

# Fair scheduling in cellular systems in the presence of noncooperative mobiles

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**Abstract**—We consider the problem of centrally controlled ‘fair’ scheduling of resources to one of the many mobile stations connected to a base station (BS). The BS is the only entity making decisions in this framework based on truthful information from the mobiles on their radio channel. We study the well-known family of parametric  $\alpha$ -fair scheduling problems from a game-theoretic perspective in which some of the mobiles may be noncooperative. We first show that if the BS is unaware of the noncooperative behavior from the mobiles, the noncooperative mobiles become successful in snatching the resources from the other cooperative mobiles, resulting in unfair allocations. If the BS is aware of the noncooperative mobiles, a new game arises with BS as an additional player. It can then do better by neglecting the signals from the noncooperative mobiles. The BS, however, becomes successful in eliciting the truthful signals from the mobiles only when it uses additional information (signal statistics). This new policy along with the truthful signals from mobiles forms a Nash Equilibrium (NE) called a Truth Revealing Equilibrium. Finally, we propose new iterative algorithms to implement fair scheduling policies that robustify the otherwise non-robust (in presence of noncooperation)  $\alpha$ -fair scheduling algorithms.

## I. INTRODUCTION

Short-term fading arises in a mobile wireless radio communication system in the presence of scatterers, resulting in time-varying channel gains. Various cellular networks have downlink shared data channels that use scheduling mechanisms to exploit the fluctuations of the radio conditions (e.g. 3GPP HSDPA [2] and CDMA/HDR [8] or 1xEV-DO [1]). A central scheduling problem in wireless communications is that of allocating resources to one of many mobile stations that have a common radio channel. Much attention has been given to the design of efficient and fair scheduling schemes that are centrally controlled by a base station (BS) whose decisions depend on the channel conditions of each mobile. The BS is the only entity taking decisions based on truthful information from the mobiles on their radio channel. These networks use various fairness criteria ([6], [4]), called generalized  $\alpha$ -fair criteria, to design a class of parametric scheduling algorithms (which we henceforth call as  $\alpha$ -fair scheduling algorithms or  $\alpha$ -FSA). One special case, proportional fair sharing (PFS), has been intensely analyzed as applied to the CDMA/HDR system. See [10], [8], [7], [19], [3], [9], [16]. These results are applicable to the 3GPP HSDPA system as well. Kushner & Whiting [14] analyzed the PFS algorithm using stochastic approximation techniques and showed that the asymptotic

averaged throughput can be driven to optimize a certain system utility function (sum of logarithms of offset-rates). See also Stolyar [20]. All the above methods depend crucially on truthful reporting by the mobiles. For example, in the frequency-division duplex system, the BS has no direct information on the channel gains, but transmits downlink pilots, and relies on the mobiles’ reported values of gains on these pilots for scheduling. A cooperative mobile will truthfully report this information to the BS. A noncooperative mobile will however send a signal that is likely to induce the scheduler to behave in a manner beneficial to the mobile.

In [11] we analyzed efficient scheduling in the presence of noncooperation using a signaling game approach ([21]). On the other hand, for  $\alpha$ -fair scheduler, the BS utility is not expected utility, but is a concave combination of the users’ expected utilities and hence cannot be modeled by a signaling game. Further,  $\alpha$ -fair scheduler has an inherent feedback in its structure (more details in section II) and this feedback makes the study difficult and different from the above paper. This paper has contributions to three main areas:

**Networking Aspects:** (1) We identify cases where noncooperation results in an unfair bias in the channel assignments in favor of noncooperative mobiles, if the base station is unaware of the noncooperative behavior. (2) We characterize the limitation of the base station, and obtain conditions under which even when it is aware of noncooperation, it is not able to share fairly the resources. (3) We show that the ability to achieve fair sharing, in the presence of noncooperation, depends on the parameter  $\alpha$ . (4) We design robust iterative algorithms that, under suitable conditions, fairly share the resources even in the presence of noncooperative signaling.

**Game theoretical modeling:** (1) We model a noncooperative mobile as a rational player that wishes to maximize its throughput. Since the  $\alpha$ -fair assignment is related to the maximization of a related utility function, one can view the BS as yet another player. We thus have a game model even if there is a single noncooperative mobile. (2) We formulate three games of which one is a concave game. The formulation of the games turn out to be surprisingly complex. Except for the special case of  $\alpha = 0$  (where the game can be shown to be equivalent to a matrix game), the games are defined over an infinite set of actions. We are able however to prove existence and characterize equilibrium policies for two games. (3) The third game arises when the BS is unaware of noncooperation.

The BS merely responds to mobiles' signals, but in an optimal way. We could model this as a hierarchical game where the mobiles are involved in a game played at the higher level and the BS optimizes some utility at the lower level, unaware of the rationality of the mobiles. (4) To analyze iterative algorithms, we consider a stochastic game with asymptotic time limits of the iterative algorithm as cost criteria.

**Design of the networking protocols based on stochastic approximation techniques.** (1) We analyze the parametric  $\alpha$ -fair scheduling algorithm ( $\alpha$ -FSA) of [14] in presence of noncooperation. We identify its robustness properties as a function of  $\alpha$ . (2) Using the knowledge of channel and signal statistics, one can control the excess utilities that the mobiles would have otherwise obtained by noncooperation. This is the basic idea behind robust policies. We then use a stochastic approximation approach to combine estimation (which replaces the knowledge required) and control in order to design robust fair scheduling algorithms.

### A Motivating example

We consider two users sharing a common channel. User 1 has two channel states with utilities 7 and 3 occurring with probabilities 0.33 and 0.67 respectively. User 2 has constant channel with utility 4. The BS has to assign the channel to one of the two users for every realization of the channel state. Every such assignment rule results in a pair of users's average utilities. The BS uses an  $\alpha$ -fair scheduler (described in the next section) to fair share these average utilities. First we assume that both users cooperate and report their individual channel states correctly. In figures 2 and 1 we plot the average utilities obtained by users under  $\alpha$ -fair scheduler as a function of the fairness parameter  $\alpha$ . We make the following observations: (1) For every  $\alpha$ , the BS always allocates the channel to user 1 if he is in good state. (2) For  $\alpha = 0$ , the expected share of user 1 ( $7 * 0.33$ ) is less than that of the user 2 ( $((1 - 0.33) * 4)$ . This corresponds to efficient scheduling point. (3) For small values of  $\alpha$ , BS allocates the channel to user 1 only when he is in good state. (4) The expected share of user 1 increases while that of the user 2 decreases as  $\alpha$  increases and eventually become equal. To achieve this, the BS starts allocating channel to user 1 even when in bad state with increasing probability.

The above scenario depends crucially on the truthful reporting of channel by the user 1. Now, we consider the scenario when user 1 is noncooperative and tries to increase his utility. He declares to be in good state 7 when actually in bad state 3 with probability  $\delta$ . BS now observes the user 1 to have good channel with better probability  $0.33 + 0.67\delta$  and will schedule as before but based on reported channel conditions. In figures 1 and 2 we plot the resulting expected utilities of both the users as a function of fairness  $\alpha$  for  $\delta = 0.1$  and  $\delta = 0.5$ , respectively. We observe that the utility of user 1 for small values of  $\alpha$  is improved in comparison with cooperative utility. This also reduces the utility of the user 2 below its cooperative share, resulting in unfair allocations. This effect is seen for all values of  $\alpha$  less than  $\alpha = 1.75$ ,  $\alpha = 6.85$  respectively for  $\delta = 0.5$ ,  $\delta = 0.1$ . However, for  $\alpha$  greater than

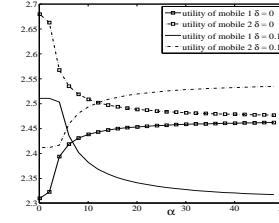


Fig. 1. User utilities versus  $\alpha$  for  $\delta = 0.1$

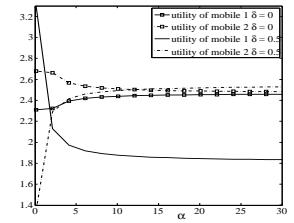


Fig. 2. User utilities versus  $\alpha$  for  $\delta = 0.5$

the above values, user 1 loses; in fact its utility gets below its cooperative share, while that of the user 2 is much above the latter's cooperative share. The above example indicates the  $\alpha$ -fair scheduler: (1) might be robust against noncooperation for large values of  $\alpha$ ; (2) fails for smaller values of  $\alpha$ ; (3) the larger the  $\delta$  the larger the amount of gain at  $\alpha = 0$ ; (4) the larger the  $\delta$  the smaller the  $\alpha$  until which the mobile gains. As  $\alpha$  increases, the two user utilities converge towards equal values at a rate that directly depends upon the difference at  $\alpha = 0$ . This is the reason for the above observation. An important point to note here is that, *there is no threshold of  $\alpha$  beyond which the scheduler will be robust to all types of noncooperation, i.e., for all values of  $\delta$* . However one can guess that for max-min fairness ( $\alpha = \infty$ ) the scheduler will be robust. The study of this noncooperation and design of robust policies will be the focus of this paper.

## II. THE PROBLEM SETTING AND $\alpha$ -FAIR SCHEDULER

**The Downlink:** We consider the downlink of a wireless network with one base station (BS). There are  $M$  mobiles competing for the downlink data channel. Time is divided into small intervals or slots. In each slot, one of the  $M$  mobiles is allocated the channel. Each mobile  $m$  can be in one of the states  $h_m \in \mathcal{H}_m$ , where  $\mathcal{H}_m$  is finite valued. We assume fading characteristics to be independent across the mobiles. Let  $\mathbf{h} := [h_1, h_2, \dots, h_M]^t$  be the vector of channel gains in a particular slot. The channel gains are distributed according to:  $p_{\mathbf{h}}(\mathbf{h}) = \prod_{i=1}^M p_{h_i}(h_i)$ , where  $\{p_{h_m}; m \leq M\}$  represents the statistics of the mobile channels. When the mobile's channel state is  $h_m$ , it can achieve a maximum utility given by  $f(h_m)$ . An example is the rate  $f(h_m) = r(m) = \log(1 + h_m \text{SNR})$  where SNR captures the nominal received signal-to-noise ratio under no channel variation, and  $h_m$  is the channel power gain. We assume  $f > 0$ .

**The decision rule:** In every slot, the BS has to make scheduling decisions, i.e., allocate the downlink slot to one of the  $M$  users, based on the current realization of the channel state vector  $\mathbf{h}$ . For any set  $\mathcal{C}$ , let  $\mathcal{P}(\mathcal{C})$  be the set of probability measures on  $\mathcal{C}$ . With that definition, a BS's decision variable is a *function*  $\beta$  that assigns to any given  $\mathbf{h}$  an element in  $\mathcal{P}(\{1, 2, \dots, M\})$ , the probability distribution over the set of

users. Thus,  $\beta(m|\mathbf{h})$  is the probability that the BS schedules transmission to mobile  $m$  given  $\mathbf{h}$ .

**The  $\alpha$ -fairness criterion and scheduler:** We introduce the well known generalized  $\alpha$ -fair criterion<sup>1</sup> ([4]) where the quantity that we wish to share fairly is the expectation of the random (instantaneous) utilities corresponding to the assignment by the scheduler to the mobiles:

$$G^\alpha(\beta) := \sum_{m=1}^M \Gamma^\alpha(\theta_m(\beta)) \quad (1)$$

where  $\theta_m(\beta) := \mathbb{E}_{\mathbf{h}}[f(h_m)\beta(m|\mathbf{h})]$  is the expected share of mobile  $m$  under policy  $\beta$  and where the  $\alpha$ -fair function is

$$\Gamma^\alpha(u) := \begin{cases} \log(u), & \text{for } \alpha = 1 \\ \frac{u^{1-\alpha}}{1-\alpha}, & \text{for } \alpha \neq 1. \end{cases}$$

One can view the scheduler  $\beta(\cdot|\cdot)$ , as a vector in  $\mathcal{R}^B$  space with  $B := M|\mathcal{H}|$ , where  $|\mathcal{H}|$  is the cardinality of product space  $\mathcal{H} = \prod_{m=1}^M \mathcal{H}_m$ . The domain of optimization is

$$\mathcal{D} := \left\{ \beta(\cdot|\cdot) : \sum_{m=1}^M \beta(m|\mathbf{h}) = 1, \beta(m|\mathbf{h}) \geq 0 \text{ for all } \mathbf{h}, m \right\}.$$

The objective function  $G^\alpha$  given by (1) is concave and continuous in  $\beta$  for each fixed  $\alpha$ , while the domain  $\mathcal{D}$  is compact and convex. Hence *there always exists a cooperative  $\alpha$ -fair scheduling BS strategy  $\beta^*$* :

$$\beta^*(\cdot|\cdot) \in \arg \max_{\beta \in \mathcal{D}} G^\alpha(\beta). \quad (2)$$

**Remarks II-1:** We may view the BS's schedule as a static optimization problem that corresponds to a single choice of  $\beta$ . Notice that the optimal schedule  $\beta^*$  maximizes some function of the *expected* shares of utilities. This expected share depends on assignments at all channel states, and is therefore a joint optimization problem. This feature arises when  $\alpha > 0$ . When  $\alpha = 0$  the problem is separable, and the solution  $\beta^*(\cdot | \mathbf{h})$  for a given  $\mathbf{h}$  depends only on that  $\mathbf{h}$ . However, for  $\alpha > 0$ , the implicit equation (3) below highlights a certain 'feedback' that is absent when  $\alpha = 0$ . This makes the present study significantly different from our previous work on efficient scheduling with strategic mobiles ([11]).

We now show a key (feedback) property of  $\alpha$ -fair schedulers. Define  $\beta^*$  as the vector fixed point, if it exists, that satisfies the following:

$$\beta^*(m|\mathbf{h}) = \frac{1_{\{m \in \arg \max_j d\Gamma^\alpha(\theta_j(\beta)) f(h_j)\}}}{|\arg \max_j d\Gamma^\alpha(\theta_j(\beta)) f(h_j)|}, \quad (3)$$

where  $d\Gamma^\alpha(\theta_j(\beta)) := \frac{d\Gamma^\alpha}{du} \Big|_{u=\theta_j(\beta)}$  is the derivative of  $\Gamma^\alpha$  with respect to (w.r.t.)  $u$ , evaluated at  $\theta_j(\beta)$ .

**Lemma 1:** If there is a  $\beta^*$  satisfying (3), then  $\beta^*$  is a global maximizer of the objective function in (2) over domain  $\mathcal{D}$  and hence is an  $\alpha$ -fair solution.

Let  $\Theta := [\theta_1, \dots, \theta_M]^T$ ,  $\Theta(\beta) := [\theta_1(\beta), \dots, \theta_M(\beta)]^T$  and  $\Theta(\mathcal{D}) := \{\Theta(\beta) : \beta \in \mathcal{D}\}$ . The map  $\Theta \mapsto \sum_m \Gamma^\alpha(\theta_m)$

<sup>1</sup>Each  $\alpha$  defines a scheduler. The system designer chooses an appropriate  $\alpha$  based on his desired tradeoff between system efficiency and fairness.

is strictly concave. Hence, there exists an unique maximizer (of the expected assigned shares) over the convex set  $\Theta(\mathcal{D})$ :

$$\Theta^* = \max_{\Theta \in \Theta(\mathcal{D})} \sum_m \Gamma^\alpha(\theta_m). \quad (4)$$

Hence, if there is a  $\beta^*$  satisfying (3), then  $\Theta^* = \Theta(\beta^*)$ . Further, any  $\bar{\beta}^*$  which is a global maximum of the objective function (2) satisfies the 'efficiency' property : whenever  $f(h_m) > f(h'_m)$ , either  $\bar{\beta}^*(m|h_m, \mathbf{h}_{-m}) = \bar{\beta}^*(m|h'_m, \mathbf{h}_{-m}) \in \{0, 1\}$  or  $\bar{\beta}^*(m|h_m, \mathbf{h}_{-m}) > \bar{\beta}^*(m|h'_m, \mathbf{h}_{-m})$  (5)

for all  $\mathbf{h}_{-m} \in \Pi_{j \neq m} \mathcal{H}_j$  and for all  $m$ . ■

**Proof :** Please refer to Appendix B in the full version [13].

The above Lemma 1 gives the exact characterization of an optimal solution for the  $\alpha$ -fairness problem (3). It further talks about the efficiency of every possible  $\alpha$ -fair solution (5): the assignment for particular state ( $h_m$ ) for any mobile  $m$  increases with increase in the utility ( $f(h_m)$ ) of the state. This property is used in the analysis under noncooperation. A part of Lemma 1, regarding the possible solution (3), when restricted to proportional fairness, is already stated in [15].

**Remarks II-2:** The solution (3) explicitly shows the feedback we mentioned in Remark II-1. This solution has already been used in practical scenarios ([15]) to achieve 'fair' scheduling: The  $\alpha$ -fair solution for the dynamic setting with ergodic channel states is the optimal  $\beta$  that fair shares the time average utilities over a single realization of a whole sample path.<sup>2</sup> In fact, the solution (3) under ergodicity can be implemented by the following procedure : 1) At any time slot  $k$ , obtain the scheduling decision using the current channel vector  $\mathbf{h}_k$  and using the time averaged assigned utilities obtained until the last step,  $\{\theta_{m,k-1}\}$ , in place of  $\{\theta_m(\beta^*)\}$  in (3); 2) Update (in the obvious way) the time averaged assigned utilities up to step  $k$ ,  $\{\theta_{m,k}\}$ , using the current decision.

### III. PROBLEM FORMULATION UNDER NON COOPERATION

In every slot, the BS needs to know  $\mathbf{h}$  for optimal scheduling. In practice, mobile  $m$  estimates channel  $h_m$  using the pilot signals sent by BS. We assume perfect channel estimation. The mobiles send signals  $\{s_m\}$  to BS as indications of the channel gains. Thus BS does not have direct access to channel state  $\mathbf{h}$ , but instead relies on the mobiles for this information. If the mobiles are strategic, they can signal a better channel condition to grab the channel even when their channel is bad. The main purpose of this paper is to study the effect of this noncooperation on  $\alpha$ -fair scheduler (2). We assume that signals are chosen from the channel space itself, i.e.,  $s_m \in \mathcal{H}_m$  for all mobiles. We consider two types of scenarios :

**Hierarchical game G1:** The BS is unaware of the possible noncooperative behavior from the mobiles and applies the  $\alpha$ -fair scheduler (2) to the signals  $\mathbf{s} = [s_1, \dots, s_M]^T$  (as if they were the true channel values). The mobiles, being aware of BS's scheduler, signal to optimize their own goals.

<sup>2</sup>For stationary and ergodic channels with finite  $E_{\mathbf{h}}[g(\mathbf{h})]$ ,  $\lim_{K \rightarrow \infty} \frac{1}{K} \sum_{k=1}^K g(\mathbf{h}_k) = E_{\mathbf{h}}[g(\mathbf{h})]$ . We are interested in a particular function  $g(\mathbf{h}) = f(h_m)\beta(m|\mathbf{h})$  whose average is exactly  $\theta_m(\beta)$ .

When the base station is unaware of this strategic behavior, we model this game as a hierarchical game with two levels: where leaders, the noncooperative mobiles, are involved in a game problem, while BS, the follower does the optimization. In this game, there is no common knowledge: the base station does not know the rationality of the mobiles. This game is related to that discussed by Aumann in [5] through many examples. For several years it has been thought that the assumption of common knowledge of rationality for the players in the game was fundamental. It turns out that, in  $N$ -player games, common knowledge of rationality is not needed as an epistemic condition for equilibrium strategies (see [5]).

*A game approach:* The BS is modeled as an additional player in a one-shot game. When the BS becomes aware of the possible noncooperation, it would implement better policies (Section V-A). However when the BS has to base its decision only on the signals from the mobiles it will not be successful in compelling the mobiles to reveal the truthful signals (Section V-A). We model this as  $M_1 + 1$  player game G2. In Section V-B we construct more intelligent BS policies that are robust to noncooperation, but require more information. The new robust BS policies and the mobile policy of truthful signaling form a Nash Equilibrium. We refer to this game as game G3.

We introduce important concepts and definitions that are used in the paper. These are specific to the second scenario. Corresponding definitions and concepts may vary for the other scenario and the differences are explained in Section IV.

**Common Knowledge:** Channel statistics  $\{p_{h_m}; m \leq M\}$  and the information about which mobiles are noncooperative is common knowledge (i.e., known to all the mobiles and the BS, and further, every one knows that every one else knows this, and so on). If the BS does not know which mobiles are cooperative, it will treat every mobile as noncooperative. When BS uses more intelligent policies (as in game G3) it can also detect the mobiles that are noncooperative.

**Mobile Policies:** Some mobiles (with indices  $1 \leq m \leq M_1$  where  $0 \leq M_1 \leq M$ ) are assumed to be noncooperative. A policy of mobile  $m$  is a function  $\{\mu_m(\cdot|h_m)\}$  that maps a state  $h_m$  to an element in  $\mathcal{P}(\mathcal{H}_m)$ .

**BS Policies:** A policy of the BS is a function which maps every signal vector  $\mathbf{s}$  to a scheduler  $\beta \in \mathcal{P}(\{1, 2, \dots, M\})$ . These policies are used in a major part of the paper, while more complicated policies are considered in section V-B.

**Utilities for a given set of strategies:** The instantaneous/sample utility of the mobile  $m$  depends only upon the true channel  $h_m$  and the BS decision  $\beta$  and is given by :  $U_m(s_m, h_m, \beta) = 1_{\{\beta=m\}} \min\{f(h_m), f(s_m)\}$ <sup>3</sup>.

Define the following to exclude mobile  $m$ :

$$\begin{aligned} \mathbf{h}_{-m} &:= [h_1, \dots, h_{m-1}, h_{m+1}, \dots, h_M], \\ p_{\mathbf{h}_{-m}}(\mathbf{h}_{-m}) &:= \Pi_{j \neq m} p_{h_j}(h_j), \\ \mu_{-m}(\mathbf{s}_{-m}|\mathbf{h}_{-m}) &:= \\ &\quad \Pi_{j \neq m; j \leq M_1} \mu_j(s_j | h_j) \Pi_{j \neq m; j > M_1} \delta(h_j = s_j). \end{aligned}$$

<sup>3</sup>The mobile achieves rate  $f(h_m)$  even if the BS allocates it a higher rate  $f(s_m)$  because of the inflated signal  $s_m$  sent by the noncooperative mobile. The justification for this is provided in detail in Appendix C of [13], [11].

Also define,  $\mu = \{\mu_m; m \leq M_1\}$  to represent strategy profile:

$$\mu(\mathbf{s}|\mathbf{h}) := \Pi_{1 \leq j \leq M_1} \mu_j(s_j | h_j) \Pi_{j > M_1} \delta(h_j = s_j).$$

With the above definitions, each noncooperative user chooses its strategy  $\mu_m$  so as to maximize its own utility:

$$U_m^\alpha(\mu, \beta) = \mathbb{E}_{\mathbf{h}} \left[ \sum_{\mathbf{s}} U_m(s_m, h_m, m) \beta(m | \mathbf{s}) \mu(\mathbf{s} | \mathbf{h}) \right]. \quad (6)$$

Under the  $\alpha$ -fair criterion (1), the natural selection of utility for BS is:

$$U_{BS}^\alpha(\mu, \beta) = \sum_m \Gamma^\alpha(U_m^\alpha(\mu, \beta)). \quad (7)$$

Throughout we write  $i = \arg \max S$  to mean  $i \in \arg \max S$ . By  $j := \arg \max S$  we mean that  $j$  is a chosen element of  $\arg \max S$ .

**ASA, ATA Utilities:** When mobile signals do not match the true channel values, the game under consideration will have two important average utilities for any given strategy profile  $(\mu, \beta)$  : (1) average signaled utilities under assignment  $\beta$  (ASA) utility, which a (more intelligent) BS can observe, and (2) average true and assigned (ATA) utility, which is the true average utility gained by the mobile and whose value cannot be estimated (as long as the mobile is noncooperative) at the BS. These are defined by

$$U_m^{ASA}(\mu, \beta) := \mathbb{E}_{\mathbf{h}} \left[ \sum_{\mathbf{s}} f(s_m) \mu(\mathbf{s} | \mathbf{h}) \beta(m | \mathbf{s}) \right] \quad (8)$$

$$U_m^{ATA}(\mu, \beta) := \mathbb{E}_{\mathbf{h}} \left[ \sum_{\mathbf{s}} \min\{f(h_m), f(s_m)\} \mu(\mathbf{s} | \mathbf{h}) \beta(m | \mathbf{s}) \right].$$

Indeed, the utility of mobile  $m$  is ATA utility,  $U_m^\alpha = U_m^{ATA}$ .

**Truth Revealing Strategy:** In the following, by truth revealing strategy at mobile  $m$  we mean the strategy

$$\mu_m^T(s_m | h_m) = 1_{\{s_m = h_m\}}, \text{ for all } h_m, s_m \in \mathcal{H}_m,$$

which reflects the true channel state. Define  $\mu^T := (\mu_1^T, \dots, \mu_M^T)$ . Under truthful strategies  $\mu^T$ , ATA and ASA utilities coincide. For any BS policy  $\beta$ , if the strategy profile  $(\mu^T, \beta)$  forms a Nash Equilibrium (NE), then the NE is called a Truth Revealing Equilibrium (TRE).

**Cooperative Shares:** Best response of BS to truthful signals  $\mu^T$  is any maximizer  $\beta^*$  of  $G^\alpha$  (1). By Lemma 1, the best response results in unique maximum average ATA utilities,

$$\theta_m^{\alpha c} := \theta_m(\beta^*) = U_m^\alpha(\mu^T, \beta^*), \quad (9)$$

which will be referred as Cooperative Shares.

**Contrast between hierarchical optimization and the game perspective:** Recall that computing a fair assignment by the BS involves maximization of (1). Thus in the first scenario, when mobiles choose profile  $\mu$ , the unaware BS fair shares ASA utilities under  $\mu$  by maximizing (11). However, what needs fair sharing is the ATA utilities. This is achieved via the game perspective, wherein the rational BS tries to fair share the ATA utilities gained by the mobiles.

#### IV. SCHEDULING UNDER NONCOOPERATION : HIERARCHICAL GAME PROBLEM G1

We consider the scenario in which the BS is unaware of the presence of noncooperative mobiles. As in the cooperative setting, the BS schedules (using optimal scheduler (2)) the channel to one of the mobiles using the mobile signals, assuming them to reflect the channel state perfectly. The mobiles, aware of BS's scheduler, send signals to maximize their utilities.

**Utilities of G1:** For any given mobile strategy profile  $\mu$ , let the induced signal probabilities be represented by  $p_s$ , i.e.,  $p_s(s) = \sum_h p_h(h)\mu(s|h)$ . Since the BS observes  $p_s$  (instead of  $p_h$ ), it assumes the expected shares of mobile  $m$  to be  $\theta_m(\mu, \beta) := \mathbb{E}_{p_s}[f(s_m)\beta(m|s)]$  and hence maximizes,

$$U_{BS}^{ASA}(\beta, \mu) = \sum_m \Gamma^\alpha(\theta_m(\mu, \beta)). \quad (10)$$

Obviously,  $\theta_m(\mu, \beta)$  are the ASA utilities.

**Stackelberg Equilibrium for G1** is a profile  $(\beta^*, \mu^*)$  with

$$\beta_\mu^* = \arg \max_{\beta} U_{BS}^{ASA}(\beta, \mu) \quad (11)$$

$$\mu_m^* = \arg \max_{\mu_m} U_m^{ATA}((\mu_m, \mu_{-m}^*), \beta_{(\mu_m, \mu_{-m}^*)}^*) \text{ for all } m.$$

We now present some examples in which a user  $m$  deviates unilaterally from  $\mu^T$  and increases its utility above its cooperative share, thus resulting in unfair allocations. These examples do not have TRE for G1. In particular we consider  $\alpha$ -fair scheduler given by (3). This scheduler is a widely used practical solution (see Remark II-2),  $\alpha$ -FSA being one of them.

**A. Asymmetric Case : Proportional fair scheduler ( $\alpha = 1$ )**

We continue with the motivating example given in Section I. User 1 has a single state with utility  $a$ . User 2 has 2 states with respective utilities given by  $rb, b$  and with  $r > 1$ . The respective probabilities to be in one of these states are  $p, (1-p)$  with  $p \in (1/(1+r), 1/2)$ .

Using (3), one can easily estimate  $\beta^*$  and  $\{\theta_m(\beta^*)\}$  to be:  $\beta^*(2|a, rb) = 1, \beta^*(1|a, b) = 1, \theta_1(\beta^*) = a(1-p)$  and  $\theta_2(\beta^*) = rbp$ . It is important to note here that  $\beta^*$  satisfying (3) exists only if  $p \in (1/(1+r), 1/2)$  as in this case :

$$\begin{aligned} d\Gamma^\alpha(\theta_2(\beta^*))rb &= \frac{rb}{rbp} > \frac{a}{a(1-p)} = d\Gamma^\alpha(\theta_1(\beta^*))a \\ d\Gamma^\alpha(\theta_2(\beta^*))b &= \frac{b}{rbp} < \frac{a}{a(1-p)} = d\Gamma^\alpha(\theta_1(\beta^*))a. \end{aligned}$$

Suppose user 2 signals  $rb$  (when actually in state  $b$ ) with probability  $q$ , i.e.,  $\mu_2(rb|b) = q$ . Then user's maximum ASA rates (with  $\beta_\delta^* = \beta^*$ ) are  $U_1^{ASA}(q, \beta_q^*) = (1-p-q)a, U_2^{ATA}(q, \beta_q^*) = rb(p+q)$  respectively whenever

$$\frac{rb}{(p+q)rb} > \frac{a}{a(1-p-q)} > \frac{b}{rb(p+q)}.$$

With this, the mobile 2 obtains an improved ATA utility  $U_2^{ATA}(q, \beta_q^*) = rbp + bq > \theta_2(\beta^*)$ . The maximum possible value of  $q$  is  $q = (0.5 - p)$ . ■

**Extension to general  $\alpha$ :** One can extend the above to general  $\alpha$ , an  $\alpha$ -fair scheduler satisfying (3) exists if,

$$(rb)^{\alpha-1}p^\alpha < a^{\alpha-1}(1-p)^\alpha < r(rb)^{\alpha-1}p^\alpha.$$

From above, as  $\alpha$  increases,  $p$  for which (3) exists reduces and thus given  $(a, r, b, p)$ , there exists a maximum  $\alpha_{max}$ , beyond which there does not exist  $\alpha$ -fair scheduler of the type (3). However another type of  $\alpha$ -fair scheduler exists. For example, under max-min fairness (when  $\alpha = \infty$ ,  $\theta_1^* = \theta_2^*$ ), an  $\alpha$ -fair scheduler  $\{\beta^*(1|rb, a), \beta^*(1|b, a)\}$  is given by:

$$\beta^*(1|rb, a) = \frac{a}{rbp + ap}; \quad \beta^*(1|b, a) = 0, \text{ if } a(1-p) < rbp,$$

$$\beta^*(1|b, a) = \frac{a(1-p) - rbp}{(b+a)(1-p)}; \quad \beta^*(1|rb, a) = 0, \quad \text{otherwise.}$$

When  $\alpha$ -fair scheduler (3) exists the noncooperative mobile benefits: the maximum  $q(\alpha)$  satisfies:

$$(p + q(\alpha))^\alpha (rb)^{\alpha-1} = a^{\alpha-1}(1-p - q(\alpha))^\alpha.$$

For example with  $a = 4, r = 3, b = 3, p = 0.33$  the maximum  $\alpha$  for which  $\alpha$ -fair scheduler (3) exists is 7.9 and user 1 can benefit by signaling with  $q = .05$  for all  $\alpha \leq 4$ .

Generalization to more states and general  $\alpha$ : Consider two asymmetric users under the following assumptions :

**N.1** The cooperative  $\alpha$ -fair solution  $\beta^*$  (3) exists and without loss of generality let  $1 = \arg \max_m \theta_m^{\alpha c}$ .

**N.2** There exists an  $i > 1$  such that

$$\eta := \inf_{j > 1, h_j \in \mathcal{H}_j} d\Gamma^\alpha(\theta_1^{\alpha c})f(h_{1,i-1}) - d\Gamma^\alpha(\theta_j^{\alpha c})f(h_j) > 0,$$

where  $\mathcal{H}_1 = \{h_{1,1}, \dots, h_{1,N_1}\}$  are arranged such that  $f(h_{1,1}) > f(h_{1,2}) > \dots > f(h_{1,N_1})$ .

**Lemma 2:** Under N.1-N.2, there exist a policy  $\mu_1^\delta$  for mobile 1 that is not truth revealing, such that its ATA utility  $U_1^{ATA}(\mu_1^\delta, (f, \beta_{\mu_1^\delta}^*))$  is larger than its cooperative share  $\theta_1^{\alpha c}$ .

**Proof :** The policy for mobile 1 is given by

$$\mu_1^\delta(h_{1,i}|h_{1,i'}) = \begin{cases} 1_{\{i=i'\}} & \text{if } i' \neq i^* \\ \delta & \text{if } i' = i^* - 1 \text{ and } i = i^* \\ 1 - \delta & \text{if } i' = i = i^*, \end{cases}$$

where  $i^*$  is the largest  $i$  satisfying N.2. That this policy is not truth revealing is verified in Appendix B of [13]. ■

#### B. Symmetric Case

We consider a simple symmetric two mobile example. The mobiles have two states with utilities  $a_1, a_2$  occurring respectively with probabilities  $p_1, p_2$ . Let  $a_1 = ra_2, p_1 = pp_2$  with  $r > 1, p > 0$ . Under truthful signaling, by Lemma 1, an  $\alpha$ -fair optimal BS policy (for any  $\alpha$ ) is given by:  $\beta^*(1|a_1, a_1) = 1/2 = \beta^*(1|a_2, a_2), \beta^*(1|a_1, a_2) = 1, \beta^*(1|a_2, a_1) = 0$  with equal cooperative shares  $\theta_1(\beta^*) = \theta_2(\beta^*) = \left(\frac{p_2}{2} + p_1p_2\right)a_1 + p_2^2 \frac{a_2}{2}$ . Without loss of generality say mobile 1 deviates unilaterally from his truthful strategy with  $\mu_1(a_1|a_2) = t$ . If mobile 1 was successful, his reported rate would be greater than  $\theta_1(\beta^*)$ ; he would obtain this rate only when his declared state is  $a_1$  while that of mobile 2 is  $a_2$ . Thus, mobile 1 will be successful with maximum ASA utilities ( $\alpha = 1$ ):

$$U_1^{ASA} = (p_1a_1 + p_2ta_1)p_2 = (p+t)p_2^2a_1 \text{ and}$$

$$U_2^{ASA} = 1p_1a_1 + p_2(1-t)p_2a_2 = (pr + (1-t)p_2)p_2a_2$$

and the corresponding ATA utility

$$U_1^{ATA} = (p_1 a_1 + p_2 t a_2) p_2 = (pr + t)p_2^2 a_2$$

if the following conditions are met:

$$\frac{a_1}{U_1^{ASA}} > \frac{a_2}{U_2^{ASA}} \text{ and } \theta_1(\beta^*) < U_1^{ATA}, \text{ i.e., if } t \text{ satisfies:}$$

$$\frac{1}{(p+t)p_2} > \frac{1}{(pr + (1-t)p_2)} \text{ and } \frac{p^2 r + 1}{2} < t \blacksquare$$

### C. Robustness at large $\alpha$

For small values of  $\alpha$  the  $\alpha$ -fair scheduler fails. However we see a different phenomenon at higher  $\alpha$ . As  $\alpha$  increases to infinity, the 'fairness' increases and the expected shares, i.e., ATA utilities of all the mobiles tend to become equal ([17]), provided all the mobiles signal truthfully. However, in presence of noncooperation, it will be the ASA utilities that tend to become equal for higher values of  $\alpha$ . This results in all the cooperative mobiles (for whom ATA and ASA utilities are equal) getting equal ATA shares that are bigger than those for the noncooperative (for whom ATA are strictly less than ASA utilities) mobiles. Thus the  $\alpha$ -fair scheduler (2) tends to become robust towards noncooperation as fairness factor  $\alpha$  increases, inspite of the BS's unawareness of the noncooperation.<sup>4</sup> This effect is seen in the motivating example as well as in Figure 3 given in a later section. The noncooperative mobile's ATA utility diminishes as  $\alpha$  increases and goes below its cooperative share beyond  $\alpha = 1.2$  and further, the cooperative mobile gets more share than its cooperative share for these large values of  $\alpha$ .

## V. SCHEDULING UNDER NONCOOPERATION : GAME THEORETIC STUDY

In this section the BS knows about noncooperative behavior of mobiles and is considered as an additional player which results in an  $M_1 + 1$  player game.

### A. BS Scheduling policies of section IV : Game G2

In contrast to section IV, the BS knows the mobiles that are noncooperative. The resulting game is a one-shot concave game: the utility of mobile  $m$  (6) is linear in its policy  $\mu_m$  while that of the BS (7) is continuous and concave in its policy  $\beta$ . By [18], this game always has a NE<sup>5</sup>  $(\mu^*, \beta^*)$  which satisfies, for all  $m$ ,  $\mu_m^* = \arg \max_{\mu_m} U_m^\alpha((\mu_m, \mu_{-m}^*), \beta^*)$  and  $\beta^* = \arg \max_{\beta} U_{BS}^\alpha(\mu^*, \beta)$ . We obtain a 'babbling' equilibrium. This game does not have a TRE.

<sup>4</sup>However, we could only establish that max-min fairness is robust. Whether there is a finite threshold for  $\alpha$  beyond which the  $\alpha$ -fair scheduler is robust to strategic behavior is not known.

<sup>5</sup>Note that when adding further concave constraints the game remains concave even if the constraints are coupled [18]. We thus obtain equilibrium also for constrained versions of the game. An example constraint is one where the (possible weighted) sum of throughputs is bounded by a constant.

1) *G2 has Babbling NE* : We will now show that this game has a Nash equilibrium where the BS neglects the signals from the noncooperative users. Let  $\mathbf{h}^{>M_1} := [h_{M_1+1}, \dots, h_M]^t$  represent the channel states of the cooperative mobiles. With  $\theta_m^{>M_1}(\beta) := E_{\mathbf{h}}[f(h_m)\beta(m|\mathbf{h}^{>M_1})]$ , the BS maximizes:

$$\sum_m \Gamma^\alpha(\theta_m^{>M_1}(\beta)). \quad (12)$$

Note that  $\theta_m^{>M_1}(\beta) = E[f(h_m)]E_{\mathbf{h}^{>M_1}}[\beta(m|\mathbf{h}^{>M_1})]$  for any noncooperative mobile. As in Lemma 1, there always exists a  $\beta$  maximizing (12). Denote one such  $\beta$  as  $\beta^{>M_1*}$ . Choose any mobile profile  $\mu = ((\mu_1(.|h_1), \dots, \mu_{M_1}(.|h_{M_1}))$  which satisfies for all  $m \leq M_1$ ,  $\mu_m(s_m|h_m) = 0$  for all  $h_m, s_m$  with  $f(s_m) < f(h_m)$ . It is easy to see that this  $(\mu, \beta^{>M_1*})$  forms a Nash Equilibrium. Note that a noncooperative mobile  $m$  can obtain the utility  $\theta_m^{>M_1}(\beta^{>M_1*})$  only if it signals better than its channel true value (only then do we have  $\min\{f(h_m), f(s_m)\} = f(h_m)$ ) and hence the requirement of above condition on the set of mobile strategies.

This is a NE at which the BS ignores the signals from the noncooperative mobiles and is similar to the Babbling equilibrium defined in the context of signaling games ([21]).

2) *G2 has No TRE* : We now examine the existence of the desired TRE. If the  $M_1 + 1$  player game were to have a TRE, the corresponding (equilibrium) strategy of the BS should be the best response to mobile's truthful strategies  $\mu^T$  and hence will be a maximizer of  $U_{BS}^\alpha(\mu^T, \beta) = G^\alpha(\beta)$ . This best response indeed equals one of the maximizers of Lemma 1, which satisfies the efficiency property (5). Using this property one can show that the game G2 has no TRE (details in [13]).  $\blacksquare$

Thus the BS, even when aware of the noncooperation, is not successful in eliciting truthful signals. In the following we construct more intelligent policies which induce a TRE.

### B. Robust BS Policies : Game G3 has TRE

BS can estimate statistics  $p_s$  after sufficient observation of the mobile signals. We use  $p_s$  to build robust policies for BS which give us the desired TRE. The policy of BS now maps every ordered pair of signal and signal statistics  $(s, p_s)$  to an ordered pair  $(\Phi, \beta) = \{(\phi_m(s, p_s), \beta(.|s, p_s))\}$  with allocation  $\phi_m(s) \leq f(s_m)$  for all  $m$ . All the utilities will change appropriately to include  $\Phi$ ; for example,  $U_m^\alpha(\mu, (\Phi, \beta)) = \mathbb{E}_{\mathbf{h}}[\sum_s \min\{\phi_m(s), f(h_m)\}\mu(s|\mathbf{h})\beta(m|s)]$

A profile  $(\mu_1^*, \dots, \mu_{M_1}^*, (\Phi^*, \beta^*))$  is a NE for G3 if,

$$\begin{aligned} \mu_m^* &= \arg \max_{\mu_m} U_m^\alpha((\mu_m, \mu_{-m}^*), (\Phi^*, \beta^*)) \text{ for all } m, \\ (\Phi^*, \beta^*) &= \arg \max_{(\Phi, \beta)} U_{BS}^\alpha(\mu^*, (\Phi, \beta)). \end{aligned} \quad (13)$$

BS can then estimate the ASA utilities for any scheduling policy and for any mobile profile  $\mu$  as:

$$U_m^{ASA}(\mu, (\Phi, \beta)) = U_m^{ASA}(p_s, (\Phi, \beta)) := \mathbb{E}_s[\phi(s)\beta(m|s)].$$

In the above the expectation is w.r.t.  $p_s$ . It can also estimate their cooperative shares  $\{\theta_m^{\alpha c}\}$  of (9) using its prior knowledge: the channel statistics. We now propose a robust policy at the

BS which uses both these average utilities. The key idea is to design a policy at BS which does not allow the (average) utility of any mobile  $m$  to be greater than  $\theta_m^{\alpha c}$ .

When a noncooperative mobile uses a signaling strategy to improve its ATA utility  $U_m^{ATA}$ , even its ASA utility  $U_m^{ASA}$  improves. The BS can estimate  $U_m^{ASA}$  of each of the mobiles and hence can sense the increase in the noncooperative mobile's ASA utility in comparison to its cooperative share. Hence, *BS can detect the mobiles that are noncooperative*. The BS can further ensure that none of the mobiles is allocated more than its corresponding cooperative share by allocating only a fraction and not the total signaled utility at every sample. The fraction to be allocated is set based on the present excess over the cooperative share, as follows:

$$\phi_m(s_m, p_s, \beta) := (f(s_m) - (U_m^{ASA}(p_s, (\Phi, \beta)) - \theta_m^{\alpha c}) \Delta) \mathbb{1}_{\{(f(s_m) - (U_m^{ASA}(p_s, (\Phi, \beta)) - \theta_m^{\alpha c}) \Delta) > 0\}} \quad (14)$$

for some large value of  $\Delta$ . Hence, to ensure that none of the mobiles get more ASA utility than its cooperative share, BS needs to choose  $\Phi = \{\phi_m\}$  that satisfies

$$U_m^{ASA}(p_s, (\Phi, \beta)) = \mathbb{E}_s [\phi_m(s_m, p_s, \beta) \beta(m|s)]. \quad (15)$$

Both (15) and (14) are satisfied if there exists a fixed point  $U_m^{ASA} = U_m^{ASA}(p_s, \beta)$  that satisfies

$$\begin{aligned} U_m^{ASA} &= \mathbb{E}_s [\phi_m \beta(m|s) \mathbb{1}_{\{\phi_m > 0\}}]; \\ \phi_m &:= f(s_m) - (U_m^{ASA} - \theta_m^{\alpha c}) \Delta. \end{aligned} \quad (16)$$

For all  $s_m$  and  $U_m^{ASA}$ ,  $\phi_m \beta(m|s) \mathbb{1}_{\{\phi_m > 0\}} \leq C_f + \theta_m^{\alpha c} \Delta$ , where  $C_f$  represents the upper bound on  $f$ . Hence the right hand side of the first equation in (16) is bounded and continuous w.r.t.  $U_m^{ASA}$  by bounded convergence theorem. Thus there exists an  $U_m^{ASA}$  satisfying the fixed point equation (16) by Brouwer fixed point theorem<sup>6</sup>.

With the above allocation, ATA utility of mobile  $m$  is

$$U_m^{ATA}(\mu, (\Phi, \beta)) = \mathbb{E}_{h,s} [f_m^{gain}(h_m, s_m, p_s, \beta) \beta(m|s)] \quad (17)$$

where  $f_m^{gain}(h_m, s_m, p_s, \beta) := \min\{f(h_m), \phi_m(s_m, p_s, \beta)\}$ . It can be shown from (16) that for any strategy profile  $(\mu, \beta)$   $U_m^{ASA}(p_s, (\Phi, \beta)) - \theta_m^{\alpha c} \leq \frac{C_f}{\Delta} \leq o(1/\Delta)$ , see [13] for a proof of this fact, and hence,

$$U_m^{ATA}(\mu, (\Phi, \beta)) \leq U_m^{ASA}(\mu, (\Phi, \beta)) \leq \theta_m^{\alpha c} + o(1/\Delta).$$

The above holds as  $f_m^{gain}(h_m, s_m, \mu, \beta) \leq \phi_m(s_m, \mu, \beta)$ . In other words, *with new allocation (16) at BS, no mobile can gain  $o(1/\Delta)$  more than its cooperative share for any  $(\mu, \beta)$* .

Further, if BS uses any  $\alpha$ -fair scheduler  $\beta^*$  of (2) along with allocation policy (16), it is easy to check that the truthful strategies'  $U_m^{ASA}(\mu^T, \beta_1^*) = U_m^{ATA}(\mu^T, \beta_1^*) = \theta_m^{\alpha c}$  for all  $m$ . Also there exists only one fixed point,  $\theta_m^{\alpha c}$ , with  $(\mu^T, \beta^*)$  (details in [13]). We have thus proved:

**Theorem 1:** If BS knows cooperative shares  $\{\theta_m^{\alpha c}\}$  and the signal statistics  $\{p_s\}$ , the  $M_1+1$  player strategic game has an  $\epsilon$ -NE, i.e., TRE:  $(\mu^T, (\{\phi_m(s_m, p_s, \beta^*)\}, \beta^*(m|s)))$ . ■

<sup>6</sup>Brouwer fixed point theorem: Any continuous function  $f$  from a closed ball of a  $\mathcal{R}^n$  to itself has a fixed point, i.e., an  $x^*$  such that  $x^* = f(x^*)$ .

In the coming sections, we will turn our attention to iterative algorithms which can achieve a desired level of 'fairness' even in the presence of some noncooperative mobiles. We begin this task by first studying  $\alpha$ -FSA ([14]).

## VI. FAIR SCHEDULER ALGORITHM ( $\alpha$ -FSA)

From this section onwards the channel states  $h$  are continuous random variables with stationary rates across time,  $\{r_{m,k}\}_{k \geq 1} = \{f(h_{m,k})\}_{k \geq 1}$  for all  $m$ , satisfying the assumptions of Appendix A of [13]<sup>7</sup>.

The  $\alpha$ -fair scheduler (3) always exists for continuous and integrable rates by Brouwer's fixed point theorem (see [13]). We outlined an algorithm to implement  $\alpha$ -fair scheduler (3) in Remark II-2. The  $\alpha$ -FSA ([14]), a stochastic approximation based fair scheduling algorithms, exactly follows this outline (with  $\Theta_k^\alpha := [\theta_{1,k}^\alpha, \dots, \theta_{M,k}^\alpha]$ ,  $\mathbf{r}_k := [r_{1,k}, \dots, r_{M,k}]$ ):

$$\begin{aligned} \theta_{m,k}^\alpha &= \theta_{m,k-1}^\alpha + \epsilon_k [I_m^\alpha(\mathbf{r}_k, \Theta_{k-1}^\alpha) r_{m,k} - \theta_{m,k-1}^\alpha] \\ I_m^\alpha(\mathbf{r}, \Theta) &= \mathbb{1}_{\{m=\arg \max_j d \Gamma^\alpha(d_j + \theta_j) r_j\}} \\ &= \mathbb{1}_{\{m=\arg \max_j r_j (\theta_j + d_j)^{-\alpha}\}} \end{aligned} \quad (18)$$

where  $d_m$  are small positive constants (added for stability). While making decisions  $\{I_m^\alpha\}$ , if there are more than one users attaining maximum, one of the maximizers is chosen by the BS randomly. In [14, Th. 2.2], the authors show that  $\{\theta_{m,k}^\alpha\}$  of (18), with  $\alpha \leq 1$ , converges weakly to the unique limit point  $\Theta^*$  that satisfies  $\mathbb{E}[r_m I_m^\alpha(\mathbf{r}, \Theta^*)] = \theta_m^*$  for all  $m$ . A close look at this limit point (when we neglect  $\{d_m\}$ ) reveals that  $I_m^\alpha(\mathbf{r}, \Theta^*)$  is the  $\alpha$ -fair scheduler (3) and that  $\Theta^*$  are the unique cooperative shares,  $\{\theta_m(\beta^*)\} = \{\theta_m^{\alpha c}\}$ . Thus,  $\alpha$ -FSA weakly converges to the unique point (cooperative shares) that maximizes the  $\alpha$ -fair criterion (1).

### A. Convergence of $\alpha$ -FSA in presence of noncooperation

The  $\alpha$ -FSA uses signaled rates,  $r_{m,k}^s := f(s_{m,k})$  and  $\mathbf{r}_k^s = [r_{1,k}^s, \dots, r_{M,k}^s]^t$  to make decisions, as in Section IV:

$$\theta_{m,k}^\alpha = \theta_{m,k-1}^\alpha + \epsilon_k [I_m^\alpha(\mathbf{r}_k^s, \Theta_{k-1}^\alpha) r_{m,k}^s - \theta_{m,k-1}^\alpha].$$

These signaled rates reflect the statistics  $p_s$  (instead of  $p_h$ ). Again, there is weak convergence, but to a different attractor corresponding to  $p_s$ . It is very easy to see as in Section IV that, *when mobiles are noncooperative with profile  $\mu$ ,  $\alpha$ -FSA converges weakly to unique maximum ASA rates  $\{U_m^{ASA}(\mu, \beta_\mu^*)\}$  with  $\beta_\mu^*$  defined by (11)*.

<sup>7</sup>For understanding the asymptotic limits of the dynamic algorithms of this section we will need the results corresponding to the static settings of Section II. But, all the results of Section II correspond to discrete channel states and rates. We assume that even for the more general case under study in this section, an  $\alpha$ -fair solution of the form (3) exists and that the corresponding shares  $\{\theta_m^{\alpha c}\}$  are unique as in Lemma 1. Sufficient conditions for this to occur are under study. This result is required for showing that  $\alpha$ -FSA asymptotically converges to the cooperative shares (i.e., limits maximize the  $\alpha$ -fair criterion) for all  $\alpha$ . In [14] Theorem 2.3 does this job approximately at least for  $\alpha \leq 1$ : any other assignment rule results in a limit  $\Theta$  with  $\sum_m \Gamma^\alpha(\theta_m)$  less than that corresponding to scheduler  $\{I_m^\alpha\}$  of  $\alpha$ -FSA (18). The simulations of this section also confirm the results we obtained based on this assumption.

## B. Failure of $\alpha$ -FSA in presence of noncooperation

As noted above, the  $\alpha$ -FSA (18) converges to the maximum ASA utility (under  $\mu$ ) which need not be equal to the ATA utility, in the presence of noncooperation. However, to understand the behavior of (18) in presence of noncooperation, one needs to study the asymptotic true utilities gained by the mobiles under (18). Towards this, we consider a second iteration running in parallel with (18), in which the instantaneous signaled utility  $r_{m,k}^s$  replaced by the true instantaneous utility  $\bar{r}_{m,k} := \min\{r_{m,k}, r_{m,k}^s\}$ :

$$\bar{\theta}_{m,k}^\alpha = \bar{\theta}_{m,k-1}^\alpha + \epsilon_k [I_m^\alpha(\mathbf{r}_k^s, \Theta_{k-1}^\alpha) \bar{r}_{m,k} - \bar{\theta}_{m,k-1}^\alpha]. \quad (19)$$

As in [14], one can show that  $\bar{\theta}_{m,k}$  converges weakly to  $U_m^{ATA}(\mu, \beta_\mu^*)$ , the ATA utility under  $\beta_\mu^*$ .

Thus, the asymptotic limits of  $\alpha$ -FSA equal maximum ASA utilities of section IV while the true utility adaptation (19) converges to the corresponding ATA utilities. These time limits will thus have all the properties of section IV: the  $\alpha$ -FSA will fail for small  $\alpha$  and will be robust for large  $\alpha$  as discussed in section IV. The only difference here is that the channel rates are continuous.

*Numerical examples :* Two asymmetric users are considered in Figure 3. Let  $Z(\sigma^2)$  be a Rayleigh random variable with density  $f_Z(z; \sigma^2) = ze^{-z^2/2\sigma^2}$ . Channel state of user 1 is conditional Rayleigh distributed, i.e.,  $h_1 \sim f_Z(z; 1)1_{\{z \leq 2\}}dz/P(Z(1) \leq 2)$ . User 2 has almost a constant channel,  $h_2 - 0.45 \sim f_Z(z; 0.05)1_{\{z \leq 2\}}dz/P(Z(0.05) \leq 2)$ . The utilities are the achievable rates  $f(h) = \log(1+h)$ . User 1 is noncooperative with  $s_1(h) = h(1-\delta) + 2\delta$  with  $\delta = 0.9$ . We plot the limit of the  $\alpha$ -FSA, the limits of true utility adaptation (19) as function of  $\alpha$ <sup>8</sup>. We also plot the cooperative shares, obtained by the limits of  $\alpha$ -FSA with  $\delta = 0$ . We observe that the cooperative shares tend towards equal values as  $\alpha$  tends to infinity. User 1 is successful in gaining more utility in comparison to its cooperative share for all  $\alpha$  less than 1.2. Beyond 1.2, user 1 actually loses and the loss increases as  $\alpha$  increases. The observations are similar to that in the motivating example of Section I and indicate that  $\alpha$ -FSA is robust only for large  $\alpha$ . More examples, inferences, including the ones for symmetric case can be found in [13].

## VII. ROBUST $\alpha$ -FAIR ALGORITHMS : ROBUST FAIR SA

We saw that  $\alpha$ -FSA fails in the presence of noncooperative users. Hence, we propose a robustification of  $\alpha$ -FSA against noncooperation using the policies of subsection V-B. The policies of subsection V-B do not allow the ATA utility of any user to go above the cooperative share. Nevertheless, when a user is noncooperative, these policies may still result in a loss for the cooperative users: the noncooperative user can still grab the channel from other users without getting a gain, because of the robust allocation policies (16). To avoid this problem,

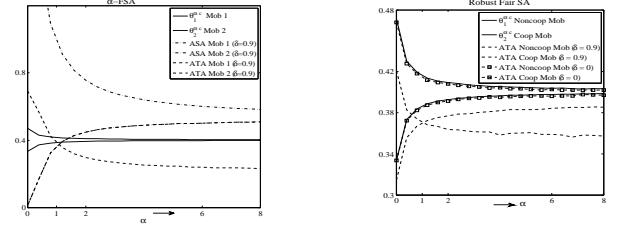


Fig. 3.  $\alpha$ -FSA : Maximal ASA and corresponding ATA shares versus  $\alpha$ .

Fig. 4. Robust Policy: ATA shares corresponding to Maximal ASA utilities versus  $\alpha$ .

one may robustify the decisions as well:

$$\begin{aligned} \theta_{m,k+1}^\alpha &= \theta_{m,k}^\alpha + \epsilon_k [\phi_{m,k+1}^\alpha I_m^\alpha(\Phi_{k+1}^\alpha, \Theta_k^\alpha) - \theta_{m,k}^\alpha] \\ \phi_{m,k+1}^\alpha &= \max\{0, r_{m,k+1}^s - (\theta_{m,k}^\alpha - \theta_m^{\alpha c}) \Delta\}, \end{aligned} \quad (20)$$

where  $\Phi_k^\alpha := [\phi_{1,k}^\alpha, \dots, \phi_{M,k}^\alpha]$  are robust allocations. As before, to analyze (20), we need to estimate the average of actual utilities,  $\hat{r}_{m,k+1}^\alpha = \min\{r_{m,k+1}, \phi_{k+1}^\alpha\}$ :

$$\hat{\theta}_{m,k+1}^\alpha = \hat{\theta}_{m,k}^\alpha + \epsilon_k [\hat{r}_{m,k+1}^\alpha I_m^\alpha(\Phi_{k+1}^\alpha, \Theta_k^\alpha) - \hat{\theta}_{m,k}^\alpha]. \quad (21)$$

### A. Analysis :

We analyze the robustness of the proposed algorithm using game theoretical tools. Fix any  $\alpha$ . We consider a  $M+1$  player game with utilities defined by:

$$U_m := \lim_{k \rightarrow \infty} \hat{\theta}_{m,k+1}^\alpha \text{ for all } m \text{ and } U_{BS} := \sum_m \Gamma^\alpha(U_m).$$

We analyze the limits of (21) using ODE approximation methods (for e.g., [14]). As a first step, we obtain the following ODE approximation result (proof available in [13]).

**Theorem 2:** Assume that algorithms (20), (21) satisfy assumptions A.1, A.2, and A.3 of Appendix A of [13]. For any initial conditions,  $(\Theta_k^\alpha, \hat{\Theta}_k^\alpha)$  converge weakly to the set of limit points of the solution of the ODE (for all  $m \leq M$ ):

$$\dot{\theta}_m = \bar{h}_m(\Theta) - \theta_m, \quad \bar{h}_m(\Theta) = \mathbb{E}[\phi_m^\alpha I_m^\alpha(\Phi^\alpha, \Theta)], \quad (22)$$

$$\dot{\hat{\theta}}_m = \bar{\hat{h}}_m(\Theta) - \hat{\theta}_m, \quad \bar{\hat{h}}_m(\Theta) = \mathbb{E}[\hat{r}_m^\alpha I_m^\alpha(\Phi^\alpha, \Theta)]. \quad (23)$$

These conclusions hold whenever  $\epsilon_n \rightarrow 0$ ,  $\sum_n \epsilon_n = \infty$  and for some  $\alpha_n \rightarrow \infty$ ,  $\lim_n \sup_{0 \leq l \leq \alpha_n} |\epsilon_{n+l}/\epsilon_n - 1| = 0$ . ■ Hence, one can upper bound utilities  $\{U_m\}$  by upper bounding all the attractors of ODE (23). Any attractor  $\Theta^*$  of the ODE (22) satisfies  $\theta_m^* - \theta_m^{\alpha c} = \frac{\mathbb{E}[\phi_m^\alpha I_m^\alpha(\Phi^\alpha, \Theta^*)] - \theta_m^{\alpha c}}{1 + \Delta \mathbb{E}[I_m^\alpha(\Phi_m^\alpha, \Theta^*)]}$ . Thus,  $\theta_m^* \leq \theta_m^{\alpha c} + o(1/\Delta)$ . Further, any attractor of ODE (23) satisfies  $\hat{\theta}_m^* = \bar{\hat{h}}_m(\Theta^*)$  leading to  $\hat{\theta}_m^* \leq \theta_m^*$ . Thus for any  $\mu$ ,

$$U_m \stackrel{w}{=} \hat{\theta}_m^* \leq \theta_m^* \leq \theta_m^{\alpha c} + o(1/\Delta). \quad (24)$$

So, none of the users, no matter what strategy they use or the others use, can gain more than  $\theta_m^{\alpha c}$ .

<sup>8</sup>The authors in [14] analyzed these algorithms only for  $\alpha \leq 1$ . However numerical simulations appear to indicate their validity for all values of  $\alpha$ .

Under  $\mu^T$ ,  $\theta^{\alpha c}$  is a zero of RHS of both the ODEs (22), (23). It will indeed be an attractor (see [13])<sup>9</sup>. Thus  $U_m = \theta_m^{\alpha c}$  for all  $m$  under  $\mu^T$ . Thus the robust policy (20) at BS together with the truth-revealing policy of users forms an  $\epsilon$ -NE.

#### Numerical examples

We continue with the example of Figure 3 (in which  $\alpha$ -FSA failed) in Figure 4. We use Robust Fair SA in place of  $\alpha$ -FSA. We set  $\Delta = 1000$ . We plot only the ATA utilities for both values of  $\delta = 0$ ,  $\delta = 0.9$ . We do not plot the ASA utilities (time limits of  $\{\theta_{m,k}\}$ ) in this figure as these utilities for all the cases are very close to cooperative shares  $\Theta^{\alpha c}$ . We see that this policy is indeed robust : 1) the time limit of the asymptotic true (ATA) utilities are lesser than the cooperative shares for the noncooperative mobile. 2) It is also lesser for cooperative mobile, but the gap between the cooperative shares and the ATA utilities is much lesser for a cooperative mobile (plots corresponding to  $\delta = 0.9$ ); 3) when all the mobiles are cooperative both the ASA as well as ATA utilities are close to the cooperative shares for all the mobiles (plots with  $\delta = 0$ ).

### VIII. CONCLUSIONS

We studied centralized downlink transmissions in a cellular network in the presence of noncooperative mobiles. Using  $\alpha$ -fair scheduler, the BS has to assign the slot to one of the many mobiles based on truthful information from mobiles about their time-varying channel gains. A noncooperative mobile may misrepresent its signal to the BS so as to maximize his throughput. We modeled a noncooperative mobile as a rational player who wishes to maximize his throughput. For this game, we identified several scenarios related to the awareness of BS. When the BS is unaware of this noncooperative behavior, we model this game as hierarchical game with two levels. We identify that, the presence of noncooperative users, results in an  $\alpha$ -fair bias in the channel assignment for small values of  $\alpha$ . As  $\alpha$  increases, an  $\alpha$ -fair scheduler becomes more and more robust to noncooperation irrespective of the awareness of BS and a max-min fair scheduler is always robust. When the BS is aware of the noncooperative mobiles, we characterized a babbling equilibrium which is obtained when both the BS and the noncooperative players make no use of the signaling opportunities. This game has no TRE. Using additional knowledge of the statistics of the signals observed at the BS, we built new robust policies to elicit the truthful signals from mobiles and achieve a Truth Revealing Equilibrium. We then studied the popular, iterative fair scheduling algorithm (which we called  $\alpha$ -FSA) analyzed by Kushner and Whiting in [14]. We showed that  $\alpha$ -FSA fail under noncooperation. Finally, we proposed iterative robust fair sharing to robustify the  $\alpha$ -FSA in the presence of noncooperation.

#### Acknowledgements

This project was supported by the Indo-French Centre for the Promotion of Advanced Research (IFCPAR), project 4000-IT-1 and by the INRIA association program DAWN. The

french co-authors have also been supported by the Bionets European project and the RNRT-ANR WiNEM project

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<sup>9</sup>That it is the only attractor for  $\mu^T$  is shown in the numerical examples.