Spatial queueing analysis for mobility in pico cell networks

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Abstract

In this work, we characterize the performance of pico cell networks in presence of moving users. We model various traffic types between base-stations and mobiles as different types of queues. We derive explicit expressions for expected waiting times, service times and drop/block probabilities for both fixed as well as random velocity of mobiles. We obtain (approximate) closed form expressions for optimal cell sizes when the velocity variations of the mobiles is small for both non-elastic as well as elastic traffic. We conclude from the study that, if the call is long enough, the optimal cell size depends mainly on the velocity profile of the mobiles, its mean and variance. It is independent of the traffic type or duration of the calls. Further, for any fixed power of transmission, there exists a maximum velocity beyond which successful communication is not possible. This maximum possible velocity increases with the power of transmission. Also, for any given power, the optimal cell size increases when either the mean or the variance of the mobile velocity increases.

I. Introduction

Since the time 3G technology was designed and deployed, various other technologies have appeared. The ambitious objectives in terms of quality of service offered by 3G technology turned to be quite expensive, which made the 3G technology vulnerable to cheaper competing technologies such as WIFI. Pico cell technology has been recently proposed as an alternative that offers some basic connectivity and mobility support, and is yet sufficiently simple so as to be economically competitive ([8], [6]). To prevent a large number of handovers that would result from the small size of the cells ([11]), it has been proposed to group together a number of pico cells in one virtual Macrocell and to restrict the effort of preventing losses due to the handover only to those handovers that occur between Pico cells of the same virtual cell. In between the Pico cells some fast switching mechanisms are proposed such as frequency following mechanism where the frequency used by a mobile follows it from one pico cell to the next. This requires reserving the same channel for a user in the entire Macrocell.

In this paper we consider a large macrocell divided into a number of pico cells and study the impact of mobility on such systems, especially the effect of frequent handovers. We assume that the ongoing call is never dropped at the pico cell boundary, however base station switching (BSS) at any pico cell boundary requires some fixed amount of information (in terms of bytes) to be exchanged. There is however a possibility of calls being dropped at macrocell boundaries. We further assume that the active users cross macrocell boundaries at maximum once. The handovers at the macrocell boundaries are modeled as independent Poisson process.

This paper has several goals. First, to model the system so as to predict its performance measures. We are thus interested in developing tools in spatial queuing that take into account not only the instantaneous geometry but also the way it varies in time. It should thus account for the impact of the speed of the users. We model the macrocells by various types of queues and well known results from queuing theory are used to obtain performance measures like expected waiting times, service times, drop or blocking probabilities, etc [16]. We shall use these results for preliminary dimensioning purposes in planning the Pico cell network catering to pedestrian and vehicular mobility, typical of urban and sub-urban areas. We derive closed form expressions of useful performance metrics considering

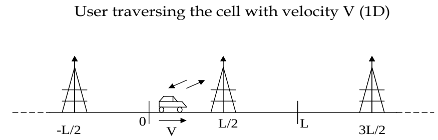

Fig. 1. User moving with velocity V along an infinite line

free space path loss, handover constraints, traffic type etc. We also obtain closed form expressions for optimal cell sizes, optimal for various performance metrics, when all the users move at the same fixed velocity. To derive these performance measures, we would require the *moments* of the time taken by the system to serve [1] the customers, which in our case will equivalently be the time the macrocell spends on a call. We derive the expressions for these service times, during which the information is exchanged between the moving user and the set of appropriate base stations (which it encounters during its journey), using variable rate of transmission. We further simplify the expressions for service times under the following assumption: in a typical pico cell network, a moving user would have traversed across a number of cells before the completion of call. In this system, arrivals occur in space and the service times depend on the position, movement of the user and the serving base station(s). The queues modeling these systems are referred to as spatial queues and have been used in interesting applications ([2], [14],[10]). We make the following theoretical and or simulation based observations: 1) Maximum possible velocity: For any fixed power of transmission P, there exists a maximum velocity $V_{lim}(P)$, beyond which successful communication is not possible; 2) Larger cells for larger velocities: Given P, the optimal cell size increases with an increase in the highest velocity that the system has to support. This is true as long as the highest velocity is less than $V_{lim}(P)$. However the system cannot cater for velocities above the limit $V_{lim}(P)$, even if one increases the cell size indefinitely; 3) Insensitivity to application: The optimal cell size remains the same for non-elastic as well as elastic calls for big file sizes, as long as rest of the parameters remain the same.

We describe our system in Section II and service times are discussed in Subsection III-A. The NES, ES calls are modeled by appropriate queuing models and performance measures are derived in Subsections III-D, III-E respectively. It is also shown that for both the types of calls, one equivalently need to optimize the expected service time. This optimization is carried out in Subsection III-C. The numerical examples are provided in Section IV.

II. System Model

We consider a macrocell, $[-D, D]$, divided into a number of pico cells of length L. Each pico cell has a base station (BS) located at the center and all these BS communicate to a central unit (CU), which controls the entire system. *We assume that there is no interference between any two transmissions.*

Traffic Types: Define the waiting time as the duration between the arrival of the call and the instance its service starts. We consider two types of traffic: elastic (ES) and Non-elastic (NES). The non-elastic traffic is very sensitive to the waiting time (multimedia streaming, voice calls etc). A call of this type will be blocked if not picked up within a very small waiting time. The elastic traffic (data traffic) is less sensitive so it is never blocked. Expected waiting time is the appropriate performance measure for the ES calls while the probability of a call being blocked, P_B and the drop probability, i.e., the probability that an ongoing call is dropped before finishing, P_D are important performance measures for NES calls. Systems are designed with more stringent requirements on P_D than P_B.

Arrivals: The two (ES, NES) arrivals are modeled by two independent Poisson arrivals with rates λ. Every arrival is associated with Marks (X, V, S): $X \in [-D, D]$ the location of arrival, S the file size requirement and V the user velocity, distributed respectively according to $P_{n,X}, P_{n,V}$ and $P_{n,S}$ with respective densities $f_{n,X}, f_{n,V}$ and $f_{n,S}$. Let $P_n := (P_{n,X}, P_{n,V}, P_{n,S})$. We *assume symmetry in both directions, i.e., that* $P_n((X, V, S) \in A) = P_n((X, V, S) \in -A)$ *for all Borel sets* A. In this paper we thus *calculate and analyze without loss of generality (w.l.g.) for* $V > 0$.

[1]Throughout we use the terms from queuing theory like arrivals, service times etc.

Handovers: In this paper we assume that handovers are always successful at pico cell boundaries. However, we do not assume the same for macrocell boundaries, i.e., a crossover into a new macrocell results in a successful handover only if the new macrocell has free servers. We model each handover into a macrocell as a Poisson arrival, stochastically independent of the new call arrivals. We further assume that there can be at maximum one handover, i.e., the calls get finished before reaching the second macrocell boundary. This simplifies the analysis to a good extent and is quite a good assumption as the macrocells are typically large in dimension. We consider generalization of this assumption in our future work.

Radio Conditions: The BS communicates with the mobiles using a wireless link, at a rate that depends upon the distance between the two. Since our primary focus is on mobility we implicitly consider pico cells deployed outdoors, for example urban, suburban scenarios. Hence we can assume significant line of sight signal. Further, pico cells being small in size, it will be sufficient to consider only the direct path for communication. A user located at x communicates with BS of cell m using unit transmit power (when receiver noise variance is one) at rate [2]given by,

$$\bar{R}(x;m) := 1_{\{|x-(mL-\frac{L}{2})|\leq d_0\}} + 1_{\{|x-(mL-\frac{L}{2})|>d_0\}} {d_0}^{\beta} \left| x - \left(mL - \frac{L}{2}\right)\right|^{-\beta}, \tag{1}$$

where $\beta \geq 1$ represents path loss factor and $d_0 > 0$ is small distance up to which there is no propagation loss. The above model is valid for systems with low signal to noise ratios, where in the rates are directly given by the SNRs.

III. System Analysis

The users are moving continuously with a fixed but random velocity. The macrocell can handle at maximum K parallel calls. Transmission always occurs at fixed power P. Since Pico cells are small in size, the movement of the users results in frequent handovers. The number of handovers will be quite large that it would be complicated to design a reliable system without redundancy: We assume that every BS can also handle K parallel calls[3]. This ensures that, once a call is picked up it is not dropped at any pico cell boundary: when a user crosses over to a new BS, the new BS would at maximum be handling $K-1$ calls and hence will have at least one free server. However it is important to note that the maximum power used at any time in the system equals KP. We further assume that : (1) Every BSS (base station switching at a pico cell boundary) requires fixed B_h bytes of information to be communicated (independent of the user's velocity), after which the user's service is resumed by the BS of the cell it just entered; (2) The user is served by the BS of the pico cell in which it is moving, as it is physically nearest to this BS.

A. *Time required for communicating S bytes:*

Define by $B_c(S, X, V)$ the time required to communicate a packet of length S bytes to a user located at X (when the service starts) and moving with velocity V. If the user can communicate at a fixed rate r bytes/sec then the communication time would have been S/r. The maximum rate at which a user can communicate with the BS in cell

[2]The analysis will go through for any other rate functions, for example like $R(x;1) = (1 + (x - L/2)^2)^{-\beta/2}$ ([15]), $R(x;1) = log\left[1 + (1 + (x - L/2)^2)^{-\beta/2}\right]$ ([15]), $R(x;1) = log\left[1 + \left(d_0^{\beta}|x - L/2|^{-\beta} \;\; 1_{\{|x-L/2|>d_0\}} + 1_{\{|x-L/2|\leq d_0\}}\right)\right]$ etc. Some of the simplifications that we obtain in subsequent sections, may not be possible with these rate functions. However one can always conduct monte carlo simulations to obtain the required inferences.

[3]In practical systems, each BS will have M backup servers to manage handovers. This means each BS can handle M parallel calls. In general M need not be equal to K, however M has to be chosen large enough to ensure negligible call drops at pico cell boundaries, taking into consideration the large number of handover associated with pico cells. With this large enough M the system's behavior will be close to the system considered in this paper (the case with $M = K$).

m is given by (1). This position dependent rate varies: minimum when the user is at the cell edges and increases as the user moves towards the cell center. This poses a need to calculate the communication time considering the variable rates. The location of the user (under service) will change according to $X(t) = X + Vt$ (Figure 1). At time t, if the user is in cell m, i.e. if $X(t) \in [(m-1)L, mL]$, it communicates with the BS of m^{th} cell. Hence the user gets service at time varying rate given by $R(t; X, V) := P\bar{R}(X(t); m)$ if $t \in \left[\frac{(m-1)L-X}{V}, \frac{mL-X}{V}\right]$. Without loss of generality we consider the users, whose communication started in the first pico cell, i.e., with $X \in [0, L]$. The communication time B_c required by the user, i.e., the time required to communicate S bytes satisfies :

$$S = \int_0^{B_c} R(t; X, V) dt. \tag{2}$$

Let

$$g(l) := \int_0^{l/V} P\bar{R}(Vt; 1) dt = P \int_0^l \bar{R}(l'; 1) \frac{dl'}{V},$$

represent the number of bytes communicated while the mobile traverses interval $[0, l]$. Note that (when $L > 2d_0$),

$$g(L) = \frac{Pd_0^\beta}{V} \int_0^{L/2-d_0} \left(\frac{L}{2} - l\right)^{-\beta} dl + \frac{Pd_0^\beta}{V} \int_{L/2+d_0}^{L} \left(l - \frac{L}{2}\right)^{-\beta} dl + \frac{2Pd_0}{V},$$

For example when the pathloss factor $\beta = 2$,

$$g(L) = \frac{4Pd_0}{V} - \frac{4Pd_0^2}{LV}.$$

For any m, the number of bytes communicated as the user traverses through m^{th} pico cell (by change of variable $l = X + Vt - (m-1)L$),

$$\int_{\frac{(m-1)L-X}{V}}^{\frac{mL-X}{V}} R(t; X, V) dt = \int_{\frac{(m-1)L-X}{V}}^{\frac{mL-X}{V}} P\bar{R}(X + Vt; m) dt = \frac{P}{V} \int_0^L \bar{R}(l; 1) dl = g(L)$$

and thus is independent of m. Out of this number, B_h number of bytes are dedicated for BSS. Hence, irrespective of the cell which the user traverses, $g(L) - B_h$ number of bytes are transmitted during the user's journey via one pico cell. Thus the communication time can have three components : 1) Time taken in the originated cell $(L - X)/V$, 2) Time taken to travel N full cells, where (with $\lfloor t \rfloor$ representing the largest integer in t)

$$N = N(S, X, V) := \left\lfloor \frac{(S - (g(L) - g(X)))}{g(L) - B_h} \right\rfloor$$

represents the number of cells traveled during the communication of S bytes. 3) Time taken in the cell in which the call terminates: time taken to communicate the leftover bytes $S_l := S - (g(L) - g(X)) - N(g(L) - B_h)$. Further a call can be handled only if the bytes that can be communicated while traversing through a cell $g(L)$, is greater than the number of bytes required for BSS B_h. From (2), the communication time $B_c(S, X, V)$ can be calculated as:

$$B_c(S, X, V) = \begin{cases} \frac{1}{V} \arg\inf_{l \in (X, L]} \{(g(l) - g(X)) \geq S\} & \text{if } S < (g(L) - g(X)) \\ \infty & \text{if } B_h > g(L) \\ \frac{L-X}{V} + N\frac{L}{V} + \frac{1}{V} \arg\inf_{l \in (0, L]} \{g(l) - B_h \geq S_l\} & \text{else.} \end{cases}$$

As g is continuous and monotonically increasing function, g^{-1} exists and hence :

Theorem *1:* Time to communicate S bytes with a user initially located at X and moving with velocity V is,

$$B_c(S, X, V) = \begin{cases} \frac{g^{-1}(S+g(X);V)-X}{V} & \text{if } S < (g(L) - g(X)) \\ \infty & \text{if } \quad B_h > g(L) \\ \frac{(L-X)+NL+g^{-1}(S_l+B_h;V)}{V} & \text{else, } \quad \text{where} \end{cases}$$

$$g^{-1}(s; v) = \begin{cases} \frac{L}{2}(1 - e^{-\frac{sv}{Pd_0}}) & \text{if} \quad \beta = 1 \quad \frac{sv}{Pd_0} < log\left(\frac{L}{2d_0}\right) \\ \frac{L}{2} + \frac{2d_0^2 e^{-2}}{L} e^{\frac{sv}{Pd_0}} & \text{if} \quad \beta = 1 \quad \frac{sv}{Pd_0} > log\left(\frac{L}{2d_0}\right) + 2 \\ \frac{L}{2} + \frac{sv}{P} - d_0 log\left(\frac{L}{2d_0}\right) - d_0 & \text{if} \quad \beta = 1 \quad \text{else} \\ \frac{L}{2} - \left(\frac{sv(\beta-1)}{Pd_0^\beta} + \left(\frac{L}{2}\right)^{-\beta+1}\right)^{\beta-1} & \text{if} \quad \beta > 1 \quad \frac{sv(\beta-1)}{Pd_0^\beta} < d_0^{-\beta+1} - \left(\frac{L}{2}\right)^{-\beta+1} \\ \frac{L}{2} + \left(\frac{2\beta d_0^{-\beta+1}}{\beta-1} - \frac{sv(\beta-1)}{Pd_0^\beta} - \left(\frac{L}{2}\right)^{-\beta+1}\right)^{\beta-1} & \text{if} \quad \beta > 1 \quad \frac{sv(\beta-1)}{Pd_0^\beta} > \frac{(\beta+1)d_0^{-\beta+1}}{\beta-1} - \left(\frac{L}{2}\right)^{-\beta+1} \\ \frac{L}{2} + \frac{d_0^\beta}{\beta-1}\left(\frac{sv(\beta-1)}{Pd_0^\beta} + \left(\frac{L}{2}\right)^{-\beta+1} - d_0^{-\beta+1}\beta\right) & \text{if} \quad \beta > 1 \quad \text{else.} \quad \square \end{cases}$$

Approximation : In pico cell based systems, user traverses a large number of pico cells while receiving service. Hence the communication time can be approximated by the product of number of cells $S/(g(L) - B_h)$ and the time taken for traveling each cell L/V:

$$B_c(S, X, V) \approx \frac{S}{g(L) - B_h} \frac{L}{V} \text{ when } g(L) > B_h. \tag{3}$$

In Figure 2 we show that this approximation is very good. We plot the expected value of actual communication time and the expected value of the approximation, for two different velocity ranges. As expected the approximation is very good, in fact for all velocity profiles (one can hardly distinguish the two lines in the figure).

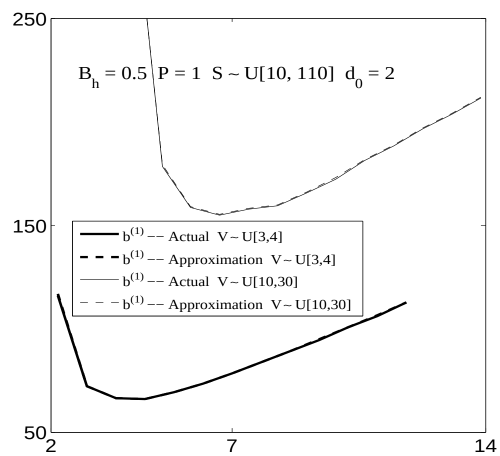

Fig. 2. Accuracy of Communication time approximation

If a call is originated at position X and moves with velocity $V \neq 0$, then in case of ES applications, the call will be picked up at a position X_s ($= X + VW$; W the waiting time) different from X. It is difficult to estimate X_s and the time taken to communicate S bytes, B_c, actually depends upon X_s but not on X. However with the above approximation, $B_c(S, X_s, V) = B_c(S, V)$, *i.e., B_c does not depend upon the location of the user when its communication started.*

Maximum velocity handled by the system : Communication time is finite only if the number of bytes transfered $g(L)$ per cell is strictly greater than the bytes required for BSS, B_h. Hence, the communication time is finite with probability one if and only if $Prob(B_h > g(L)) = P_{n,V}(VB_h > \eta(L)) = 0$, where

$$\eta(L) := Vg(L) = P\int_0^L \bar{R}(l;0)dl.$$

Note that η is only a function of P, L and hence that this probability depends only upon the velocity profile V, cell size L and the transmit power P. Because of the path loss, for a fixed P, $\eta(L)$ increases as L increases and finally saturates. Thus

$$\eta_\infty := \lim_{L\to\infty} \eta(L) < \infty.$$

As $Prob(B_h > g(L)) = Prob\,(VB_h > \eta(L))$, there exists a cell size L with $P(B_h > g(L)) = 0$ if and only if

$$Prob\left(V > \frac{\eta_\infty}{B_h}\right) = 0.$$

Hence for a given power P, the system can handle all those velocity profiles which satisfy (4). Thus with a given power P, the system can handle all velocities that are strictly less than

$$V_{lim} := \frac{\eta_\infty}{B_h} = \frac{P}{B_h}\left(2d_0 + 2d_0^\beta \int_{d_0}^\infty l^{-\beta}dl\right). \tag{4}$$

We thus have,

Theorem *2:* When $\beta = 1$, for any transmit power P, system can handle all velocity profiles. When $\beta > 1$, there exists a bound $V_{lim}(P) < \infty$ (increasing linearly with P) on the maximum velocity that can be handled by system

$$V_{lim}(P) := \eta_\infty/B_h \text{ with } \eta_\infty := \lim_{L\to\infty}\eta(L) = 2P\left(d_0 + d_0^\beta\int_{d_0}^\infty l^{-\beta}dl\right) = \frac{2Pd_0\beta}{\beta - 1} \quad \square$$

Henceforth we consider only the velocity profiles with all velocities lesser than (4), infact it should satisfy:

$$Prob(V < V_{max}) = 1, \text{ where } V_{max} < V_{lim}(P). \tag{5}$$

B. Service time : The time of the macrocell spent for user's service

The user reaches the boundary of the macrocell starting from a point X in time $B_\partial(X,V) := (D-X)/V$ (w.l.g. when $V > 0$). The macrocell has to serve the user either until all its S bytes are transmitted (which takes time B_c) or till the user reaches the boundary. Thus, the overall service time requirement of the user in a macrocell is $B_D := \min\{(D - X_s)/V, B_c(V,S)\}$, where X_s is the user position when its service starts. As noted above, B_c does not depend upon X_s. For NES applications $X_s = X$ the position of arrival. For ES applications, it is difficult to estimate X_s, instead we approximate X_s with X, i.e., $B_D(X,V,S) \approx \min\{B_\partial(X,V), B_c(V,S)\}$. The error in this approximation is given (with W representing the waiting time) w.l.g for $V > 0$ by:

$$E_{rr} = W1_{\left\{B_c > \frac{D-X}{V}\right\}} + \left(B_c - \frac{D-X_s}{V}\right)1_{\left\{\frac{D-X_s}{V} < B_c < \frac{D-X}{V}\right\}} \text{ and so for any } k,\ E[E_{rr}^k] \le E\left[W^k 1_{\left\{B_c > \frac{D-X}{V} - W\right\}}\right].$$

The *error is small either whenever the number of servers is large (so waiting times are small) or when the macrocell is large in size.*

Distribution of handover call marks (X,S): In general their densities will be different from $f_{n,X}$, $f_{n,S}$ and $f_{n,V}$). As the users move in either direction with equal probability, $P_{h,X}$ the position of handover arrival occurs either at $-D$ or at D with half probability. If handover occurs at $-D$ the corresponding velocity will be positive, which is the case we consider w.l.g. We assume $f_{n,S}$ *is exponential*, i.e., $f_{n,S}(s) = \mu e^{-\mu s}$ for some $\mu > 0$, in which case $f_{h,S} = f_{n,S}$.

Rate of handovers: Let $\nu(v,L) := (\eta(L) - vB_h)/L$. Then $P_{ho} := Prob(B_\partial < B_c) = E_{n,X,V}[exp^{-\mu\nu(V,L)(D-X)/V}]$ gives the probability that a call is not completed in one macrocell. This precisely represents that fraction of new arrivals which get converted into handover calls. So $\lambda_{ho} := \lambda P_{ho}$ is the rate at which handovers occur.

Speed of handover arrival : A handover call arrives at $X = -D$ with velocity $v > 0$ only if a new call with velocity v is not completed before reaching the boundary. Here we use the assumption that handover occurs at maximum at one macrocell boundary, i.e., an handover call is not converted to another handover. Let $P_{ho,v} := Prob(B_\partial < B_c|v) = E_{n,X}[exp^{-\mu\nu(v,L)(D-X)/v}]$ represent the conditional probability given $V = v$. Then the handover speed distribution, $f_{h,V}(v) = f_{n,V}(v)P_{ho,v}/P_{ho}$.

C. *Moments of Service time*

We *assume* $P(V < V_{max}) = 1$ *with* $V_{max} < V_{lim}(P)$. Then the k^{th} moment of B_D exists (whenever the corresponding for S and V^{-1} exist) and equals:

$$\begin{aligned} b^{(k)} &:= E_{X,V,S}[(B_D(X,V,S))^k] \text{ where } P_{X,V,S} := \frac{\lambda(P_{n,X}, P_{n,V}, P_{n,S}) + \lambda_{ho}(P_{h,X,V}, P_{h,S})}{\lambda + \lambda_{ho}} \qquad (6) \\ &= \frac{1}{(1+P_{ho})} \int_0^\infty \int_{-D}^{D} \int_0^{V_{max}} \Big[B_D(x,v,s)^k + B_D(-D,v,s)^k P_{ho,v} \Big] f_{n,V}(v)dv \quad f_{n,X}(x)dx \quad \mu exp^{-\mu s} ds. \end{aligned}$$

By bounded convergence theorem, all $b^{(k)}$ are continuously differentiable (c.d.) in L because: 1) B_D is almost surely c.d.; 2) $P_{ho,v}$ is c.d. everywhere in L and 3) all the derivatives involved are uniformly bounded almost surely; 4) hence by virtue of mean value theorem, the terms like

$$\frac{|B_D(X,V,S;L+\delta) - B_D(X,V,S;L)|}{\delta}, \frac{|P_{ho,v}(L+\delta) - P_{ho,v}(L)}{\delta} \text{ etc}$$

can be bounded uniformly by a constant.

The number of bytes that can be communicated in a cell increases with the increase in cell size (g is clearly monotonically increasing and a continuous function of L). For any given velocity there exists a minimum cell size (the smallest cell size at which one can transmit more than B_h bytes per cell), beyond which successful communication is possible. When cell sizes are closer to this smallest one, the useful bytes transmitted per cell ($g(L) - B_h$) are very small and hence it take more time to transmit S bytes, i.e., the communication time B_c will be large. As the cell sizes increases from this smallest size, the communication time B_c starts reducing. However after some point, due to path loss, the number of bytes per cell starts saturating and hence the gain in terms of useful bytes transmitted per cell will be small in comparison with the extra time taken to traverse each cell, resulting in increasing the communication time again. Thus there exists an optimal cell size for every fixed velocity. One can extrapolate similar things even for random velocity. We find the optimal cell size $L^*_{b^{(1)}}$ and relate the same to the optimizer of more interesting performance measures for ES and NES calls in the subsequent sub-sections.

Note that

$$b^{(k)} = \frac{\Psi(L)}{1+P_{ho}} \text{ where } \Psi(L) := E_n\left[B_D(X,V,S)^k + B_D(-D,V,S)^k P_{ho,V} \right].$$

It is easy to see that the service time $B_D(x,v,s)$, $P_{ho,v}$ and P_{ho} all depend upon L only via the function ν. Hence with

$$\Theta(v) := -\frac{\partial P_{ho,v}}{\partial \nu} = \frac{\mu}{v} E_{n,X}\left[(D-X) exp^{-\mu\nu(v,L)(D-X)/v} \right]$$

$$\begin{aligned}
\frac{db^{(k)}}{dL} &= \frac{1}{1+P_{ho}} E_n\left[\frac{\partial \nu(V,L)}{\partial L}\left(\frac{\partial B_D(X,V,S)^k}{\partial \nu} + P_{ho,V}\frac{\partial B_D(-D,V,S)^k}{\partial \nu} + \frac{\partial P_{ho,V}}{\partial \nu} B_D(-D,V,S)^k\right)\right] \\
&\quad - \frac{1}{(1+P_{ho})^2}\Psi(L)\frac{dP_{ho}}{dL} \\
&= \frac{1}{1+P_{ho}} E_n\left[\frac{\partial \nu(V,L)}{\partial L}\left(-kB_D(X,V,S)^{k-1}\frac{S 1_{\{SV<(D-X)\nu(V,L)\}}}{\nu(V,L)^2}\right.\right. \\
&\qquad \left.\left. -kP_{ho,V}B_D(-D,V,S)^{k-1}\frac{S1_{\{SV<2D\nu(V,L)\}}}{\nu(V,L)^2} - B_D(-D,V,S)^k\Theta(V)\right)\right] \\
&\quad + \frac{1}{(1+P_{ho})^2}\Psi(L)E_n\left[\frac{\partial \nu(V,L)}{\partial L}\Theta(V)\right]
\end{aligned}$$

$$= E_n\left[\frac{\partial \nu(V,L)}{\partial L}\left(\frac{-k\frac{S^k 1_{\{SV<(D-X)\nu(V,L)\}}}{\nu(V,L)^{k+1}} - kP_{ho,V}\frac{S^k 1_{\{SV<2D\nu(V,L)\}}}{\nu(V,L)^{k+1}} - B_D(-D,V,S)^k\Theta(V)}{1+P_{ho}} + \frac{\Psi(L)\Theta(V)}{(1+P_{ho})^2}\right)\right].$$

Thus the derivatives will have the form

$$\frac{db^{(k)}}{dL} = E_{n,V}\left[\frac{\partial \nu(V,L)}{\partial L} E_{n,X,S}[\Gamma^k(X,V,S,\nu(V,L))]\right]$$

for some functions Γ^k. Thus for fixed velocities, i.e., when $V \equiv \bar{v}$, all the moments have unique minima $\arg\min_L b^{(k)}$ and the unique minimum $\arg\min_L b^{(k)} = \arg\min_L \nu(\bar{v}, L)$ for all k. Thus $b^{(2)}$ *and* $b^{(1)}$ *are optimized by the same cell size for fixed velocities. Further for velocity profiles with small variances, the optimizers will be equal approximately.* Hence when $P_{n,V}$ has small variance with mean $\bar{v}$ then $L^*_{b^{(1)}} := \arg\min_L b^{(1)}$ is close to $L^*_\nu := \arg\min_L \nu(\bar{v}, L)$ and hence satisfies

$$\left.\frac{\partial \nu(\bar{v},L)}{\partial L}\right|_{L=L^*_{b^{(1)}}} \approx 0 \text{ or } 2P\left(\frac{L^*_{b^{(1)}}}{d_0}\right)^{-\beta} L^*_{b^{(1)}} - \eta(L^*_{b^{(1)}}) + \bar{v}B_h \approx 0. \text{ Thus we have,}$$

Theorem *3:* For fixed velocity profile, i.e., $P_{n,V}(V=\bar{v}) = 1$ the optimal cell size for expected service time is,

$$L^*_{b^{(1)}} = 2\left(\frac{2Pd_0^\beta \frac{\beta}{\beta-1}}{2Pd_0\frac{\beta}{\beta-1} - \bar{v}B_h}\right)^{\beta-1} \text{ when } \beta > 1 \text{ and } L^*_{b^{(1)}} = 2d_0 e^{\frac{\bar{v}B_h}{2Pd_0}} \text{ when } \beta = 1.$$

The above will be approximately true for the velocity distribution $P_{n,v}$ with mean $\bar{v}$ and small variance. *From the above it is clear that L^*_b increases when the mean $\bar{v}$ increases.*

Having obtained the service times, we now turn our attention to model various configurations of the macrocell with appropriate queues to further obtain their performance measures.

D. ES Calls : Average Waiting time

Each macrocell can handle at maximum K parallel calls. The CU of the macrocell keeps a record of the users entered into the system and serves them in FIFO order via the BSs of the Pico cells. When a new user initiates a call, it is immediately picked up if there are less than K active calls in the system. If not the user will have to wait. Its service will start at the time : 1) when one of the active K users finish their service and exit 2) if there are no other waiting users arrived before it. The BS nearest to the user, at the time of its service start, will initiate the call. Hence after, its call is served (by the macrocell under consideration) as discussed in subsection III-A either till its service is over or till it reaches the macrocell boundary. When it reaches the boundary the call will be transfered to the next macrocell as a handover call and the handover call is treated by the new cell similar to that of a new call. Thus each macrocell can be modeled by a $M/G/K$ queue with service times B_D and Poisson arrivals at rate $\lambda + \lambda_{ho}$. This queue has been analyzed to a good extent and the system is stable only if $\rho := \lambda b^{(1)}/K < 1$ ([9]). For stable queues, the expected waiting time of a randomly arrived customer can be approximated by ([9]):

$$E[W]_K = \left(\frac{b^{(2)}}{2(b^{(1)})^2}\right)\left(\frac{b^{(1)}}{K(1-\rho)}\right)\left(\frac{(K\rho)^K}{K!}\right)\pi_0; \quad \pi_0^{-1} = \frac{(K\rho)^K}{K!} + (1-\rho)\sum_{i=0}^{K-1}\frac{(K\rho)^i}{i!}, \tag{7}$$

where $b^{(1)}, b^{(2)}$ are given by (6). If the system is unstable the number of waiting customers grows towards infinity and thus one should consider only the cell sizes L with $\rho < 1$. Hence, the optimal size, which minimizes (7) is

$$L^*_{ES} = \arg\min_{\{L:\rho<1\}} E[W]_K.$$

We saw in the previous section that the optimizer of $b^{(2)}$ is same as that of $b^{(1)}$ for fixed velocities and will be close to each other for smaller velocity variances. The expected waiting time (7) is continuously differentiable in both $b^{(1)}$, $b^{(2)}$. Thus (minimizer of (7) is a zero of its derivative and $E[W]_K$ depends upon L only via $b^{(1)}$, $b^{(2)}$),

Theorem *4:* Optimal cell size for a system with elastic traffic and when all users move with velocity $\bar{v}$ is

$L^*_{ES} = \arg\min_{\{L:\rho<1\}} E[W]_k = L^*_{b^{(1)}}$. In other words, $L^*_{b^{(1)}}$ minimizes both expected waiting and service times. □

Further, when the two optimizers are close, from (7) it is easy to see that the optimizer of $E[W]_K$ will be close to that of the expected service time, $b^{(1)}$. Thus for low velocity variances also,

$$L^*_{ES} \approx \arg\min_{\{L:\rho<1\}} b^{(1)} = L^*_b.$$

We see that this is true even for many general velocity profiles via numerical examples in the subsequent sections.

E. NES Calls : Block and Drop Probabilities

As before the system can handle at maximum K parallel calls. The call is picked up immediately only if the system is serving lesser than K users at the time of its arrival. If all the servers are busy it is dropped. When an active customer reaches the boundary of a macrocell, its call is continued in the next macrocell only if the new macrocell has free servers. This system can thus be modeled by an $M/G/K/K$ queue. And its call block probability is given by,

$$P_B(L) = \frac{\rho(L)^K/K!}{\sum_{k=0}^{K}\rho(L)^k/k!}; \quad \rho(L) := \frac{\lambda b^{(1)}}{K}.$$

It is interesting to note that $P_B(L)$ and $b^{(L)}$ are both differentiable w.r.t. L and further that if the derivative $db^{(1)}/dL$ is zero at a L^* so is the derivative dP_B/dL. By taking the second derivative, we can in fact show that their minimizers are the same. Hence,

$$\begin{aligned} L^*_{NES} &= \arg\min_{\{L:\rho(L)<1\}} P_B(L) \\ &= \arg\min_{\{L:\rho(L)<1\}} b^{(1)}. \end{aligned}$$

Theorem *5:* The minimizer, $L^*_{b^{(1)}}$ also minimizes the block probability, P_B, for all velocity profiles. □

Drop Probability : Under the assumptions stated earlier, only a new call can reach the boundary and not a call which was already handed over once. Further, an active call is dropped only when it reaches the macrocell boundary and the new macrocell is busy. By independence of the two events (status of the new macrocell prior to handover is independent of the call that is handed over), the drop probability is

$$P_D(L) = P_{ho} P_B(L).$$

When $X \sim \mathcal{U}[-D, D]$ and because of the assumption that call can crossover at most once (w.l.g. for $V > 0$),

$$\begin{aligned} P_{ho} = P_n\left(B_\partial(X, V) < B_c(V, S)\right) &= E_{S,V}\left[P_X(D - X < VB_c(V, S))\right] = \frac{E_{S,V}\left[\min\left\{2D, \frac{VSL}{\eta(L) - VB_h}\right\}\right]}{2D} \\ &= \frac{E_n[VB_c(V, S)]}{2D} = \frac{\mu L E_{n,V}[V/(\eta(L) - VB_h)]}{2D}. \end{aligned}$$

One can design an optimal system which jointly minimizes the two probabilities or minimizes one of the probabilities while placing a constraint on the other. Usually systems are designed with stringent requirements on P_D than on P_B. We note from the above calculations that P_D is directly proportional to P_B and will be smaller than P_B by a factor which depends on the inverse of the macrocell size: $1/2D$. Macrocells are large in dimension and hence P_D can be ensured to be within the prescribed limits (the limit is a design parameter) by directly minimizing P_B itself. Thus we propose to chose cell size L to minimize P_B and hence equivalently $b^{(1)}$:

$$L^*_{NES} := L^*_{b^{(1)}} = \arg \min_{\{L:\rho(L)<1\}} b^{(1)}.$$

Thus *for both ES and NES applications one needs to minimize the first moment of the service time, $b^{(1)}$, to obtain the optimal cell size.* This optimal cell size has been discussed in the previous section for fixed velocities and for velocities with small variances. The general situation is studied in the next section via numerical examples.

IV. Mobility Examples

In the numerical examples of this section, we consider uniformly distributed velocity profiles. The position of arrival X is also uniformly distributed in the macrocell $[-D, D]$. We use monto carlo simulation to estimate $b^{(1)}$, $b^{(2)}$ and use exhaustive search to find the optimizers. In figure 3 we plot normalized values of $b^{(1)}$, $b^{(2)}$ and $E[W]_K$ versus L. As discussed earlier we notice that the various performance measures decrease with cell size initially, reach an optimal value and increase again from then on. In fact, all the performance measures have unique minimum at the same L. We study more details of these minimizers in the following.

In figure 4 we plot the optimal cell size (optimal with respect to moments of service time $b^{(1)}$, $b^{(2)}$, block and drop probabilities P_B, P_D of NES calls and the expected waiting time $E[W]_K$ of ES calls) versus mean velocity for two different values of variance. We set $d_0 = 4$, $\lambda = 0.1$, $B_h = 2$, $P = 1$, $\mu = 5$, $K = 20$ and consider a macrocell of size $D = 1000$. We also plot L_ν, which is the minimizer of $\nu(E_n[V], L)$. For small velocity variances (curves with variance equal to 1), all the minimizers are close to L_ν. For large velocity variance, we notice that all the minimizers ($L^*_{b^{(1)}}$, $L^*_{b^{(1)}}$, $L^*_{P_B}$, $L^*_{P_D}$ and $L^*_{E[W]}$) are away from L_ν, but however are close to each other for most cases. That is, the minimizers of expected waiting time are the same as that of block as well as drop probabilities and all of them equal $L^*_{b^{(1)}}$. This suggests that *even for velocity profiles with high variances, it is sufficient to optimize the average service time $b^{(1)}$ for both ES as well as NES calls and hence the optimal cell size again remains independent of the application.* However in this case, it is not sufficient to minimize $\nu(E_n[V], L)$ but one needs to minimize $b^{(1)}$ directly. In Figure 5 we plot the various optimal cell sizes as a function of velocity variance. We set mean, $E[V] = 10$, $d_0 = 4$, $\lambda = 0.1$, $B_h = 3.12$, $P = 1$, $\mu = 5$, $K = 20$. We once again note that all the minimizers are close to each other for many cases. We also note that all the minimizers are close to L_ν for low velocity variances. We further observe that the optimal cell size increases with increase in the variance also. Thus *larger the velocities the system has to support, the larger are the optimal cell sizes.*

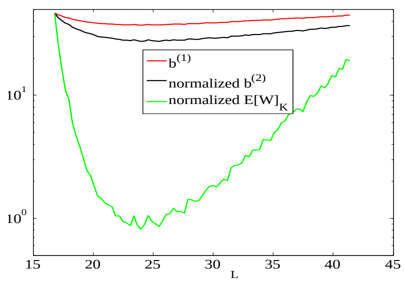

Fig. 3. Moments of the service times and the expected waiting time versus L.

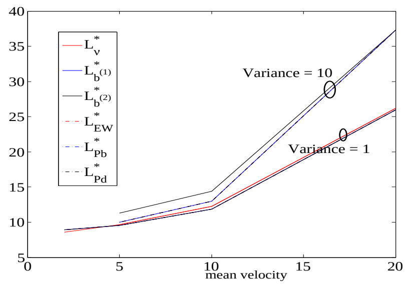

Fig. 4. Optimal cell size versus mean velocity for different variances.

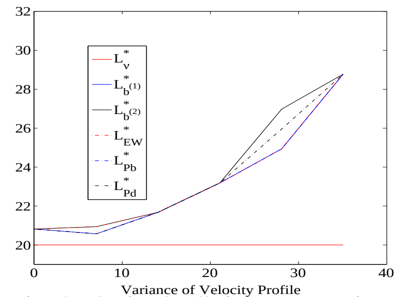

Fig. 5. Optimal cell size versus variance of the velocity.

We notice in both the figures 4 and 5 that, only the optimizer of the second moment $L^*_{b^{(2)}}$, is some times different from the rest of the minimizers. However, even when $L^*_{b^{(2)}}$ is different from $L^*_{b^{(1)}}$, the minimizer L^*_{EW} is equal to $L^*_{b^{(1)}}$ and thus for both the types of traffic $L^*_{b^{(1)}}$ gives the optimal cell size.

V. Conclusions

We looked at the problem of characterizing the performance of pico cell networks in the presence of mobility. We modeled various traffic types between base-stations and mobiles as different types of queues. We derived explicit expressions for expected waiting times, service times and drop/block probabilities for the various queuing models considered for both fixed as well as random velocity of mobiles. We showed that there exists an optimal cell size for a given velocity profile, which minimizes the service times for elastic applications as well as the drop and block probabilities of non-elastic applications. We obtained (approximate) closed form expressions for this optimal cell size when the velocity variations of the mobiles is very small. We find that if the call is long enough, the optimal cell size depends mainly on the velocity profile of the mobiles, its mean and variance.; It is independent of the traffic type or duration of the calls. We show that for any fixed power of transmission, there exists a maximum velocity beyond which successful communication between the mobile and the system is not possible. This maximum possible velocity increases with the power of transmission. Further, for any given power, the optimal cell size increases when either the mean or the variance of the mobiles velocity increases.

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