Analysis of Pico cells with Randomly Walking Users

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Abstract

In this paper, we develop a framework to analyze pico cells with randomly walking users. We model the user movements by a random walk with exponential wandering times. We partition each pico cell into disjoint regions based on the transmission rates allocated and model each rate region as an equivalent step in the random walk model. We further use queuing theoretic tools to obtain explicit expressions for the expected service time, call busy and drop probabilities. We obtain approximate closed form expressions for optimal cell size for the case of non-elastic traffic (for some asymptotic cases) and show that these values closely match the cell sizes obtained via numerical simulations. We also obtain analysis of users moving at high speeds and in a fixed direction (ex., those traveling in a car) using the theory developed in this paper. We study the influence of system parameters like, the path loss factor, speed of user movement, power budget etc, on these optimizers. We show that the optimal cell size increases with the increase in the speed of the users, decreases with increase in path loss factor, increases with increase in power budget.

Index Terms

Pico cells; Cell dimensioning; Random walk;

I. Introduction

Recent trends in mobile broadband access and services is paving the way for dense deployment of base stations, popularly known as small cell networks (SCNs) [1]. Typically small cell networks, comprising of portable pico and femto base stations serve dense urban areas, commercial and office spaces, hot-spots, etc. The design and deployment of such networks pose many a new challenges to the optimal system design. One of the key challenges for managing mobile users is handling handovers. As the cell size decreases, on one hand the frequency of the handovers increases resulting in losses and on the other hand the cell edge users obtain services at better communication rates. As the handovers increase the calls get dropped before completing the service with higher probabilities, while with better communication rates the amount of time taken for the same service reduces. Thus the performance of the system depends upon these contrasting phenomenon and one needs to address this trade-off while designing optimal systems. Virtual cells and fast base station switching [2] are some of the ideas proposed to reduce the handover losses. However, they can not completely prevent the same. In this paper we study this trade-off.

In a recent work ([4]), we used spatial queuing theory to study user mobility in small cell networks. Important system performance metrics like expected waiting time, service time, call busy and drop probabilities for various traffic types are derived and the cell sizes, which optimizes these metrics for a given user velocity profile, are computed. We dealt with high speed users moving in a fixed direction in that paper. The speed can be random but is assumed to be constant during the course of the service. This model gives an understanding of cell dimensioning with users traversing on well structured streets in urban areas and deriving service from the base stations (BS) located on street infrastructure. However, there can be many example scenarios in which the users move randomly. Typically, this happens in commercial centers, hot-spots and office spaces. The idea of the current work is to obtain optimal dimensioning rules when users move in a random manner. The analysis of the system with randomly wandering users would be way different from that of the users that traverse in a fixed direction and we use random walk model techniques to obtain these results.

Further, in [4], we considered "maximal" rates of service, i.e., we assume that the service rates can be changed continually based on the distance between the user and the serving BS and that too, to the maximum possible one (i.e., capacity). In this paper we consider a more practical scenario. We assume that the system can support one of the finite number of transmission (or service) rates and that a user derives his service at one of these rates based on his distance from the serving BS. The finite set of all possible transmission rates can further depend upon the cell dimension. Thus a cell is partitioned into as many disjoint regions as the number of possible transmission rates and we only need to know in which rate region the user is currently located. *Hence we model user movements by a random walk model, in which each step represents a rate region.* The wandering times in each region can depend upon the region itself as well as the cell dimension. We obtain performance measures like expected service time, call busy and drop probabilities, etc, further using queuing theoretic tools. We also obtain initial results for users moving in fixed directions, as in [4], but served with finite number of transmission rates.

Mathematical models that can capture the dynamics of stocks, animals, humans, traffic, etc., has been a well studied subject over the past decades. Random walks serve as a fundamental model that can explain the observed behavior of the stochastic processes in many such cases. Often, these dynamics exhibit Markovian behavior and hence random walks can be analyzed via Markov chains. The notion of time associated with random walks can be discrete or continuous. Also, the step sizes can follow a distribution (for e.g. Gaussian etc.), while the direction of the walk can be uniform over the interval $(0, 2\pi)$. Further, the steps need not be independent, but, can be correlated. Another important aspect is that, if the step sizes are very small, the dynamics can be well represented by a Wiener process, also popularly known as Brownian motion. The theory of random walks has a long history which goes back to the beginning of the last century by Karl Pearson. Feller's and Spitzer's books [9], [10] contain preliminary material on this topic.

Random walk and other mobility models are often used to study user movement in cellular networks in various contexts. In the following we list a few. An excellent survey of mobility models used in the simulations of wireless networks, is provided in [8] and the references therein. The authors in [5] develop a two dimensional random walk model to study mobility in wireless networks and use the underlying Markov chain property of this random walk to derive the cumulative distribution function of the dwell time. They present preliminary results for the case of rectangle and circular cell shapes. Their approach provides modeling mainly for distance-based criterion of boundary crossing, which can be extended to take into account the radio link propagation effects. In [6], an enhanced random mobility model to simulate user movement in wireless networks is introduced. The authors assume correlated movement and derive the model considering speed and direction change events as random processes with specific emphasis to the users border behavior. The authors in [7] define a generic mobility model: the random trip model for independent mobiles that contains random waypoint, random walk and other models. They study the necessary and sufficient condition for a stationary regime. This framework provides a rich set of well understood models that can be used to simulate mobile networks with independent node movements. While random walk and other such models have been a popular choice to simulate user movement for cellular systems, in this work, we use the random walk model to analyze wandering users in small cell networks. Assuming that at each step, the user is served by one of the transmission rates available at the base station for a duration that is exponential, we derive derive important system performance metrics using queuing theoretic tools and further use them to derive dimensioning rules in such networks.

Section II describes the system model while in Section III, theoretical analysis is presented for generic case. Section IV studies the case of random walk with exponential wandering times. The case of high speed users is discussed in section V.

II. System Model

We have a network with small cells, each of dimension L. In the case of one dimensional networks (see Figure 1), the entire network spans over a line segment say $[-D, D]$ which is divided into cells of length $2L$ while in the case of two dimensional networks (see Figure 2), each cell is a circle of radius L. Our aim is to find optimal dimension, L^*, while the network caters to moving users. Let $\eta := 1_{\{1D\}} + 21_{\{2D\}}$. We assume that the neighboring cells do not interfere each other.

Rate Regions: The cell is divided into $2N$ (or N for 2D) disjoint segments (based on the distance from Base station[1]) such that the users in a segment are served with the same transmission rate. Let $\{\mathbb{A}_n\}_{n\in\mathbb{N}}$ represent these

[1]In small cell networks (transmission at small distances), distance based propagation losses would be sufficient for deciding the theoretical rate limits as well as the practical transmission rates.

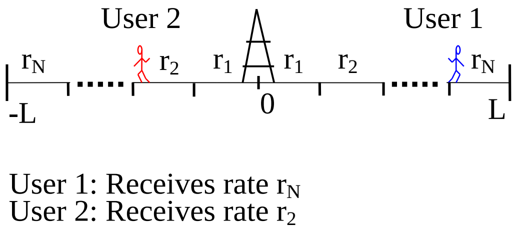

Fig. 1. One dimensional cell, rate partitioning and user's movement

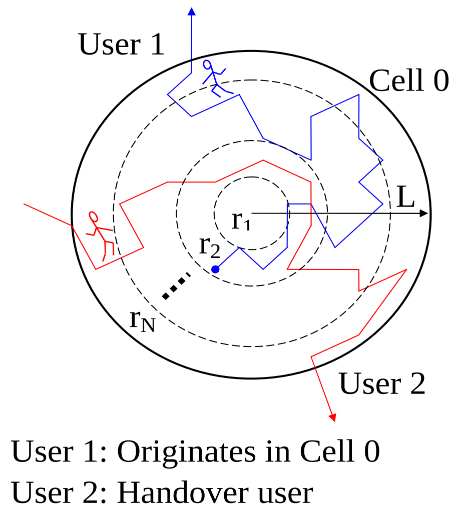

Fig. 2. Two dimensional cell, rate partitioning and user's movement

rate regions (see figures 1 and 2) where with $|.|$ representing the norm or the absolute value,

$$\mathbb{A}_n := \begin{cases} \left[\frac{(n-1)L}{N}, \frac{nL}{N}\right] 1_{\{n>0\}} + \left[\frac{nL}{N}, \frac{(n+1)L}{N}\right] 1_{\{n<0\}} & \text{if 1D} \\ \left\{\mathbf{x} \in \mathcal{R}^2 : \frac{(n-1)L}{N} \leq |\mathbf{x}| \leq \frac{nL}{N}\right\} & \text{if 2D} \end{cases} \tag{1}$$

$$\text{and } \mathbb{N} \;\; := \;\; \begin{cases} \{-N, \cdots, -1, 1, \cdots, N\} & \text{if 1D} \\ \{1, \cdots, N\} & \text{if 2D.} \end{cases} \tag{2}$$

User in region n receives service at rate $r_{|n|}$. Let $\mathbb{R} := \{r_1, \ldots, r_N\}$ represent the ensemble of all possible transmission rates. Note that this set is arranged in the decreasing order. For example in a two dimensional (circular) cell of Figure 2, each annular ring is served with a common rate and these common rates decrease as the distance from the center (where BS is located) increases. The rate at which the service is offered changes once the user switches from one region to another.

Embedded (Rate) Markov Chain: The users can be located any where in one of the rate regions $\{\mathbb{A}\}_n$. We represent the user location at time step k by Φ_k . When $\Phi_k = n$, it implies that the user is wandering in segment $\mathbb{A}_n$ and is receiving service at rate $r_{|n|}$ at time k. Let W_n represent the time for which the user remains in n^{th} region, $\mathbb{A}_n$. This represents (for any k), the actual time for which the k^{th} step lasts, given that $\Phi_k = n$. Note here, we are inherently assuming that the consequent times, the user spends in the same rate region, are identically and independently distributed (IID). However these times can depend upon the region in which the user is wandering. After wandering in a certain rate region n for time W_n the user either moves to region $n+1$ with probability p_n or to region $n-1$ with probability $1-p_n$. *Note that $p_1 = 1$ always for 2D.* That is, $\{p_n\}$ represent the transition probabilities of the embedded Markov chain $\{\Phi_k\}$.

All the quantities $\{p_n\}$, $\{W_n\}$ $\{r_n\}$ can depend upon the dimension L (which we are trying to optimize) and the dependence is shown explicitly only if required by adding L as a parameter in the usual way, for example like, $p_{n;L}$.

Arrivals: There are two types of arrivals: 1) arrivals from external world (represented explicitly by subscript e and this is done only when there is ambiguity) modeled as Poisson arrivals with parameter λ; 2) handover arrivals (always indicated using subscript h) modeled again as Poisson arrivals[2], but this stream is derived from a fraction of the stream (1) whose service is not completed at a cross over. The rate of arrivals into the cell of interest depends

[2]This is a commonly made assumption, for example see [11], [12].

upon the cell dimension L and this is shown by either λ_L (for external arrivals) or $\lambda_{h;L}$ (for handover arrivals). For external arrivals, we assume[3] $\lambda_L = \lambda L^\eta$ while $\lambda_{h;L}$ will be calculated in later sections.

Every arrival, brings along with it the marks (Φ, S), where $\Phi \in \mathbb{N}$ is the position of arrival with distribution $\Pi := \{\pi_n\}$ and S the number of bytes to be transmitted is exponentially distributed, i.e., $S \sim \mu exp^{-\mu t} dt$ for some $\mu > 0$.

Resources: A cell can attend to K parallel calls. The power per transmission, P_L, depends upon the cell dimension and this dependency will be discussed later.

An example of $\mathbb{R}$: One can *choose the set of possible transmission rates,* $\mathbb{R}$ *and* N *based on the practical channel coding schemes that are going to be used in the network design. The analysis presented can be utilized to study a system with any given* $\mathbb{R}$ *and* N. However, in this paper, we consider a specific example. This specific $\mathbb{R}$ is obtained using low SNR approximation of the following theoretical (capacity) rate[4]:

$$r(d) := P_L \left(1_{\{d \le d_0\}} + r_0 \, |d|^{-\beta} \, 1_{\{d > d_0\}} \right) \text{ with } r_0 = d_0^\beta,$$

where $r(d)$ is the rate at distance d, d_0 is a small lossless distance[5] while β is the propagation co-efficient. We consider a specific system which supports transmission at the maximum possible rate for the entire region. For example in $\mathbb{A}_n$ the farthest user will be at distance $|n|L/N$ and hence maximal transmission rate, that can be allocated, equals

$$r_n = r(|n|L/N) = r_0 P_L N^\beta L^{-\beta} |n|^{-\beta}. \tag{3}$$

Alternatively, if the system under consideration can design modulation and or channel coding schemes so as to achieve (almost) ν *percent of the theoretical rates where* $\nu < 1$ *is a fixed coefficient, then again the above rate structure is applicable (after absorbing* ν *into* r_0 *of (3)).*

Handovers: Whenever the user reaches the boundary $\{|\mathbf{x}| = L\}$ the call is handed over to the neighboring cell. The random walk pattern of the users can result in multiple hand-overs and one can avoid such situation by again using latency, i.e. for example for 1D, by assuming that the handover occurs only when the user jumps either to $(N + \delta_n)L/(2N)$ or to $-(N + \delta_n)L/(2N)$ so that there is an overlap of δ_n steps on either direction. The old BS continues to serve the user till these overlap steps are also crossed. Similarly when a call is handed over from the cell, say $[L, 3L]$, the call is handed over to the cell under consideration $[-L, L]$ when its user crosses $(N - \delta)L/(2N)$. We right now present the analysis with $\delta_n = 0$ however the analysis goes through in a similar way for $\delta_n > 0$.

Information to initiate handover: Every new connection requires s_h extra bytes to be exchanged to initiate it. The effect of these bytes (on the system performance) for a new call will be negligible (as it would be once), however one needs to consider their effect on handover calls. These bytes are usually very small in proportion to the actual bytes to be transmitted, i.e., $s_h << S$ with high probability. *We assume that these* s_h *bytes are exchanged with probability one, while the user is wandering in the last rate region (e.g.,* r_N*) itself.*

Notations: Let the flag, η, represent 1 for 1D and 2 for 2D. We denote the transpose by ^{t}. Calligraphic letters represent matrices. Mathbb letters represent sets (e.g., $\mathbb{N}$ - set of segment numbers $\mathbb{R}$ - set of all possible transmission rates, $\mathbb{A}_n$ - rate region n). The contents inside two flower brackets represent either a set or an ordered tuple (as according to convenience): for example $\{r_n\}$ represents the set $\mathbb{R}$ while $\{\pi_n\}$ represents the ordered tuple Π. Lower case letters represent time index (k) or the segment index (n). Lower case bold letters represent the vectors.

Upper case letters either represent system parameters (e.g., D - dimension of Macrocell, L - dimension of small cell, P - Power per transmission, K - Number of servers, N - Total number of possible transmission rates (number of elements in $\mathbb{R}$) , $\Pi = \{\pi_n\}_n = \{Prob($ Arrival in segment $n)\}_n$ - Vector of arrival probabilities etc.) or represent

[3]If the arrivals in the entire line segment $[-D, D]$ occur at rate λ' those in segment $[-L, L]$ occur at a smaller rate $\lambda_L = \lambda' Prob$(arrival in $[-L, L]$). For the special case of uniform arrivals (i.e., arrivals landing uniformly in $[-D, D]$) $\lambda_L = \lambda L$. Similarly for 2D, $\lambda_L = \lambda L^2$ for some $\lambda \ge 0$.

[4]For unit noise variance, capacity equals $log(1 + SNR)$, where signal to noise ratio $SNR = P_L A$, attenuation $A = 1_{\{d \le d_0\}} + (d/d_0)^{-\beta} 1_{\{d > d_0\}}$. For low SNRs, $log(1 + SNR) \approx SNR$ and hence capacity equals $P_L A$.

[5]Typically d_0 is very small and in this paper **we consider optimizing over cell sizes** $L > d_0 N$ so that $r(d) = r_0 P_L d^{-\beta}$ always.

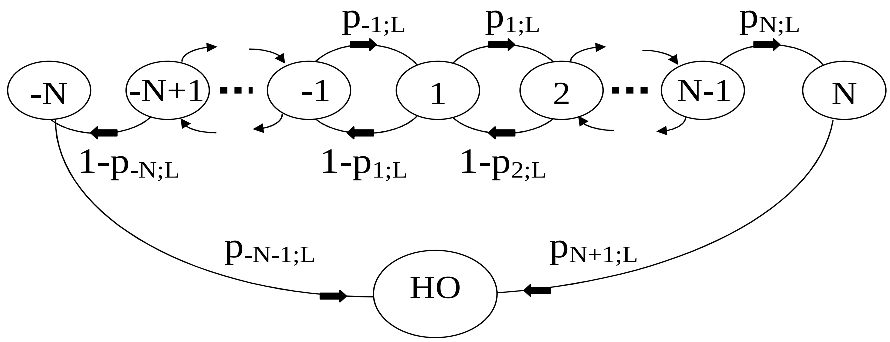

Fig. 3. Transitions of the embedded Markov chain $\{\Phi_k\}$ for 1D.

random variables (W - wandering time, S - number of bytes to be transferred, Φ - the segment in which the user is wandering etc.). When any of the above have to be indexed by n or k and further the dependency on parameter L has to be shown, then we use notation like $\pi_{n;L}$, $W_{n;L}$, P_L, Φ_k etc.

We study such small cells using queuing theoretic tools and obtain relevant performance metrics like, the expected service time, call busy and or drop probability etc. We also derive "Capacity per cell", a notion that gives the maximum number of bytes that can be transfered while the user traverses in a cell divided by the cell size.

III. Analysis

The aim of this section is to obtain performance analysis of the network under consideration and then to obtain optimal cell dimension using the performances derived. We start with analysis of the embedded Markov chain $\{\Phi_k\}$, whose transitions are as depicted in Figure 3. We obtain most of the analysis using conditional expectation techniques and the transition properties.

A. Expected service time

Let B_e represent the total amount of time for which an user derives service from the cell (in which the call originated), either before finishing his call or before being handed over to a neighboring cell. Let $\bar{b}_e$ represent its expected value while let b_n represent the same given that the call originated in region n (which happens with probability π_n). Then $\bar{b}_e = \sum_n \pi_n b_n$.

The time taken to transfer S bytes at a rate r_n equals S/r_n and hence a user wandering in region n completes his service if $W_n > S/r_n$. Thus, the probability of completing the service while the user is in region n equals,

$$\begin{aligned} q_n &= E[W_n > S/r_n] = E_{W_n}[1 - exp^{-\mu W_n r_n}] \\ &= -E\left[exp^{-\mu W_n r_n}\right], \end{aligned} \tag{4}$$

and the expected time for which the user receives the service, while moving in region n will be

$$t_n = E\left[\min\left\{W_n, \quad \frac{S}{r_n}\right\}\right] = E\left[\frac{1 - exp^{-\mu W_n r_n}}{\mu r_n}\right] = \frac{q_n}{\mu r_n}. \tag{5}$$

We assume S is exponentially distributed and hence the bytes remaining after receiving the service in the previous rate region will again be exponentially distributed with the same parameter, by memoryless property.

Now, $\{b_n\}$ can be computed by conditioning on appropriate events. While in region n, it derives service on average for t_n time and then it moves to region $n+1$ with probability p_n or to region $n-1$ with probability $1 - p_{n-1}$. If the service is not completed in region n (which happens with probability $1 - q_n$) then the remaining bytes (note S is exponential) will be served in a similar manner, in the new region entered albeit with new rate. This repeats either till the service is completed or till the user exits the cell. Thus, $\{b_n\}$ satisfy the linear equations (note $p_1 = 1$ and negative indices are not applicable for 2D):

$$\begin{aligned} b_n &= t_n + (q_n 0 + (1-q_n)p_n b_{n+1} + (1-q_n)(1-p_n)b_{n-1}), \\ b_n &= 0 \text{ when } n = N+1 \text{ or } -(N+1). \end{aligned} \tag{6}$$

The last equation indicates when the user moves out of the last rate region(s) (e.g., N) it no more derives service from the cell under consideration. In other words, $\mathcal{Z}\mathbf{b} = \mathbf{t}$ where,

$$\text{with } z_n := -(1-q_n)p_n \text{ and } \bar{z}_n := -(1-q_n)(1-p_n), \tag{7}$$

$$\begin{aligned} \mathcal{Z} &:= \begin{bmatrix} 1 & z_{-N} & 0 & & & & & \cdots & & 0 \\ \bar{z}_{-(N-1)} & 1 & z_{-(N-1)} & & & & & \cdots & & 0 \\ & & \vdots & & & & & & & \\ 0 & 0 & \cdots & & \bar{z}_1 & 1 & z_1 & 0 & \cdots & 0 \\ & & \vdots & & & & & & & \\ 0 & 0 & \cdots & & & \cdots & & & \bar{z}_N & 1 \end{bmatrix} \\ \mathbf{b} &= [b_{-N}, b_{-N-1}, \cdots, b_{-1}, b_1, \cdots, b_N]^t \\ \mathbf{t} &= [t_{-N}, t_{-N-1}, \cdots, t_{-1}, t_1, \cdots, t_N]^t. \end{aligned}$$

For 1D $\mathcal{Z}$ is a $2N \times 2N$ matrix, while the same for 2D is the right lower $N \times N$ matrix. For maintaining consistency in notations we represent the 2D matrix also by $\mathcal{Z}$. In other words $\mathcal{Z}$ is the $2N \times 2N$ size matrix as shown above for 1D while for 2D the same is the right lower $N \times N$ part of the matrix given above. In a similar way for 2D, $\mathbf{b}$ and $\mathbf{t}$ contain only the lower N elements. Matrix $\mathcal{Z}$ is invertible and hence, one can solve for $\{b_n\}$. The expected service time equals

$$\bar{b}_e = \Pi^t \mathcal{Z}^{-1}\mathbf{t} \text{ with } \Pi := [\pi_{-N}, \pi_{-N+1} \cdots, \pi_N]^t. \tag{8}$$

B. *Hand-overs (HO)*

A call that crossed over to a neighboring cell before completing its service is termed as an handover call. *We assume, the hand-overs can also be modeled by Poisson arrivals (see for e.g., [11], [12]).* These handovers, just like the external or new calls, are picked up whenever the neighboring cell has free resources and require s_h bytes of information to be exchanged for initiating the call.

Stochastic equivalence, HO-SE: Due to stationarity, *the handovers into the cell of interest (cell 0, $[-L, L]$) are stochastically same as those that go out of the same cell, cell 0*, because, for example in 1D: 1) by symmetry, the handovers from cell 0 ($[-L, L]$) to cell 1 ($[L, 3L]$) are stochastically same as those from cell -1 ($[-3L, -L]$) to cell 0; 2) the same is true for handovers when an user travels from right to left. The same is true even for 2D networks. *Using this stochastic equivalence (which we will refer henceforth as HO-SE) we calculate all the quantities related to handovers (that are required for further analysis) via fixed point equations.*

HO arrival positions: Let $\pi_{h,n}$ represent the probability that a handover call arrives at n. For 2D the handover can occur only at N and hence,

$$\pi_{h,n} = 0 \text{ for all } 1 \le n < N \text{ and } \pi_{h,N} = 1.$$

For 1D handover can occur either at N or at $-N$ and hence

$$\pi_{h,n} = 0 \text{ for all } -Nn < N \text{ and } \pi_{h,N} \neq 0, \pi_{h,-N} \neq 0.$$

For 1D networks, *we assume symmetry in either direction. That is we assume that* $p_n = 1 - p_{-n}$ *and that* $W_n \stackrel{d}{=} W_{-n}$ *(stochastically equivalent).* Thus, $\pi_{h,N} = \pi_{h,-N} = 1/2$.

$$\text{Let } \Pi_h := \begin{cases} [0.5, & 0, & \cdots, & 0, & 0.5]^t & \text{for 1D} \\ [0, & 0 & \cdots, & 0, & 1]^t & \text{for 2D.} \end{cases} \tag{9}$$

HO Arrival rate: The probability of a (possible) handover is one minus the probability of service being completed within the cell and this has to be calculated by solving linear equations as in the case of $\bar{b}_e$. Let h_n represent the overall probability of completing the service in cell 0, given the call is originated in the region n. Then $\{h_n\}$ solves (by conditioning as explained for $\{b_n\}$, see equation (6))

$$\begin{aligned} h_n &= q_n + (1-q_n)p_n h_{n+1} + (1-q_n)(1-p_n)h_{n-1} \text{ and} \\ h_n &= 0 \text{ when } n = N+1 \text{ or } -(N+1). \end{aligned}$$

That is, $\mathcal{Z}\mathbf{h} = \mathbf{q}$ where $\mathbf{h} := [h_{-N}, \cdots, h_N]^t$ and $\mathbf{q} := [q_{-N}, \cdots, q_N]^t$. Again for 2D, the vectors $\mathbf{h}$ and $\mathbf{q}$ have only the lower N elements. And so, the probability of a new arrival not completing the service before moving out of the current cell (which results in a handover) equals

$$P_{e,ho} = 1 - \sum_n \pi_n h_n = 1 - \Pi^t \mathcal{Z}^{-1}\mathbf{q}. \tag{10}$$

In other words, out of all the new or external arrivals that arrived in cell 0, $P_{e,ho}$ portion of them get handed over to a neighboring cell. Some of these handovers get converted to a handover again. The probability of this event can be calculated in a similar way and it equals (see equation (9)),

$$P_{h,ho} = 1 - {\Pi_h}^t \mathcal{Z}^{-1}\mathbf{q}.$$

Because of memory less property (as S is exponential) there is no difference in this probability (or any other quantity that we calculate further) for the first handover and for the subsequent handovers. The expected service time of a handover call (irrespective of the number of times it is already handed over) can be calculated in a similar way as done while obtaining (8) and equals:

$$\bar{b}_h = \Pi_h^t \mathcal{Z}^{-1}\mathbf{t}.$$

A handover can result in further handovers and so on. Thus (by conditioning on appropriate events) we notice that this rate $\lambda_{h;L}$ by stochastic equivalence (HO-SE), satisfies:

$$\lambda_{h;L} = \lambda_L P_{e,ho} + \lambda_{h;L} P_{h,ho} \qquad \text{and hence} \qquad \lambda_{h;L} = \frac{\lambda_L P_{e,ho}}{1 - P_{h,ho}}.$$

C. Overall service time and stability factor

Let $\bar{b}$ represent the average of the service times demanded by external as well as handover arrivals. This service time also includes, $t_h := s_h / r_N$ (note these bytes are exchanged in the exterior most rate region), defined as the time taken to serve the handover bytes s_h. Calculation of t_h depends upon the specific example and we deal with it in the subsequent sections. Further this time has to be added to the service time only if the call is a handover call and also in general t_h can effect $\bar{b}_h$. We discuss these issues subsequently. It is easy to see that $\bar{b}$ is given by,

$$\bar{b} = \frac{\lambda_L \bar{b}_e + \lambda_{h;L}(\bar{b}_h + t_h)}{\lambda_L + \lambda_{h;L}} = \frac{(\lambda_L \Pi^t + \lambda_{h;L}\Pi_h^t)\mathcal{Z}^{-1}\mathbf{t} + \lambda_{h;L} t_h}{\lambda_L + \lambda_{h;L}}.$$

Then the stability factor is

$$\rho_L = \frac{\bar{b}(\lambda_L + \lambda_{h;L})}{K}.$$

D. Busy and Drop Probability for Non elastic traffic

Non elastic traffic comprises of users demanding immediate service. These users (e.g., voice calls) drop the call if it is not picked up immediately, i.e., if all the servers are busy. The probability that a call is not picked immediately is called the Busy probability and the probability that a call that was picked up is ever dropped before completing its service is called the Drop probability. We compute both these quantities. A small cell catering to non elastic traffic can be modeled by a M/G/K/K queue (as we have done in [4]). Then using Erlang loss formula the busy probability can be calculated as,

$$P_{Busy}(L) := \frac{\rho_L^K/K!}{\sum_{k=0}^K \rho_L^k/k!}.$$

Busy probability, P_{Busy}, depends upon L only via ρ and both are differentiable in L (see [4] for similar details) and by differentiating twice one can immediately obtain the following:

Lemma 1: The optimizers of ρ and P_{Busy} are same, i.e.,

$$L_\rho^* := \min_L \rho = \min_L P_{Busy}(L) =: L^*_{P_{Busy}}. \quad \blacksquare$$

Drop probability (probability that a call that is picked up will ever be dropped) can now be calculated by conditioning.

$$P_{Drop} = P_{e,ho}(P_{Busy} + (1 - P_{Busy})P_{h,ho}P_{h,Drop})$$

where $P_{h,ho}$ and $P_{e,ho}$ are defined in previous section and where $P_{h,Drop}$ is the drop probability given that the call is a handover call, which satisfies by HO-SE:

$$P_{h,Drop} = P_{Busy} + (1 - P_{Busy})P_{h,ho}P_{h,Drop}$$

and so $P_{h,Drop} = \dfrac{P_{Busy}}{1-(1-P_{Busy})P_{h,ho}}$. Substituting,

$$P_{Drop} = \frac{P_{e,ho}P_{Busy}}{1-(1-P_{Busy})P_{h,ho}}. \tag{11}$$

E. Expected waiting time for Elastic Traffic

One can follow the approach as in ([4]) to derive the corresponding performance, the average waiting time of a call. However this is not considered in this paper.

F. Capacity per cell

We define capacity of a cell as the average number of the "maximum"[6] bytes that can be transferred when a call originates in the cell. Let c_n represent the maximum number of bytes that can be transmitted when a call originates in $\mathbb{A}n$. While staying in region n a maximum of $W_n r_n$ number of bytes can be transferred and hence c_n can be obtained using the following iteration (by same procedure as used for (6))

$$c_n = E[W_n]r_n + p_n c_{n+1} + (1 - p_n)c_{n-1}.$$

Thus the capacity of the cell and the capacity per cell equals (for 1D the length of a cell $\propto L$ while for 2D the area of the cell is $\propto L^2$)

$$C_{cap} = \Pi^t \mathcal{P}^{-1}\mathbf{r_w} \text{ and } C_{cell} := \frac{C_{cap}}{L^\eta} = \frac{1}{L^\eta}\Pi^t\mathcal{P}^{-1}\mathbf{r_w} \tag{12}$$

[6]By "maximum" we mean the theoretical maximum possible rate, given the rate partitioning. The rates given by (3) exactly represent this "maximum" rates when $\nu = 1$.

where with $\bar{p}_n := p_n - 1$ and $\hat{p}_n := -p_{-n}$,

$$\mathcal{P} := \begin{bmatrix} 1 & \hat{p}_N & 0 & & & \cdots & 0 \\ \bar{p}_{-(N-1)} & 1 & \hat{p}_{N-1} & & \cdots & & 0 \\ & & \vdots & & & & \\ 0 & 0\cdots & & \bar{p}_1 \quad 1 & \hat{p}_1 & \cdots & 0 \\ & & \vdots & & & & \\ 0 & 0\cdots & & & \cdots & \bar{p}_N & 1 \end{bmatrix}$$

$$\mathbf{c} := [c_{-N}, c_{-N-1}, \cdots, c_{-1}, b_1, \cdots, c_N]^t$$

$$\mathbf{r_w} := [r_{-N}E[W_{-N}], \cdots, r_{-1}E[W_{-1}], r_1E[W_1], \cdots, r_NE[W_N]]^t .$$

Again for 2D the quantities are reduced matrices/vectors as explained before.

G. Time to reach boundary

Expected time to reach boundary can be calculated in a similar way and it equals ($\mathbf{w}$ is a column vector of made up of $\{E[W_n]\}$)

$$\tau_L := E[T_L] = \Pi^t \mathcal{P}^{-1}\mathbf{w}. \tag{13}$$

We summarize all the expressions derived in the Table I. From this table, it is clear that analysis can be carried out and performance measures can be obtained for any system (i.e., given N, K, the set of possible transmission rates $\mathcal{R}$ etc.,) for which $\{p_n\}$ (the transition probabilities w.r.t. the rate regions) and $\{q_n\}$ (the Laplace transform of the wandering times $W_{n;L}$) can be calculated. We next consider two example user movement models and apply the analysis of this section to obtain expressions for various performance measures. We also obtain the closed form expressions for the optimizers (optimal cell dimension) of these performance measures in some cases.

TABLE I
THE VARIOUS EXPRESSIONS

$q_n = 1 - E\left[exp^{-\mu W_n r_n}\right]$	$t_n = \frac{q_n}{\mu r_n}$
$\bar{b}_e = \Pi^t \mathcal{Z}^{-1}\mathbf{t}$	$\bar{b}_h = \Pi_h^t \mathcal{Z}^{-1}\mathbf{t}$
$P_{e,ho} = 1 - \Pi^t \mathcal{Z}^{-1}\mathbf{q}$	$P_{h,ho} = 1 - \Pi_h^t \mathcal{Z}^{-1}\mathbf{q}$
$\lambda_{h;L} = \lambda_L \frac{P_{e,ho}}{1-P_{h,ho}}$	$\rho = \frac{\left(\lambda_L \Pi^t + \lambda_{h;L}\Pi_h^t\right)\mathcal{Z}^{-1}t + \lambda_{h;L}t_h}{K}$
$P_{Busy}(L) = \frac{\rho_L^K/K!}{\sum_{k=0}^K \rho_L^k/k!}$	$P_{Drop} = \frac{P_{e,ho}P_{Busy}}{1-(1-P_{Busy})P_{h,ho}}$
$C_{cell} = L^{-\eta}\Pi^t\mathcal{P}^{-1}\mathbf{r_w}$	Example, $r_n = r_0 P_L N^\beta L^{-\beta}\lvert n\rvert^{-\beta}$

IV. Random Walk with Exponential Wandering times

The users arrive in one of the rate regions n, wander in that region for time $W_{n;L}$ which is exponentially distributed (whose distribution is independent of every other process) and then switches to one of its neighboring rate regions or moves over to the next cell if region n was in the edge of the cell. The mean of the wandering time $W_{n;L}$ is proportional to the measure (length in case of 1D, area in case of 2D) of the region in which it is moving. The area of 2D annular ring n equals $\pi(L/N(n+1))^2 - \pi(L/Nn)^2 = \pi L^2/N^2(2n-1)$. That is (recall $\eta = 1$ for 1D and 2 for 2D),

$$E[W_{n;L}] = \frac{1}{\omega_L} = \frac{L^\eta(2n^{\eta-1}-1)}{\omega} \text{ for some } \omega > 0.$$

This dependence upon the cell size L ensures that the mean variations of the mobility model remains (almost) same irrespective of the cell size. In this case (from (4), (5) and (7)),

$$\begin{aligned} q_n &= \frac{\mu r_n}{\omega_L + \mu r_n}, \quad t_n = \frac{1}{\omega_L + \mu r_n}, \\ z_n &= -\frac{\omega_L}{\omega_L + \mu r_n} p_n, \text{ and } \bar{z}_n = -\frac{\omega_L}{\omega_L + \mu r_n}(1 - p_n). \end{aligned} \tag{14}$$

We assume that the arrivals position themselves uniformly across the entire system and hence $\pi_n = 1/((3-\eta)N)$. We further assume that the rates used depend upon the distance from the BS. In particular we choose the theoretical rates (in low SNR regime) as in equation (3), reduced by a ν factor (which is absorbed into r_0) as explained in section II.

A. Capacity per cell

In this case, the capacity per cell (12) simplifies to,

$$\begin{aligned} C_{cell} &= \frac{r_0 P_L L^{-\beta}}{N^{-\beta}\omega} \Pi^t \mathcal{P}^{-1} \mathbf{n}_\beta \text{ with} \\ \mathbf{n}_\beta^t &:= \begin{cases} \left[N^{-\beta}, \cdots, 2^{-\beta}, 1, 1, 2^{-\beta}, \cdots, N^{-\beta}\right] & \text{if 1D} \\ \left[1, 3 * 2^{-\beta}, \cdots, (2N-1)N^{-\beta}\right] & \text{if 2D} \end{cases} \end{aligned} \tag{15}$$

The capacity per cell represents the maximum information per cell size that can be transfered while an user moves in the cell which can support N distinct rates. If the total power in the system has to remain constant[7] then $P_L = PL^\eta$. With P_L scaling as PL^η, we notice from equation (15) that C_{cell} decreases with L (note practical values of $\beta \geq 2$, even $\beta = 2$ is not considered as a very practical value of path loss coefficient). This implies that the optimal cell size (optimizing the fundamental limit C_{cell}) is Nd_0, which is a practically infeasible cell dimension. To put it in the other way, the total power budget has to be increased with L, to design cells with practical values of cell dimension. The necessary growth rate can easily be read from (15) and hence we have,

Lemma 2 (β^+**-scaling):** Capacity per cell increases with L only if the power per transmission scales with L according to

$$P_L = PL^{\beta+\gamma} \text{ for some } \gamma > 0. \quad \blacksquare$$

To obtain this result we used *low SNR approximation of the capacity formula* $log(1 + P_L r_n) \approx P_L r_n$. *However one can easily see that Lemma 2 is true, even without this approximation.* This approximation is used only for simplifying further analysis.

The above lemma only says that the fundamental capacity can improve monotonically with cell size once you use $P_L = PL^{\beta+\gamma}$. *We call this henceforth as* β^+*-scaling.* However, this fundamental limit does not consider the losses due to handovers.

B. Drop and Busy probability:

The handover losses become significant for small cell sizes and metrics like drop probability (P_{Drop}) or busy probability (P_{Busy}) capture these losses. The rest of the section focuses on obtaining the optimal cell size for these metrics, when the power scales as in Lemma 2. We also show in some cases that the optimal cell size (for P_{Busy}) is Nd_0, if this scaling is not done. Towards the end we also consider/propose an optimal cell size that optimizes a cost combining the busy probability and the total power used.

For exponential wandering times, from (3) and (14):

$$q_n = \frac{\mu r_0 P_L L^{\eta-\beta} |n|^{-\beta}}{\mu r_0 P_L L^{\eta-\beta} |n|^{-\beta} + N^{-\beta}(2n^{\eta-1} - 1)^{-1}\omega}. \tag{16}$$

[7]The number of pico cells, when each is of dimension L, is proportional to $L^{-\eta}$ and hence total power would be proportional to $P_L L^{-\eta}$. Thus to maintain the total power constant, $P_L = PL^\eta$ for some constant $P > 0$.

We obtain further analysis in the two asymptotic limits of ω. The intermediate values of ω are studied via numerical examples. The cell sizes obtained in Lemmas 4 and 5 approximate well the optimal cell size obtained via the exhaustive search in numerical examples.

1) Low speeds (as $\omega \to 0$): From (13), the average time to reach the boundary of the cell, for exponential wandering times, equals:

$$\tau_L = \frac{L}{\omega} \Pi^t \mathcal{P}^{-1} \mathbf{1}, \text{ with } \mathbf{1}^t := [1, \cdots, 1, \cdots, 1]. \tag{17}$$

Thus, as $\omega \to 0$ the user covers the cell with large average times and hence this corresponds to the case of users moving with low speeds. In this limit, from equation (16) $q_n \approx 1$. This in turn implies $\mathcal{Z}$ is close to identity matrix and that

$$t_n \approx \frac{1}{\mu r_n} = \frac{N^{-\beta}}{\mu r_0 P_L L^{-\beta} |n|^{-\beta}}, \quad P_{e,ho} \approx 0 \approx P_{h,ho}.$$

When the users wander in the same cell for considerable amount of time, its service gets completed within one cell itself and this is the reason for no handovers (i.e, $P_{e,ho} = P_{h,ho} = \lambda_{h;L} \approx 0$). With no handovers the drop probability is zero. And further with β^+-power scaling, one can expect an improvement in busy probability as the cell size increases. Substituting the power scaling ($P_L = PL^{\beta+\gamma}$) and with $\mathbf{n}_\beta^{-1} := [N^\beta, \cdots, 1, 1, \cdots, N^\beta]^t$ (note $\lambda_L = \lambda L^\eta$),

$$\rho = \frac{\lambda L^\eta \left(\Pi^t \mathbf{n}_\beta^{-1} \right)}{K \mu r_0 P_L L^{-\beta} N^\beta} = \frac{\lambda L^{\eta-\gamma} \left(\Pi^t \mathbf{n}_\beta^{-1} \right)}{K \mu r_0 P N^\beta} \text{ and } P_{Drop} \approx 0.$$

From the above equation it is clear that the busy probability improves only if $\gamma > \eta$. We notice that as γ increases, the performance (ρ) improves for the same cell size. Hence the busy probability, P_{Busy}, also reduces with γ. However this requires the power to be boosted. One can consider a joint cost, which combines power cost, for a given γ and for an appropriate weight $a > 0$:

$$\arg\min_L \left(L^{\eta-\gamma} + aL^{\beta+\gamma} \right). \tag{18}$$

Lemma 3: In the limit $\omega \to 0$, the optimizer for the joint cost (18) combing the power spent and a factor proportional to P_{Busy} is given by (when $\gamma > \eta$),

$$L_\rho^*(\gamma) = \left(\frac{\beta + \gamma - \eta}{a(\beta + \gamma)} \right)^{1/(\beta+2\gamma-\eta)}. \quad \blacksquare$$

2) High speeds (as $\omega \to \infty$): From (17), as $\omega \to \infty$ the average time to reach the boundary decreases to 0, which implies the users are moving at high speeds. With these,

$$q_n \approx \frac{\mu r_n}{\omega_L} = L^{\eta-\beta} \frac{\mu P_L r_0 |n|^{-\beta} (2n^{\eta-1} - 1)}{N^{-\beta} \omega},$$

$(1-q_n)$ with 1, t_n with $1/\omega_L$ and $\mathcal{Z}$ with $\mathcal{P}$. Then,

$$\begin{aligned}
\bar{b}_e &\approx \frac{L^\eta}{\omega}\Pi^t \mathcal{P}^{-1}\mathbf{n}_\omega, \quad \bar{b}_h \approx \frac{L^\eta}{\omega}\Pi_h^t \mathcal{P}^{-1}\mathbf{n}_\omega \text{ with} \\
\mathbf{n}_\omega &:= \mathbf{1}1_{\{\eta=1\}} + [1,3,\cdots,2N-1]^t 1_{\{\eta=2\}} \\
P_{e,ho} &\approx 1 - \frac{\mu P_L r_0 L^{\eta-\beta}}{\omega N^{-\beta}}\Pi^t \mathcal{P}^{-1}\mathbf{n}_\beta, \\
P_{h,ho} &\approx 1 - \frac{\mu P_L r_0 L^{\eta-\beta}}{\omega N^{-\beta}}\Pi_h^t \mathcal{P}^{-1}\mathbf{n}_\beta, \\
\rho_L &= \frac{\lambda N^{-\beta}}{K\Pi_h^t \mathcal{P}^{-1}\mathbf{n}_\beta \mu r_0}\left(\check{\rho}_1 L^{\beta+\eta}P_L^{-1} + \check{\rho}_2 L^{2\eta}\right) \text{ where} \\
\check{\rho}_1 &= \Pi_h^t \mathcal{P}^{-1}\mathbf{n}_\omega \text{ and} \\
\check{\rho}_2 &= \frac{\mu r_0}{\omega N^{-\beta}}\mathbf{n}_\beta^t \mathcal{P}^{-1}\left(\Pi_h \Pi^t - \Pi\Pi_h^t\right)\mathcal{P}^{-1}\mathbf{n}_\omega.
\end{aligned}$$

The above calculations did not consider $t_h = s_h/r_N$ the time required for handovers. *By exponential nature of the wandering times, the left over time in the last region will once again be exponential and hence the remaining calculations are unchanged.* With β^+- power scaling we have,

$$\begin{aligned}
\rho_L &= \frac{\lambda N^{-\beta}\left(\check{\rho}_1 L^{\beta+\eta}P_L^{-1} + \check{\rho}_2 L^{2\eta}\right)}{K\Pi_h^t \mathcal{P}^{-1}\mathbf{n}_\beta \mu r_0} + \frac{s_h \lambda_{h;L}}{K r_N} \qquad (19) \\
&= \frac{\lambda N^{-\beta}}{K\Pi_h^t \mathcal{P}^{-1}\mathbf{n}_\beta \mu r_0}\left(\tilde{\rho}_1 L^{\eta-\gamma} + \tilde{\rho}_2 L^{-2\gamma} + \check{\rho}_2 L^{2\eta}\right) \\
\tilde{\rho}_1 &= P^{-1}\check{\rho}_1 - \frac{s_h \mu \Pi^t \mathcal{P}^{-1}\mathbf{n}_\beta}{N^{-\beta}}, \quad \tilde{\rho}_2 = \frac{s_h \omega}{P^2 r_0}.
\end{aligned}$$

For large values of ω, $\check{\rho}_2$ is small and hence we have

$$\rho_L \approx \frac{\lambda N^{-\beta}}{K\Pi_h^t \mathcal{P}^{-1}\mathbf{n}_\beta \mu r_0}\left(\tilde{\rho}_1 L^{\eta-\gamma} + \tilde{\rho}_2 L^{-2\gamma}\right).$$

We *see that Lemma 2 is affirmed again, i.e., the optimizer of* ρ *(and hence that of* P_{Busy}*) equals trivial one* Nd_0 *if* $\gamma \le 0$. When $\gamma > 0$, by differentiating twice (first derivative is zero and second derivative is positive at minimizer) we obtain:

Lemma 4: For large values of ω, cell size optimizing busy probability, $L^*_{P_{Busy}} = Nd_0$ if $\gamma \le 0$. Whenever $0 < \gamma < \eta$,

$$L^*_\rho = L^*_{P_{Busy}} = \left(\frac{2\gamma\tilde{\rho}_2}{(\eta-\gamma)\tilde{\rho}_1}\right)^{1/(\eta+\gamma)}. \quad \blacksquare$$

If s_h, the handover bytes are negligible, then (since $\check{\rho}_2$ is very small for large ω) the load factor ρ varies with L predominantly via the term $L^{\eta-\gamma}$. This again implies that load factor decreases with cell size, only when $\gamma > \eta$ and so again one can optimize a joint cost combining the power cost as in Lemma 3. Cell size optimizing the drop probability, can be obtained similarly (proof in Appendix A):

Lemma 5: In the limit $\omega \to \infty$, the cell dimension that optimizes the drop probability is (whenever $K(\eta-\gamma) > \eta+\gamma$)

$$L^*_{P_{Drop}} = \left(\frac{((2K+1)\gamma+\eta)\tilde{\rho}_2}{(K(\eta-\gamma)-(\eta+\gamma))\tilde{\rho}_1}\right)^{1/(\eta+\gamma)}. \quad \blacksquare$$

Properties of the optimizers: We observe from Lemmas 4 and 5 that the optimal cell size: 1) decreases with increase in pathloss factor β ($\tilde{\rho}_1 \uparrow$ with $\beta \uparrow$); 2) increases with γ, the power scaling factor; 3) increases with increase in ω (from (17), when $\omega \uparrow$ "speed" of user $\uparrow$).

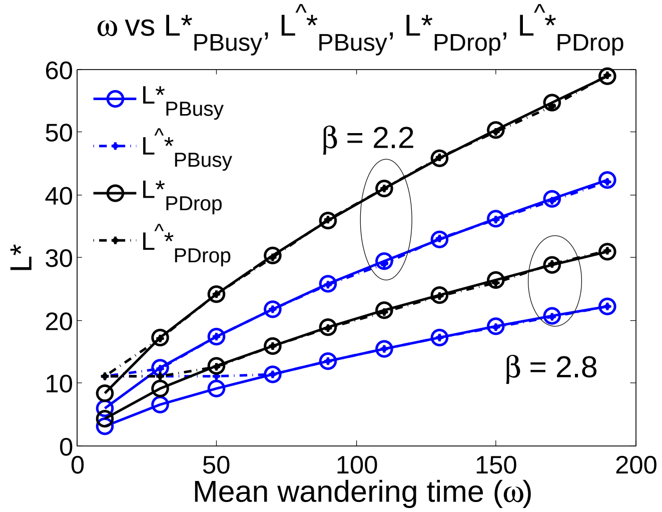

Fig. 4. Mean wandering time ω vs L*

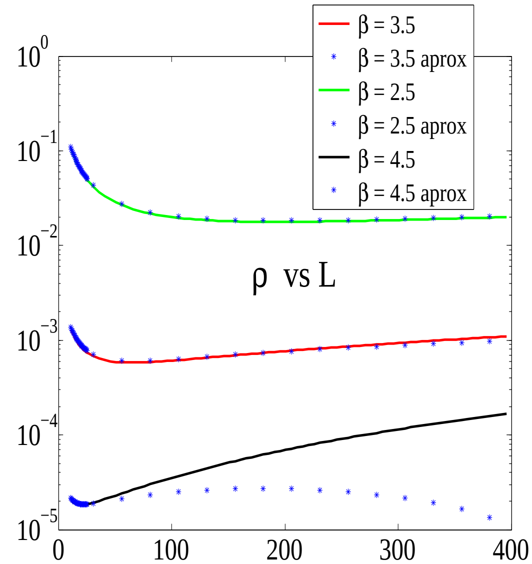

Fig. 5. ρ_L vs L

Joint Cost: One can optimize the following joint cost comprising of ρ and the power spent:

$$\min_{L} \left(\tilde{\rho}_1 L^{\eta-\gamma} + \tilde{\rho}_2 L^{-2\gamma} + aPL^{\beta+\gamma} \right).$$

3) Numerical examples: We obtain the optimizers for the general case of ω via numerical examples. We estimate the optimizers for the performance metrics given in Table I, after substituting the values of q_n, t_n etc., with equations (14), via grid search method. We compare the estimated optimizers (shown in figures with $\hat{}^{*}$ symbols) with that of the Lemmas 4 and 5 (shown in figures as *). From figure 4, we see that the computed optimizers are close to the numerically estimated ones for both the values of β (2.2 and 2.8), for both P_{Drop} as well as P_{Busy} and for large values of ω. For small values of ω ($\omega < 70$ for $\beta = 2.8$ and $\omega < 25$ for $\beta = 2.2$) we notice the approximation is no more good. In this example we set, $P = 0.0001$, $\gamma = 0.5$, $K = 10$, $\mu = 0.5$, $d_0 = 5$, $s_h = 0.01$, $N = 2$, $\eta = 1$ and $\lambda = 10^{-6}$.

In figure 5 we plot the high speed approximation for ρ given by (19) and the actual value of ρ as given in Table I after substituting (16). In this example we set $\gamma = 0.5$, $\mu = 10$, $\omega = 0600$, $s_h = 0.00001$, $P = .00000001$, $\lambda = .000001$, $K = 20$, $\eta = 1$ and N. We notice that the approximation is very close to the actual value, however the approximation error increases with increase in β the path loss coefficient, which once again confirms the closeness of the two sets of the optimizers of Figure 4 for large values of ω.

From these numerical examples, we again see that, the optimal cell size decreases with increase in path loss factor as well as with decrease in speed of the user given in terms of ω.

V. High Speed users (cars) moving in one direction

In this section we depart from randomly wandering users and study the case of users moving in a fixed direction. Interestingly the analysis of the previous section can still be used for this case and we obtain some initial results for this scenario, using the theory already developed. The users are moving in one direction (in a 1D cell) and at high speeds, which can vary slightly. This example arises when a user driving in a car derives his service from portable base stations which are installed on street infrastructure (like lamp posts). This scenario is exactly similar to the case in our previous work ([4]), but for one major difference. In [4], it is assumed that the rate of communication can be changed continually. This in some sense gives a "maximal" performance: if one can change rate of communication continually and that too, to the maximum possible one (i.e., capacity) then one obtains the best performance. But in reality this is not possible and we now consider similar situation but with maximum N different possible rates of communication as in the previous sections.

The users can move in one of the two directions with equal probability, i.e., with half probability. We assume symmetry in both the directions and hence any performance (e.g., busy probability, drop probability etc.), conditioned on the direction of the user, will be equal for both the directions. Thus, unconditional performance would be the same as the performance given a direction, say left to right. *Without loss of generality we assume the users are moving from left to right.* In every segment (say n), the user moves with constant speed $V_{n;L}$ and we assume that $V_{n;L}$ is independent of $V_{n';L}$ whenever $n \neq n'$. We also assume that the distribution of $V_{n;L}$ is same for all n and L. We assume uniform arrivals. Thus the wandering time in each segment equals,

$$W_{n;L} = \frac{L}{NV_{n;L}} \text{ for all } n.$$

The user is always moving from right to left (without loss of generality). Thus $p_n = 1$ for all n. It is easy to see that,

$$q_n = Prob\left(\frac{S}{r_n} < \frac{L}{NV_{n;L}}\right) = 1 - E\left[e^{-\frac{\mu r_n L}{NV_{n;L}}}\right].$$

The users are moving in high speeds (which are more or less constant) and so it is appropriate to assume that V is uniform between V_{max} and V_{min} (with V_{max} close to V_{min} and both away from 0). It is difficult to obtain the Laplace transform and hence q_n for such cases. However, with high values of V (for all realizations) one can approximate, $1 - q_n \approx 1$,

$$q_n \approx \frac{\mu r_n L}{N} E[1/V] \text{ and } t_n \approx E[W_{n;L}] = \frac{L}{N} E[1/V].$$

Thus, this case will be same as that in section IV-B2 (High speeds with exponential wandering times) with

$$\omega = \frac{N}{E[1/V]} \text{ and } \mathcal{P} = \begin{bmatrix} 1 & -1 & 0 & \cdots & & 0 \\ 0 & 1 & -1 & \cdots & & 0 \\ & & \vdots & & & \\ 0 & 0\cdots & & \cdots & 0 & 1 \end{bmatrix}.$$

By substituting these into the previous analysis (see Table I):

$$\begin{aligned} b_n &= \sum_{k \geq n}^{N} t_n, \quad \bar{b}_e = \sum_{n=-N}^{N} \pi_n b_n, \quad \bar{b}_h = b_{-N}, \\ P_{e,ho} &= 1 - \sum_n \pi_n \sum_{k \geq n} q_k \text{ and } P_{h,ho} = 1 - \sum_{k \geq -N} q_k. \end{aligned}$$

There will however *be a difference because of handover bytes s_h, as the wandering times are no more memory less.* We assume $L/(NV_{max}) > t_h$ so that the handover gets completed in the exterior rate region (e.g., r_{-N}) itself. Under this assumption, the analysis would still be applicable if we reduce the wandering time in the $\mathbb{A}_{-N}$ rate region, to $W_{-N;L} = L/(NV) - t_h$ for handover calls. This results in only the following changes,

$$\begin{aligned} \bar{b}_h &= b_{-N-1} + E\left[\frac{L}{NV} - t_h\right] + t_h = b_{-N} \text{ and} \\ q_{-N} &= 1 - E\left[e^{-\mu r_{-N}\left(\frac{L}{NV} - t_h\right)}\right] \approx \frac{\mu r_{-N} L}{N} E[1/V] - \mu s_h, \end{aligned}$$

i.e., the average service time is not changed, however the possibility of service being completed, q_{-N} is reduced.

We can complete the analysis as in the previous section and obtain the following (with $\pi_n = 1/2N$ for all n),

$$\begin{aligned}
P_{e,ho} &= 1 - a_1 L^{1+\gamma} \left(\sum_n \pi_n \sum_{k \geq n} |k|^{-\beta} \right), \text{ with} \\
a_1 &= \frac{\mu E[1/V] r_0 P}{N^{-\beta+1}} \\
P_{h,ho} &= 1 - a_1 L^{1+\gamma} \left(\sum_n |n|^{-\beta} \right) + \mu s_h \\
\bar{b}_e &= \frac{L E[1/V]}{N} \sum_n \pi_n (N - n) = L E[1/V] \\
\bar{b}_h &= 2 L E[1/V].
\end{aligned}$$

We can easily show (with $\pi_n = 1/2N$ for all n) that,

$$\sum_{k \geq -N} |k|^{-\beta} - 2 \sum_n \pi_n \sum_{k \geq n} |k|^{-\beta} = \frac{-1}{N} \sum_{k \geq -N} |k|^{-\beta}.$$

Then (with $a_2 := a_1 \sum_n |n|^{-\beta}$),

$$\begin{aligned}
\rho &= \frac{\lambda L^2 E[1/V] \left(2 - \mu s_h - a_1 L^{1+\gamma} \frac{1}{N} \sum_n |n|^{-\beta} \right)}{K(1 - P_{h,ho})} \\
&= \frac{\lambda L^2 E[1/V] N \left(2N - N\mu s_h - a_2 L^{1+\gamma} \right) K^{-1}}{a_2 L^{1+\gamma} - \mu s_h}.
\end{aligned}$$

Therefore,

$$\frac{d\rho}{dL} = \frac{\lambda E[1/V] K^{-1} N}{\left(a_2 L^{1+\gamma} - \mu s_h \right)^2}$$
$$\left[\left(a_2 L^{1+\gamma} - \mu s_h \right) \left(2(2N - N\mu s_h) L - a_2 (3+\gamma) L^{\gamma+2} \right) - L^2 \left(2N - N\mu s_h - a_2 L^{1+\gamma} \right) a_2 (1+\gamma) L^{\gamma} \right]$$

Thus $d\rho/dL$ is zero if and only if

$$(1-\gamma) a_2 (2N - N\mu s_h) L^{3+\gamma} - 2 a_2^2 L^{3+2\gamma} - 2\mu s_h (2N - N\mu s_h) L + \mu s_h a_2 (3+\gamma) L^{\gamma+2} = 0$$

and so for large file sizes, i.e., with μ very small, one can neglect the terms with μ^2 (i.e., the second and the fourth terms)

Lemma 6: For cars moving on streets, whenever $\gamma < 1$ and with μ small,

$$L^*_{P_{Busy}} = \left(\frac{2\mu s_h N^{\beta-1}}{(1-\gamma) \mu E[1/V] r_0 P \sum_n |n|^{-\beta}} \right)^{\frac{1}{\gamma+2}}. \blacksquare$$

In [4] while dealing with a similar situation, but with continuum of rates, we showed that the optimal cell size is larger when the system has to support users with larger velocities. Here again, we notice that as $E[1/V]$ decreases, the optimal cell size increases. These are preliminary results and we plan to study this scenario in depth (the effects of β^+ scaling, $L^*_{P_{Drop}}$, expected waiting times in the case of elastic traffic etc.,) in future and obtain a complete comparison with the results of [4].

Conclusions

We obtained the performance analysis of small cell networks catering to randomly wandering users. We modeled the user movements by a random walk, in which each step corresponds to a rate region, where the rate regions are obtaining by partitioning the cell based on the service rates. With exponential wandering times, in each rate region, we obtained important performance measures like capacity per cell, busy and drop probabilities etc. We showed that the fundamental capacity per cell decreases monotonically with cell size, unless the power budget is increased (by a factor greater than the path loss factor, β) with cell size. We also showed that without β^+ power scaling, the optimal cell size, optimizing the busy probability, would be trivial (equal to the lossless distance).

We obtain closed form expressions for optimal cell sizes, with β^+ power scaling, in the two asymptotic regimes of the user speeds (speed tending to zero and infinity). We also obtained the optimizers for intermediate values of speeds via numerical simulations and established the following: 1) Optimal cell size increases with speed, ω; 2) decrease with path loss factor β and 3) increases with the power scaling factor γ.

We then obtained some initial results for high speed users moving in fixed direction as in [4], but receiving service at one among a finite number of service rates. We then proposed a further optimization of a joint cost comprising of total power budget and the busy probability.

These are initial results and the theory developed in this paper can be used for studying many more example scenarios. One can also extend the analysis of this paper to more complicated and accurate user movement models.

Appendix A

Proof of Lemma 5: By differentiating and simplifying (as $K/\rho >> 1$),

$$\frac{dP_{Busy}}{d\rho} = P_{Busy}\frac{K}{\rho} - P_{Busy}\frac{\sum_{m=0}^{K-1}\frac{\rho^m}{m!}}{\sum_{m=0}^{K}\frac{\rho^m}{m!}} \approx P_{Busy}\frac{K}{\rho}.$$

From table I, since $P_{h,ho}P_{Busy} << 1 - P_{h,ho}$ (these probabilities are small usually of the orders 10^{-3} or lesser):

$$P_{Drop} \approx P_{Busy}\frac{P_{e,ho}}{1-P_{h,ho}}.$$

Hence (for large ω) ,

$$\begin{aligned}
\frac{dP_{Drop}}{dL} \quad &\approx \quad P_{Busy}\left(\frac{d\left(\frac{P_{e,ho}}{1-P_{h,ho}}\right)}{dL} + \frac{P_{e,ho}}{1-P_{h,ho}}\frac{K}{\rho}\frac{d\rho}{dL}\right) \\
&\approx \frac{P_{Busy}}{\frac{\mu Pr_0}{\omega N^{-\beta}}\Pi_h^t\mathcal{P}^{-1}\mathbf{n}_\beta}\Big(- (\eta+\gamma)L^{-\eta-\gamma-1} \\
&\qquad + L^{-\eta-\gamma}K\frac{(\eta-\gamma)\tilde{\rho}_1 L^{\eta-\gamma-1} - 2\gamma\tilde{\rho}_2 L^{-2\gamma-1}}{\tilde{\rho}_1 L^{\eta-\gamma} + \tilde{\rho}_2 L^{-2\gamma}}\Big) \\
&= \frac{P_{Busy}L^{-\eta-\gamma-1}}{\frac{\mu Pr_0}{\omega N^{-\beta}}\Pi_h^t\mathcal{P}^{-1}\mathbf{n}_\beta\left(\tilde{\rho}_1 L^{\eta-\gamma} + \tilde{\rho}_2 L^{-2\gamma}\right)} \\
&\qquad \left((K(\eta-\gamma) - (\eta+\gamma))\tilde{\rho}_1 L^{\eta-\gamma} - (2K\gamma+\eta+\gamma)\tilde{\rho}_2 L^{-2\gamma}\right) \\
&= \frac{\lambda P_{Busy}L^{-\eta-\gamma-1}\omega N^{-2\beta}}{\mu^2 Pr_0^2\left(\Pi_h^t\mathcal{P}^{-1}\mathbf{n}_\beta\right)^2\rho} \\
&\qquad \left((K(\eta-\gamma) - (\eta+\gamma))\tilde{\rho}_1 L^{\eta-\gamma} - (2K\gamma+\eta+\gamma)\tilde{\rho}_2 L^{-2\gamma}\right).
\end{aligned}$$

The first term in the last equation is always non zero and so the derivative is zero if and only if the second term is zero. Further, the second derivative is positive at that zero. ∎

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