lambda_{ho})b^{(1)}}{K} < 1.$$

For stable queues, the expected waiting time of a randomly arrived customer can be approximated by ([2]):

$$E[W]_K = \left(\frac{b^{(2)}}{2(b^{(1)})^2}\right)\left(\frac{b^{(1)}}{K(1-\rho)}\right)\left(\frac{(K\rho)^K}{K!}\right)\pi_0; \quad \pi_0^{-1} = \frac{(K\rho)^K}{K!} + (1-\rho)\sum_{i=0}^{K-1}\frac{(K\rho)^i}{i!}. \quad (13)$$

where $b^{(1)}, b^{(2)}$ are given by (11). If the system is unstable the number of waiting customers grows towards infinity and thus one should consider only the cell sizes L with $\rho<1$. Hence, the optimal size, which minimizes (13) is

$$L^*_{ES} := \arg\min_{\{L:\rho<1\}} E[W]_K.$$

We saw in the previous section that the optimizer of $b^{(2)}$ is same as that of $b^{(1)}$ for fixed velocities and will be close to each other for smaller velocity variances. In a similar way the same thing is true for ρ, i.e., for fixed velocities,

$$L^*_\rho := \arg\min_{\{L:\rho<1\}} \rho = L^*_{b^{(1)}}.$$

The expected waiting time (13) is continuously differentiable in $b^{(1)}$, $b^{(2)}$ and ρ. Thus (minimizer of (13) is a zero of its derivative and $E[W]_K$ depends upon L only via $b^{(1)}$, $b^{(2)}$, ρ),

Theorem 4. *Optimal cell size for a system with elastic traffic, with $P_{n,V}(V = \bar{v}) = 1$ is,*

$$L^*_{ES} = \arg \min_{\{L:\rho<1\}} E[W]_k = L^*_{b(1)}.$$

In other words, $L^*_{b(1)}$ minimizes both expected waiting time and the expected service time. □

Also from (13), when $L^*_{b(1)}$ and $L^*_{b(2)}$ are close, it is easy to see that the optimizer of $E[W]_K$ will be close to that of the expected service time, $b^{(1)}$. Thus even for low velocity variances,

$$L^*_{ES} \approx \arg \min_{\{L:\rho<1\}} b^{(1)} = L^*_{b(1)}.$$

We see that this is true even for many general velocity profiles in examples section 5.

3.8. NES Calls : Block and Drop Probabilities

As before the system can handle at maximum K parallel calls. The call is picked up immediately (by the BS of the Picocell in which the call is originated) only if the Macrocell is serving lesser than K users at the time of its arrival. If all the K servers are busy it is dropped. When an active customer reaches the boundary of a Macrocell, its call is continued in the next Macrocell only if the new Macrocell has free servers. Each Macrocell can thus be modeled by an $M/G/K/K$ queue. And its call block probability is given by the Erlang Loss formula (ρ was defined in previous section),

$$P_B(L) = \frac{\rho(L)^K/K!}{\sum_{k=0}^K \rho(L)^k/k!}.$$

It is interesting to note that $P_B(L)$ and ρ are both differentiable w.r.t. L and further that if the derivative $d\rho/dL$ is zero at a L^* so is the derivative dP_B/dL. By taking the second derivative, we can in fact show that their minimizers are the same. Hence,

$$L^*_{P_B} = \arg \min_{\{L:\rho(L)<1\}} P_B(L) = L^*_\rho.$$

Further at fixed velocities, $L^*_\rho = L^*_{b(1)}$ and so we have,

Theorem 5. *The minimizer, $L^*_{b(1)}$ also minimizes the block probability, P_B, for fixed velocities. For any velocity profile, L^*_ρ also minimizes the block probability.* □

Drop Probability : Under the assumptions stated earlier, only a new call can reach the boundary and not a call which was already handed over once. Further, an active call is dropped only when it reaches the Macrocell boundary and the new Macrocell is busy. By independence of the two events (status of the new Macrocell prior to handover is independent of the call that is handed over), the drop probability is

$$P_D(L) = P_{ho} P_B(L).$$

One can design an optimal system, catering to NES calls, either by jointly minimizing both the block and drop probabilities or by minimizing one of the probabilities while placing a constraint on the other. Usually systems are designed with stringent requirements on P_D than on P_B. We note from the above calculations that P_D is directly proportional to P_B and will be smaller than P_B by a factor P_{ho}. We make here a common assumption that the location of the call arrivals is uniformly distributed, i.e. that $X \sim \mathcal{U}[-D, D]$. Under this assumption:

$$\begin{aligned} P_{ho} &= P_n\left(B_\partial(X,V) < B_c(V,S)\right) = E_{n,S,V}\left[P_X(D - X < VB_c(V,S))\right] \\ &= \frac{E_{n,S,V}\left[\min\left\{2D, \frac{VSL}{\eta(L)-VB_h}\right\}\right]}{2D} \\ &= E_{n,V}\left[e^{-\mu 2D(\eta(L)-VB_h)/VL}\right] + \frac{E\left[1 - e^{-\mu 2D(\eta(L)-VB_h)/VL}\left(1 + \frac{2D\mu(\eta(L)-VB_h)}{VL}\right)\right]}{2D\mu} \\ &< E_{n,V}\left[e^{-\mu 2D(\eta(L)-VB_h)/VL}\right] + \frac{1}{2D\mu}. \end{aligned}$$

Thus P_{ho} decreases with $2D$, the Macrocell size. Macrocells are large in dimension and hence P_D can be ensured to be within the prescribed limits (the limit is a design parameter) by directly minimizing P_B itself. Thus we propose to choose cell size L to minimize P_B and hence equivalently ρ:

$$L^*_{NES} := L^*_\rho = \arg\min_{\{L:\rho(L)<1\}} \rho(L) \approx L^*_{b^{(1)}}.$$

Remark 2. *Thus for both ES and NES applications one needs to minimize the first moment of the service time, $b^{(1)}$, to obtain the optimal cell size. This optimal cell size has been discussed in the previous section for fixed velocities and for velocity profiles with small variances. The general situation is studied in the next section via numerical examples.*

4. Mobility on a street grid

We assume that users are moving in a rectangular grid overlaying a large area $[-D, D] \times [-D, D]$ with grid size b, d as shown in Figure 3. This example is typical of urban areas where the streets are in a criss-cross manner (this is a well known model, see for example [3, 7]) and hence is an interesting example for study.

In this case, we assume that the location of arrival X is uniformly distributed on the lines, i.e, $X \sim \mathcal{U}[\mathcal{G}]$, where the grid

$$\mathcal{G} := \cup_{i=1}^{D/d}[-D, D] \times \left\{id + \frac{d}{2}\right\} \;\; \cup \;\; \cup_{i=1}^{D/b}[-D, D]\left\{ib + \frac{b}{2}\right\} \times [-D, D].$$

A one dimensional vector V represents the speed of the vehicle, which is uniformly distributed, i.e., $V \sim \mathcal{U}[0, V_{max}]$. It's direction depends upon the position of arrival X: it is horizontal if X is on horizontal line and is vertical if on vertical line. One can easily extend the analysis to include zig-zag paths. In either case we assume it be equi-probable in the two possible directions; towards left or right in case of horizontal line and towards up or down in case of vertical line.

Any Picocell is a line segment of a street and a base station is placed at the center of this cell (if we neglect the small number of Picocells that might possibly span across two intersecting streets). The mobiles may change directions as they take a turn, but the rate they obtain with their BS once again follows periodic pattern as explained in section 3.1. Hence the time to reach the boundary B_∂, the time to serve S bytes B_S and hence the service time B_D are just the same as those derived in the previous sections. Thus *the analysis and the results of all the previous sections is applicable to the grid structure. So in the grid structure, the 2 dimensional analysis actually boils down to one dimensional analysis itself. We extended the analysis to a hexagonal 2D cellular structure in which the users can move in any direction and at any point in the cell in [13]. The system in [13] also considers the possible drops at Pico boundaries as well as works with a more distributed call management system as done in the coming section 6 for a one dimensional model.*

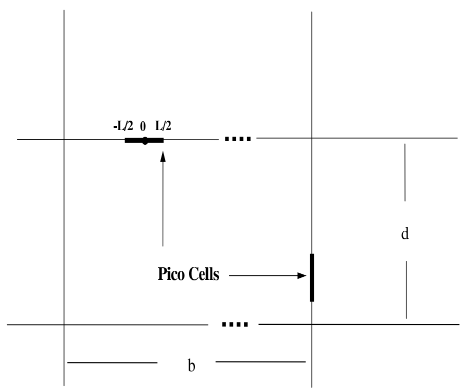

Figure 3: 2D network for rectangular-grid small cell networks

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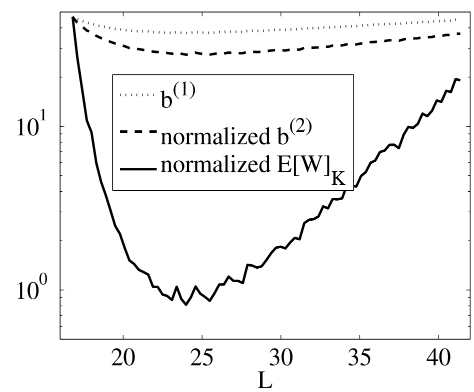

Figure 4: Moments of the service time and the expected waiting time versus L.

5. Mobility Examples

In the numerical examples of this section, we consider uniformly distributed velocity profiles. The position of arrival X is also uniformly distributed in the Macrocell $[-D, D]$. We use Monte Carlo simulations to estimate $b^{(1)}$, $b^{(2)}$ and use exhaustive search to find the optimizers. In figure 4 we plot normalized values of $b^{(1)}$, $b^{(2)}$ and $E[W]_K$ versus L. As discussed earlier we notice that the various performance measures decrease with cell size initially, reach an optimal value and increase again from then on. In fact, all the performance measures have unique minimum at the same L. We study more details of these minimizers in the following.

In figure 5 we plot the optimal cell size (optimal with respect to moments of service time $b^{(1)}$, $b^{(2)}$, block and drop probabilities P_B, P_D of NES calls and the expected waiting time $E[W]_K$ of ES calls) versus mean velocity for two different values of variance. We set $d_0 = 5$, $\lambda = 0.1$, $B_h = 2$, $P = 1$, $\mu = 5$, $K = 20$ and consider a Macrocell of size $D = 1000$. We also plot L_ν, which is the maximizer of $\nu(E_n[V], L)$. For small velocity variances (curves with variance equal to 1), all the minimizers are close to L_ν. For large velocity variance, we notice that all the minimizers ($L^*_{b^{(1)}}$, $L^*_{b^{(1)}}$, $L^*_{P_B}$, $L^*_{P_D}$ and $L^*_{E[W]}$) are away from L_ν, but however are close to each other for most cases. That is, the minimizers of expected waiting time are the same as that of block as well as drop probabilities and all of them equal $L^*_{b^{(1)}}$. This suggests that *even for velocity profiles with high variances, it is sufficient to optimize the average service time* $b^{(1)}$ *for both ES as well as NES calls and hence the optimal cell size*

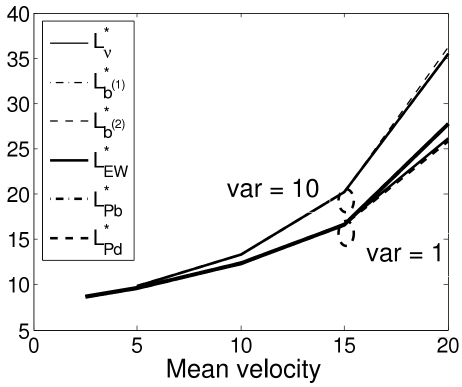

Figure 5: Optimal cell size versus mean velocity for different variances.

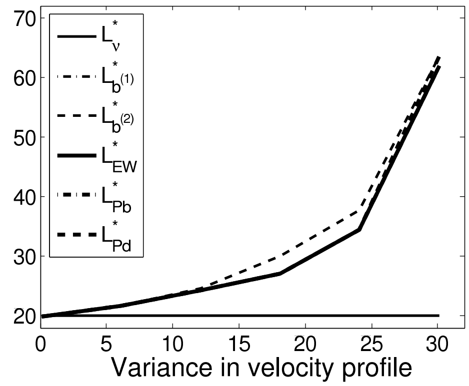

Figure 6: Optimal cell size versus variance of the velocity.

again remains independent of the application. However in this case, it is not sufficient to minimize $\nu(E_n[V], L)$ but one needs to minimize $b^{(1)}$ directly. In Figure 6 we plot the various optimal cell sizes as a function of velocity variance. We set mean, $E[V] = 10$, $d_0 = 5$, $\lambda = 0.1$, $B_h = 2$, $P = 1$, $\mu = 5$, $K = 20$. We once again note that all the minimizers are close to each other for many cases. We also note that all the minimizers are close to L_ν for low velocity variances. We further observe that the optimal cell size increases with increase in the variance also. Thus *larger the velocities the system has to support, the larger are the optimal cell sizes.*

We notice in both the figures 5 and 6 that, only the optimizer of the second moment $L^*_{b^{(2)}}$, is some times different from the rest of the minimizers. However, even when $L^*_{b^{(2)}}$ is different from $L^*_{b^{(1)}}$, the minimizer L^*_{EW} is equal to $L^*_{b^{(1)}}$ and thus for both the types of traffic $L^*_{b^{(1)}}$ gives the optimal cell size.

6. Call drops at Picocell boundaries (NES calls)

So far in our analysis, we have assumed that there are no call drops during handover when the mobile traverses across Picocells. We hence called it as just BSS (Base station switching). In practice even the handovers at Picocell boundaries can fail, even though with an extremely small probability. In this section we obtain some initial results considering possible Pico drops for the special case of uniform arrivals, uniform velocity profile and the networks catering to NES calls. We further assume that the velocity profile does not include 0 so that $E[1/V] < \infty$.

Every Pico handover, as before, needs B_h bytes to perform the BS switching and in addition has a (small) possibility of not being successful. We now call the BSS near a Picocell boundary also as a handover. The BS of the Picocell admits a call (handover or a new one), as before K calls at maximum, however some of the new calls are not picked up to reserve resources for the handover calls, now to keep the overall Drop probability under the required limit. Further in this case the call admission control can also be distributed: 1) in a centralized scheme the CU directs the Pico BS as in the previous section, either to admit or not admit the new calls (base stations possibly are using fewer number of servers); 2) in a decentralized scheme, the Pico BS always reserves K_1 servers exclusively for handover calls. That is, in a decentralized scheme, the Pico BS (independent of the other Picocells) admits a new call if more than K_1 servers are free while a handover call is admitted whenever any server is free.

In this paper we consider a call admission control scheme that is decentralized and is different from the above two schemes. A new call is *considered for picking up* with probability p independent of everything else. And a call *considering for picking up* will be picked up when any one of the servers is free. This scheme is close to the decentralized scheme described above when p is close to one, i.e., when K_1 is close to 0. The centralized scheme (with Pico drops) on the other hand has to be handled using totally different queuing models and is not considered in this paper. Note that smaller p implies larger priority to handover calls. In this case *each Picocell itself is modeled as a separate $M/G/K/K$ queue.*

Remark 3. *We considered a simple, analytically tractable and a decentralized call admission control in this section. One can analyze a system which reserves K_1 servers for the handover calls using priority queues and we are currently working towards it. On the other hand it is more difficult to analyze a system with centralized control (with possible Pico drops), in which the CU directs to admit new calls or not depending upon the total number of calls in the system. One will need the theory of interacting queues for such a study.*

6.1. Service Time : Time of the Picocell spent for user's service

The service time B_L, now represents the overall time of a Picocell used by the mobile. Thus the service time will be much smaller in comparison with the previous section and its analysis will be significantly different. There is no more periodicity while obtaining the service time, however the analysis in this case also simplifies, now due to small values of L. The service time components B_c and B_∂ are:

$$B_\partial = \frac{L-X}{V} \text{ and } B_c = \frac{g^{-1}\left(S+g(X);V\right)-X}{V} \text{ when } B_c \le B_\partial$$

with the overall service time as before equal to $B_L = \min\{B_\partial, B_c\}$. We use notation B_L instead of B_D for service time, to emphasize that this is the service time of a Picocell of length L. Further,

$$P_{ho} = Prob(B_\partial < B_c) = Prob(g(L) - g(X) < S) = E[e^{-\mu(g(L)-g(X))}] = E[e^{-\mu\frac{\eta(L)-\eta(X)}{V}}]$$

and $P_{ho,v}$ also changes similarly. However it is easy to see that as the cell size decreases towards zero the function inside the expectation converges to 1 for all the realizations and hence by BCT P_{ho}, $P_{ho,v}$ (for all v) converge to 1 as $L \to 0$. Thus for Picocells *we approximate P_{ho}, $P_{ho,v}$ by 1.*

The service time in a Picocell with high probability (the probability increasing as $L \to 0$) will be equal to the time taken to traverse the cell itself, as with high probability the user has to pass through many Picocells before completing his request and thus:

$$B_L(S, X, V) \approx \frac{L-X}{V}.$$

6.2. Pico Handovers

<u>Handover arrival rate :</u> There is a major difference in the analysis while calculating the handover rate λ_{ho}: now we should consider that the (Pico) handovers can happen many times (and not at most once as in case of a Macro handover) during a call duration. The calculations though different can easily be carried out. We introduce some new notations for this

purpose. Let $P_{ho,\partial}$, $P_{ho,\partial,v}$ respectively represent terms equivalent to P_{ho} and $P_{ho,v}$ for a call that is already handed over (at least once). It is easy to see that (when $P(V < V_{lim}(P)) = 1$) by BCT again (using equation (8)):

$$P_{ho,\partial} = Prob(g(L) < S + B_h) = E\left[e^{-\mu(g(L)-B_h)}\right] = E\left[e^{-\mu\frac{L\nu(L)}{V}}\right] \to 1 \text{ as } L \to 0$$

and similarly $P_{ho,v,\partial} \to 1$ as $L \to 0$ (for all v). By Taylor series, $e^{-x} \approx 1 - x$ for small values of x and this is used for obtaining another approximation for $P_{ho,\partial}$ again using BCT:

$$(1 - P_{ho,\partial}) - \mu L E\left[\frac{\nu(L,V)}{V}\right] \to 0 \text{ as } L \to 0. \tag{14}$$

This approximation is more appropriate when we consider terms like $1/(1 - P_{ho,\partial})$. From (8) for L small using (14),

$$1 - P_{ho,\partial} \approx \frac{\mu L \nu(\bar{v}_{inv}, L)}{\bar{v}_{inv}} \text{ where } \bar{v}_{inv} := \frac{1}{E\left[\frac{1}{V}\right]}. \tag{15}$$

Let, $\lambda_L := pL\lambda/D$ be the effective rate at which the users arrive into the Picocell of interest $[0, L]$. Note in the above that $L\lambda/D$ represents the actual rate at which the new calls arrive in $[0, L]$ while λ_L represent the arrival rate of those sampled users who are considered for picking up. The handover rate λ_{ho} in this case can be calculated as: 1) due to symmetry, the handovers from Picocell 0 ($[0, L]$) to cell 1 ($[L, 2L]$) are stochastically same as those from cell -1 ($[-L, 0]$) to cell 0; 2) the same is true for handovers when a mobile travels from right to left; 3) so, the handovers into a cell of interest (cell 0) are stochastically same as those that go out of the cell 0; 4) thus under the assumption that handovers are Poisson in nature, the handover rate should satisfy the following fixed point equation:

$$\lambda_{hL} P_{ho,\partial} + \lambda_L P_{ho} = \lambda_{hL}.$$

Using the approximation in equation (15),

$$\lambda_{hL} = \frac{\lambda_L P_{ho}}{1 - P_{ho,\partial}} \approx \lambda \frac{p\bar{v}_{inv}}{D\mu\nu(\bar{v}_{inv}, L)}. \tag{16}$$

Remark 4. *Note that* $L^*_{\lambda_{hL}} := \arg\min_L \lambda_{hL} = L^*_\nu(\bar{v}_{inv})$ *and that for fixed velocities (when* $V \equiv \bar{v}$*)* $\bar{v}_{inv} = \bar{v}$. *Thus for fixed velocities, interestingly for any value of* p*, the cell size optimizing the the handover rate* λ_{hL} *(considering the Pico drops) is the same one that is optimal for NES as well as ES calls, obtained by neglecting the Pico drops.*

Handover Speed distribution: Note in equation (16) the expectation $E[1/V]$ in the last term is w.r.t. to the distribution corresponding to the handover calls and hence one needs to calculate these distributions. The handover arrivals are either at 0 or at L with half probability and since we *are considering the positive velocities without loss of generality the position is always at* 0. The (Pico) handover speed distribution, $f_{h,V}(v)$, (after considering the drops at Picocells) once again satisfies another fixed point equation due to the statistical similarity between the arrivals into and out of the cell 0. In Appendix P, this fixed point equation is derived and the Pico handover speed distribution is shown to converge to uniform speed distribution as the cell size L tends to zero. Thus *the (Pico) handover arrivals have approximately uniform speed distribution.*

6.3. Stability Factor

The stability factor ρ in a Pico queue is calculated in Appendix P using the approximations for small cell sizes and we obtain:

$$\rho_{pico}(L) \quad = \quad \frac{(\lambda_L + \lambda_{hL})b^{(1)}}{K} \quad \approx \quad \frac{\lambda p L}{KD\mu\nu(\bar{v}_{inv}, L)}. \tag{17}$$

Define

$$L^*_{\nu/L}(v) := \arg\max_L \frac{\nu(v, L)}{L}.$$

Thus the optimal cell size for stability factor is,

$$L^*_{\rho,pico} := \arg\min_L \rho_{pico}(L) = L^*_{\nu/L}(\bar{v}_{inv}).$$

6.4. New Call Block Probability

The Picocell can be modeled by an $M/G/K/K$ queue as in the previous section. Let $P_{Busy,pico}$ represent the probability that an arrival (a new or an handover call) finds all the K servers busy in the Picocell. This probability can be calculated using the results from queuing theory as done in the previous section using Erlang Loss formula,

$$P_{Busy,pico}(L) := \frac{\rho_{pico}(L)^K/K!}{\sum_{k=0}^{K} \rho_{pico}(L)^k/k!}.$$

A new arrival is not picked up either with probability $1-p$ when the BS intentionally does not consider it for picking up or with probability $pP_{Busy,pico}$ when all the servers are busy. Thus, the new call block probability for system when considering Pico drops will be given by,

$$P_{B,pico}(L) = (1-p) + pP_{Busy,pico}(L).$$

From the above the cell size optimizing the block probability $P_{B,pico}$ will be same as that optimizing $P_{Busy,pico}$, which further is same as $L^*_{\rho,pico}$ (using the logic as in section 3.8). Thus it is clear that, $L^*_{P_{B,pico}} = L^*_{\nu/L}(\bar{v}_{inv})$ and from (17) this optimizer is a zero of

$$\frac{d\rho}{dL} = c_1 \frac{2L(\eta(L) - \bar{v}_{inv}B_h) - L^2\eta'(L)}{(\eta(L) - \bar{v}_{inv}B_h)^2},$$

where c_1 is an appropriate positive constant. Hence we have,

Theorem 6. *The cell size optimizing the block probability of the NES calls considering the possible drops at Pico boundaries, when resources are reserved for handover calls by intensionally dropping some of the new arrivals, is given by,*

$$L^*_{P_{B,pico}} \quad = \quad L^*_{\rho,pico} = L^*_{\nu/L} = \begin{cases} 2\left(\dfrac{Pd_0^\beta \frac{\beta+1}{\beta-1}}{2Pd_0\frac{\beta}{\beta-1} - \bar{v}_{inv}B_h}\right)^{\frac{1}{\beta-1}} & when \quad \beta > 1 \\ 2d_0 e^{\frac{\bar{v}_{inv}B_h - Pd_0}{2Pd_0}} & when \quad \beta = 1. \end{cases} \qquad \square$$

When this optimal cell size is compared at fixed velocities with that obtained after neglecting the Pico drops (that obtained in Theorem 3) we find that: 1) when $\beta > 1$, the two cell sizes matches except for a factor of $((\beta+1)/(2\beta))^{1/(\beta-1)}$; 2) when $\beta = 1$, the difference is in the power of the exponent, an extra $-Pd_0$ factor for the cell size with Pico drops. This differences are negligible only when β is close to 1 (but not equal to 1). The two systems use different call admission control mechanisms and hence the difference. Nevertheless *the optimal cell size of the Theorem 6 is valid for a distributed call management system and further it gives the cell size even for velocity profiles with non zero variance.*

6.5. Drop Probability

It was straight forward to calculate the drop probability for the previous case (i.e., without Pico drops) as the drop could have occurred at maximum at one Macro boundary. It is more tedious to calculate the same when drops are also possible at Pico boundary. This tedious job is carried out in Appendix P, wherein the drop probability is obtained by conditioning on two events. We obtain (see Appendix P):

$$P_{D,pico} = \frac{P_{Busy,pico}}{\frac{\mu(\eta(L)-\bar{v}_{inv}B_h)}{\bar{v}_{inv}} + P_{Busy,pico}}$$

It is interesting to see that both the call drop and the new call block probabilities depend upon the busy probability $P_{Busy,pico}$ and one can thus design a optimal system (considering Pico drops) by minimizing the busy probability or equivalently the stability factor ρ_{pico}. We thus propose to choose the optimal cell size:

$$L^*_{NES,pico} := L^*_{\rho,pico} = L^*_{P_B,pico},$$

which is obtained in Theorem 6.

7. Conclusions and Future work

We looked at the problem of characterizing the performance of Picocell networks in the presence of mobility. We modeled various traffic types between base-stations and mobiles as different types of queues. We derived explicit expressions for expected waiting time, service time and drop/block probabilities for the various queuing models considered for both fixed as well as random velocity of mobiles. We showed that there exists an optimal cell size for a given velocity profile, which minimizes the service time for elastic applications as well as the drop and block probabilities of non-elastic applications. We obtained (approximate) closed form expressions for this optimal cell size when the velocity variations of the mobiles is very small. We find that if the call is long enough, the optimal cell size depends mainly on the velocity profile of the mobiles, its mean and variance.; It is independent of the traffic type or duration of the calls. We show that for any fixed power of transmission, there exists a maximum velocity beyond which successful communication between the mobile and the system is not possible. This maximum possible velocity increases with the power of transmission. Further, for any given power, the optimal cell size increases when either the mean or the variance of the mobiles velocity increases.

The mobility models considered in this paper are suitable for modeling users traveling continuously with considerable speeds (example users traveling in a car). The movement of slower users (e.g., pedestrians) can be better captured by either Random Walk or Random Way-point model. Still better would be to consider systems with heterogeneous users (slow moving, fast moving and users that are at rest). We are currently working towards this. We considered two dimensional Picocellular networks in [13] catering to non elastic users. In that work, we brought out some issues while designing the Pico networks that are specific to two dimensional cellular networks. It would be interesting to extend those results for elastic users and further for heterogeneous users.

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Appendix M: Calculations related to Macro queue

M.1 *Proof of Theorem 2*

The communication time is finite with probability one if and only if

$$Prob(B_h > g(L)) = P_{n,V}(VB_h > \eta(L)) = 0, \quad \text{where } \eta(L) := Vg(L).$$

Note from (3) that η is only a function of P, L and hence that this probability depends only upon the velocity profile V, cell size L and the transmit power P. Because of the path loss, for any fixed P, $\eta(L)$ increases as L increases and finally saturates (when $\beta > 1$). Thus for all L (when $\beta > 1$),

$$\eta(L) \leq \eta_\infty := \lim_{L\to\infty} \eta(L) < \infty.$$

When $\beta = 1$, $\eta_\infty = \infty$ and this proves the first statement of the Theorem. As $Prob(B_h > g(L)) = Prob\,(VB_h > \eta(L))$, there exists a cell size L with $P(B_h > g(L)) = 0$ if and only if

$$Prob\left(V > \frac{\eta_\infty}{B_h}\right) = 0.$$

Thus with a given power P, the system can handle all velocities that are strictly less than

$$V_{lim} := \frac{\eta_\infty}{B_h} = \frac{2Pd_0\beta}{\beta - 1}. \quad \square$$

M.2 *Moments of service time and its derivatives*

The moments can be rewritten as,

$$\begin{aligned} b^{(k)} &= \frac{1}{(1 + P_{ho})} \int_0^\infty \int_{-D}^{D} \int_0^{V_{max}} \Big[B_D(x, v, s)^k + B_D(-D, v, s)^k P_{ho,v}\Big] \\ &\qquad\qquad f_{n,V}(v)dv \quad f_{n,X}(x)dx \quad \mu e^{-\mu s}ds. \\ &= E_n\left[\frac{B_D(x, v, s)^k + B_D(-D, v, s)^k P_{ho,v}}{1 + P_{ho}}\right]. \end{aligned}$$

By Bounded Convergence Theorem (BCT), all $b^{(k)}$ are continuously differentiable (c.d.) in L and the derivative is

$$\frac{db^{(k)}}{dL} = E_n\left[\frac{d}{dL}\left(\frac{B_D(x, v, s)^k + B_D(-D, v, s)^k P_{ho,v}}{1 + P_{ho}}\right)\right]$$

because: 1) B_D is almost surely c.d.; 2) $P_{ho,v}$ is c.d. everywhere in L; 3) P_{ho} is c.d. and 4) all the derivatives involved are uniformly bounded almost surely; 4) hence by virtue of mean value theorem, the terms like

$$\frac{|B_D(X, V, S; L + \delta) - B_D(X, V, S; L)|}{\delta}, \frac{|P_{ho,v}(L + \delta) - P_{ho,v}(L)}{\delta} \text{ etc}$$

can be bounded uniformly by a constant.

M.3 *ν has an unique maximizer:*

From equation (3), g and hence $\eta = vg$ are both concave in L on $(0, \infty)$ for every v. Thus from (8), for any fixed velocity v, ν has a unique maxima,

$$L^*_\nu(v) := \arg\max_L \nu(v, L),$$

which satisfies $\partial\nu/\partial L = 0$ (as clearly $L^*_\nu(v) > 0$ for all v).

M.4 *Derivatives $db^{(k)}/dL$ vanish only at $L^*_\nu(\bar{v})$ when $V \equiv \bar{v}$*

Define $\Psi(L) := E_n\left[B_D(X, V, S)^k + B_D(-D, V, S)^k P_{ho,V}\right]$. Then from (11)

$$b^{(k)} \quad = \quad \frac{\Psi(L)}{1 + P_{ho}}.$$

From (4), B_c depends upon L only via the function ν given by (8) and hence so is the service time $B_D(x, v, s) = \min\{B_c(v, s), B_\partial(x, v)\}$ for all x, v, s. Similarly from (10) and (9), $P_{ho,v}$ and P_{ho} depend upon L only via the function ν. Hence with

$$\Theta(v) \quad := \quad -\frac{\partial P_{ho,v}}{\partial \nu} \quad = \quad \frac{\mu}{v} E_{n,X}\left[(D - X)e^{-\mu\nu(v,L)(D-X)/v}\right]$$

$$\begin{aligned}
\frac{db^{(k)}}{dL} \quad &= \quad \frac{1}{1+P_{ho}} E_n\left[\frac{\partial\nu(V,L)}{\partial L}\left(\frac{\partial B_D(X,V,S)^k}{\partial\nu} + P_{ho,V}\frac{\partial B_D(-D,V,S)^k}{\partial\nu} + \frac{\partial P_{ho,V}}{\partial\nu}B_D(-D,V,S)^k\right)\right] \\
&- \quad \frac{1}{(1+P_{ho})^2}\Psi(L)\frac{dP_{ho}}{dL} \\
&= \quad \frac{1}{1+P_{ho}} E_n\left[\frac{\partial\nu(V,L)}{\partial L}\left(-kB_D(X,V,S)^{k-1}\frac{S1_{\{SV<(D-X)\nu(V,L)\}}}{\nu(V,L)^2}\right.\right. \\
&\qquad - \left.\left. kP_{ho,V}B_D(-D,V,S)^{k-1}\frac{S1_{\{SV<2D\nu(V,L)\}}}{\nu(V,L)^2} - B_D(-D,V,S)^k\Theta(V)\right)\right] \\
&+ \quad \frac{1}{(1+P_{ho})^2}\Psi(L)E_n\left[\frac{\partial\nu(V,L)}{\partial L}\Theta(V)\right] \\
&= \quad E_n\left[\frac{\partial\nu(V,L)}{\partial L}\left(\frac{-k\frac{S^k 1_{\{SV<(D-X)\nu(V,L)\}}}{\nu(V,L)^{k+1}} - kP_{ho,V}\frac{S^k 1_{\{SV<2D\nu(V,L)\}}}{\nu(V,L)^{k+1}} - B_D(-D,V,S)^k\Theta(V)}{1+P_{ho}}\right.\right. \\
&\qquad\qquad \left.\left. + \frac{\Psi(L)\Theta(V)}{(1+P_{ho})^2}\right)\right].
\end{aligned} \tag{18}$$

Thus the derivatives will have the form

$$\frac{db^{(k)}}{dL} = E_{n,V}\left[\frac{\partial\nu(V,L)}{\partial L}E_{n,X,S}[\Gamma^{(k)}(X, V, S, \nu(V, L))]\right] \tag{19}$$

for some functions $\Gamma^{(k)}$. Thus for fixed velocities, i.e., when $V \equiv \bar{v}$

$$\frac{db^{(k)}}{dL} = \frac{\partial\nu(\bar{v}, L)}{\partial L}E_{n,X,S}[\Gamma^{(k)}(X, \bar{v}, S, \nu(\bar{v}, L))]. \tag{20}$$

Claim : The term $E_{n,X,S}[\Gamma^{(k)}(X, \bar{v}, S, \nu(\bar{v}, L))]$ is strictly negative.

Proof of Claim: For any velocity v, $B_D(X, v, S) \leq B_D(-D, v, S)$ for all (X, S). Thus, for fixed velocities,

$$\Psi(L) \leq (1 + P_{ho,\bar{v}}) E_{n,S}[B_D(-D, \bar{v}, S)^k].$$

Further $P_{ho} = E_{n,V}[P_{ho,V}] = P_{ho,\bar{v}}$. Hence the sum of the last two inner terms of the equation (18),

$$E_{n,X,V}\left[-\frac{B_D(-D,V,S)^k \Theta(V)}{1+P_{ho}} + \frac{\Psi(L)\Theta(V)}{(1+P_{ho})^2}\right] \leq 0.$$

The remaining two inner terms of (18) are always negative and this proves the Claim. $\square$

From (20), by the virtue of the Claim, derivative $db^{(k)}/dL$ is zero only at a zero of $\partial\nu/\partial L$. But $\partial\nu/\partial L$ has a unique zero at the maximizer $L^*_\nu(\bar{v})$. Thus $L^*_\nu(\bar{v})$ is the only zero of all the service time moments.

Appendix P: Calculations for Pico queue

P.1 *Pico Handover Speed Distribution*

In the following the event $\{\partial \ \ arrival\} = \{\partial\}$ means that the arrival in cell 0 was at the boundary (i.e., it was an handover arrival from cell -1), the event $\{int \ \ arrival\} = \{int\}$ meant a new arrival in cell 0 and the event $\{ho\}$ implies the cell 0 active user has reached the next boundary before finishing his service (i.e, has to be handed over to cell 1). Note that an handover from cell 0 occurs either due to an already handed over call or new call, whose service could not be completed before reaching the (other) boundary of cell 0 and thus:

$$Prob(ho) = \frac{\lambda_L P_{ho} + \lambda_{hL} P_{ho,\partial}}{\lambda_L + \lambda_{hL}}.$$

Note that $Prob(ho) \to 1$ as $L \to 0$ (as both P_{ho}, $P_{ho,\partial}$ converge to 1). With the above notations, the fixed point equation for the handover speed density can be obtained as:

$$\begin{aligned}
f_{h,V}(v) &= Prob(V \in dv | ho \ \ from \ \ cell \ \ 0) \\
&= Prob(V \in dv \ \ int \ \ arrival | ho) + Prob(V \in dv \ \ ho \ \ arrival | ho) \\
&= \frac{Prob(ho \ \ V \in dv \ \ int) + Prob(V \in dv \ \ \partial \ \ ho)}{Prob(ho)} \\
&= \frac{Prob(ho | int \ \ v) Prob(V \in dv | int) Prob(int)}{Prob(ho)} \\
&\qquad + \frac{Prob(ho | \partial \ \ v) Prob(V \in dv | \partial) Prob(\partial)}{Prob(ho)} \\
&= \frac{P_{ho,v} f_{n,V}(v)}{Prob(ho)} \frac{\lambda_L}{\lambda_L + \lambda_{hL}} + \frac{P_{ho,v,\partial} f_{h,V}(v)}{Prob(ho)} \frac{\lambda_{hL}}{\lambda_L + \lambda_{hL}}
\end{aligned}$$

Solving (when new call speed density $f_{n,V}(v) = 1/V_{max}$ for all $v \leq V_{max}$) the handover speed density is,

$$f_{h,V}(v) = \frac{\frac{\lambda_L}{Prob(ho)(\lambda_L + \lambda_{hL}) V_{max}} P_{ho,v}}{1 - \lambda_{hL} \frac{P_{ho,v,\partial}}{Prob(ho)(\lambda_L + \lambda_{hL})}}$$

Thus as L tends to zero ($P_{ho,v}, P_{ho,v,\partial}, Prob(ho), P_{ho} \to 1$), the speed of an handover arrival will tend to uniform distribution. This effect is seen faster if the packet sizes S are larger.

P.2 *Pico Stability Factor*

The stability factor ρ in a Pico queue is given by:

$$\begin{aligned}
\rho_{_{pico}}(L) &= \frac{(\lambda_L+\lambda_{hL})b^{(1)}}{K} = \frac{1}{K}E_V\left[\lambda_L E_X[E_S[B_L(S,X,V)]] + \lambda_{hL}E_S[B_L(S+B_h,0,V)]\right] \\
&\approx \frac{1}{K}\left(\lambda_L(L-E[X]) + L\lambda_{hL}\right)E\left[\frac{1}{V}\right] \;=\; \frac{1}{K}\left(\lambda_L\frac{L}{2}+L\lambda_{hL}\right)E\left[\frac{1}{V}\right] \\
&= c_0 L(\lambda_L + 2\lambda_{hL}) \overset{\text{as } P_{ho}\approx 1}{\approx} c_0 L\lambda_L\frac{1-P_{ho,\partial}+2}{1-P_{ho,\partial}} \approx \frac{2c_0\lambda_L L}{1-P_{ho,\partial}}
\end{aligned}$$

where $c_0 = 1/(2K)E[1/V]$ is a constant independent of L. Using the approximation in (15),

$$\rho_{_{pico}}(L) = \frac{\lambda p L}{KD\mu\nu(\bar{v}_{inv},L)}. \tag{21}$$

P.3 *Drop probability*

The drop probability $P_{D,pico}$ considering possible Pico drops can be calculated as below:

$$\begin{aligned}
P_{D,pico} &= Prob(\text{ Call ever Dropped before completion}|\text{ Call is picked up}) \\
&= P_{ho}\left(P_{ho,D}(1-P_{Busy,pico}) + P_{Busy,pico}\right) + (1-P_{ho})(0)
\end{aligned}$$

where $P_{ho,D}$ is defined as the probability of call drop at any of the future instances of handovers, given that the current handover (first handover in the context of the above equation) is successful. Because of the memoryless nature of S, this probability does not depend upon the number of the handover. Probability $P_{ho,D}$ can be calculated by first conditioning on the event that the call is completed in the current cell (call it as $\mathcal{C}$) and then on the event that the call is not picked up in the next cell (call it as S). Note that $P_{ho}(\mathcal{C}^c) = P_{ho,\partial}$ and $P_{ho}(\mathcal{S}|\mathcal{C}^c) = P_{ho,fail} = 1 - P_{Busy,pico}$. Thus, by conditioning

$$\begin{aligned}
P_{ho,D} &= P_{ho}(\text{ Call dropped } \cap \mathcal{C}) + P_{ho}(\text{ Call dropped } \cap \mathcal{C}^c) \\
&= 0 + P_{ho,\partial}P_{ho}(\text{ Call dropped }|\mathcal{C}^c) \\
&= P_{ho,\partial}\left(P_{ho}(\text{ Call dropped } \cap \mathcal{S}^c|\mathcal{C}^c) + P_{ho}(\text{ Call dropped } \cap \mathcal{S}|\mathcal{C}^c)\right) \\
&= P_{ho,\partial}\left(P_{ho,D}(1-P_{Busy,pico}) + 1P_{Busy,pico}\right) \\
&\overset{Solving}{=} \frac{P_{Busy,pico}P_{ho,\partial}}{1-P_{ho,\partial}(1-P_{Busy,pico})} \text{ and hence,} \\
P_{D,pico} &= \frac{P_{ho}P_{Busy,pico}}{1-P_{ho,\partial}(1-P_{Busy,pico})} \approx \frac{P_{Busy,pico}}{1-P_{ho,\partial}+P_{Busy,pico}}. \qquad (22) \\
&= \frac{P_{Busy,pico}}{\frac{\mu L\nu(\bar{v}_{inv},L)}{\bar{v}_{inv}} + P_{Busy,pico}} \\
&= \frac{P_{Busy,pico}}{\frac{\mu(\eta(L)-\bar{v}_{inv}B_h)}{\bar{v}_{inv}} + P_{Busy,pico}}
\end{aligned}$$