ENERGY EFFICIENT CHANGE DETECTION OVER A MAC USING PHYSICAL LAYER FUSION

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ABSTRACT

We propose a simple and energy efficient distributed Change Detection scheme for sensor networks based on Page's parametric CUSUM algorithm. The sensor observations are IID over time and across the sensors conditioned on the change variable. Each sensor run CUSUM and transmits only when the CUSUM is above some threshold. The transmissions from the sensors are fused at the physical layer. The channel is modeled as a Multiple Access Channel (MAC) corrupted with IID noise. The fusion center which is the global decision maker, performs another CUSUM to detect the change. We provide the analysis and simulation results for our scheme and compare the performance with an existing scheme which ensures energy efficiency via optimal power selection.

Key words: CUSUM, Decentralized Change Detection, Reflected Random Walk, Brownian Approximation, Sensor Networks.

1. INTRODUCTION

Sensor networks are often deployed for monitoring and control of systems where human intervention is not desirable or feasible. We are interested in the problem of intrusion detection in a geographical area using sensor networks. The sensor network has the responsibility to detect this intrusion via periodic monitoring of the area and running some algorithm in a distributed fashion. It is assumed that the entry of an intruder will statistically affect the observations taken by the sensors. Thus, the algorithms developed for detection of change ([9, 11]) can be useful.

There are broadly two approaches to change detection ([11], [9]). Shiryaev [11] obtained an optimal algorithm while assuming that the change occurs at a random time with a geometric distribution (this is called the Bayesian setting). Page [9] did not assume any statistics for the unknown time of change, and developed the CUSUM algorithm. This was later shown to be asymptotic optimal in the Min-Max sense by [4]. Moustakides [8] later on showed that it minimizes the worst case delay.

In our application using sensor networks, two extra issues are involved. One is that the observations are made by many sensors distributed in space. These observations need to be sent to a fusion node to make the final decision. Secondly, each sensor is an inexpensive node with limited computational resources and has very little power. Thus the following variants of the change detection algorithm have been studied in this scenario:

1. Each sensor sends a quantized version of its observations to the fusion center. The fusion center, based on the data transmitted, makes a decision on the change ([14, 15]).
2. Each sensor runs a test based on algorithms of Shiryaev or Page and performs a local detection. The local decisions are then transmitted to the fusion center for it to fuse and make the final decision ([5, 7]).

A specific case of the first version is solved in ([15]) under the Bayesian setting with geometric change variable. If Page's CUSUM is used at each sensor to run the local test and no assumption is made about the change, then [5] has proved that it is asymptotically optimal (in the min-max sense) for the fusion center to declare a change at the first instant at which all the sensors declare a change. Recently Moustakides has shown that the CUSUM is Min-Max optimal when used in either of the above two scenarios.

In all the above strategies the change detection problem is solved at the link layer, by assuming a perfect physical layer (reliable communication) and energy is conserved by sending fewer bits. The problems with this type of formulation are:

1. In all of these approaches the sensor nodes transmit their observations to the fusion node at each time instant. By sending fewer bits the energy is conserved. However in change detection scenarios often the change occurs rarely. One can potentially save considerably by sending observations only when a change is detected by a sensor.
2. Even if the communication between the sensor and the fusion center is reliable (this can be ensured for example by using large enough transmit power), the sensors still have to contend with the MAC. With a large number of sensors, this delay can be significant.

In [10], the second effect is mitigated by making decision in each slot at the fusion center by using only the first few bits arriving in the slot. Information from other sensors are ignored. In [6], the information from all the sensors is sent simultaneously using orthogonal codes. In [16], Gaussian MAC is used to fuse the information from different sensors without using orthogonal codes. They also try to minimize directly the energy used in transmission. This provides them with significant gain in performance over the previous studies.

However, because in [16] also, every sensor transmits all its observations, the energy can be saved as mentioned above by sending data only when change occurs.

We propose an energy efficient scheme called DualCUSUM, which uses parametric CUSUM at the sensors (a sensor sends observations only when it senses change by CUSUM), physical layer fusion at the channel (to reduce MAC delay) and one more parametric CUSUM at the fusion center (to exploit all the observations obtained by the fusion center till that time). As compared to [16], we have two improvements. Not sending information from sensor all the time and using CUSUM at the fusion node, exploiting all past information. No one seems to have used CUSUM at the sensors as

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well as at the fusion center. We provide the detailed analysis, show that our algorithm is more efficient than that of [16] and obtain an optimization algorithm to fine tune our algorithm.

Section 2 explains the model and introduces the notation, Section 3 analyzes the performance. Section 4 provides the optimization algorithm and Section 5 concludes the paper.

2. OUR MODEL AND ALGORITHM

In a geographical area L sensors are deployed providing completely overlapping coverage. Let $X_{k,l}$ be the observation made at sensor l at time k and it transmits $Y_{k,l}$. The fusion center receives at time k,

$$Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_{MAC,k},$$

where, $Z_{MAC,k}$ is iid MAC noise (assuming synchronization). Observe that this already models the physical layer fusion at the MAC.

The distribution of the observations at each sensor changes at a random time T with a known distribution. Before the change $\{X_{k,l}, k \geq 1\}$ are iid with density f_0 and after the change with density is f_1. The objective of the fusion center is to detect this change as soon as possible at time τ (say) using the messages transmitted from all the L sensors, subject to an upper bound on the False Alarm (FA) probability $P(\tau < T)$ and the average energy used. Then, the general problem is:

$$\min E_{DD} = E[(\tau - T)^+], \tag{1}$$

$$\text{Subj to } P_{FA} = P(\tau < T) \leq \alpha \ \ \& \ \ E\left[\sum_{k=1}^{\tau} Y_{k,l}^2\right] \leq \mathcal{E}_0, 1 \leq l \leq L.$$

Since, sensor observation noise and the MAC noise are often Gaussian, we will pay particular attention to this case (although all our analysis is valid for general distribution). Then, we will assume that $X_{k,l} \sim N(\theta_k, \sigma^2)$, where $\theta_k = m_0$ before the change and $\theta_k = m_1$ after the change. Also, then $\{Z_{MAC,k}\}$, the MAC noise will be assumed iid Gaussian with mean 0 and variance σ_M^2.

The following algorithm does not provide an optimal solution to (1) but uses several desirable features to provide better performance than the ones we are aware of.

DualCUSUM

1. Sensor l uses parametric CUSUM (as defined in [9]),

$$W_{k,l} = \max\left(0, W_{k-1,l} + \xi_{k,l}\right), W_{0,l} = 0, \tag{2}$$

where, $\xi_{k,l} = \log\left[f_1\left(X_{k,l}\right)/f_0\left(X_{k,l}\right)\right]$.

2. Sensor l transmits in slot k only if $W_{k,l} > \gamma$. This is the energy saving step.

3. Physical layer fusion (as in [16]) reducing transmission delay:

$$Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_{MAC,k} = \sum_{l=1}^{L} bI_{\{W_{k,l} > \gamma\}} + Z_{MAC,k}, \tag{3}$$

where, $b > 0$ is a design parameter.

4. Change detection at fusion center via CUSUM:

$$F_k = \max\left\{0, F_{k-1} + \log\frac{g_I(Y_k)}{g_0(Y_k)}\right\}. \tag{4}$$

For $Z_{MAC,k} \sim N(0, \sigma_M^2)$, we set $g_0 \sim N(0, \sigma_M^2)$ and $g_I \sim N(Ib, \sigma_M^2)$, with I being a design parameter.

5. The fusion center declares a change at time $\tau(\beta, \gamma, b, I)$ when F_k crosses a threshold β:

$$\tau(\beta, \gamma, b, I) = \inf\{k : F_k > \beta\}.$$

Figure (1) compares optimal DualCUSUM (obtained via algorithm in Section 4) with the scheme in [16] and the optimal centralized Shiryaev scheme via simulation for the parameters: $L = 2, I = 1, f_0 \sim N(0,1), f_1 \sim N(0.75, 1)$, $Z_{MAC,k} \sim N(0,1)$, $T \sim \text{Geom}(\rho = 0.05)$ and $\mathcal{E}_0 = 7.61$. Clearly DualCUSUM performs much better than [16] and the performance tends to improve as P_{FA} decreases. This motivates us to study DualCUSUM further.

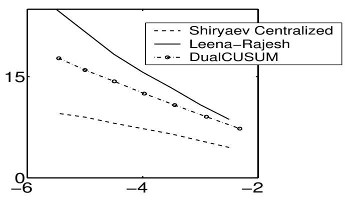

Fig. 1. $\ln(P_{FA})$ (x axis) vs E_{DD} comparison with [16]

In Section 3, we obtain theoretical analysis of DualCUSUM and get:

$$(\beta^*, \gamma^*, b^*, I^*) = \arg\min_{(\beta,\gamma,b,I)} E_{DD}(\beta, \gamma, b, I) \tag{5}$$

subj to $P(\tau(\beta, \gamma, b, I) < T) \leq \alpha$ and energy $\mathcal{E}_{avg}(\beta, \gamma, b, I) \leq \mathcal{E}_0$.

3. ANALYSIS

We first provide the false alarm analysis.

3.1. Analysis at the local sensor

At sensor l, $Y_{k,l} = 1$ only when $W_{k,l} > \gamma$. Since $\{W_{k,l}, k \geq 1\}$ is a reflected Random Walk, with negative drift before the change, the upcrossing of γ by $W_{k,l}$ can be given by a Poisson process with rate λ_γ where ([13])

$$\lambda_\gamma = \frac{\exp\left[-\sum_{n=1}^{\infty} 2n^{-1}\mathbf{Q}(S_{n,l})\right]}{\int_{-\infty}^{\infty}\left(\log\frac{f_1(u)}{f_0(u)}\right) f_1(u) du} e^{-\gamma}, \tag{6}$$

$\mathbf{Q}(S_{n,l}) = (\mathbf{P}_\infty(S_{n,l} > 0) + \mathbf{P}_1(S_{n,l} \leq 0))$, $\mathbf{P}_\infty, \mathbf{P}_1$ respectively representing the probability measures under no change and change at time index 1 and $S_{n,l} = \sum_{k=1}^{n} \xi_{k,l}$, For Gaussian f_0 and f_1, $S_{n,l}$ is Gaussian and hence λ_γ can be easily computed.

Distribution of the batch :

Next, we compute the sojourn time of $W_{k,l}$ above γ after each upcrossing of γ (called a batch size in the following).

Define, $\nu_{\gamma,l} := \inf\{k \geq 1 : W_{k,l} \geq \gamma\}$. Note that $\nu_{\gamma,l} \sim exp(\lambda_\gamma)$ (equation (6)). The reflected random walk $\{W_{k,l}\}_{k \geq \nu_{\gamma,l}}^{\tau_{0,l}}$ with $\tau_{0,l} := \inf\{k : k > \nu_{\gamma,l}; W_{k,l} \leq 0\} - \nu_{\gamma,l}$, is given by an ordinary random walk. Further, with large values of γ, $\tau_{0,l}$ is sufficiently large. Thus, using Donsker's theorem [1] we approximate (with large N):

$$\begin{aligned}\{W_{k+\nu_{\gamma,l},l}\}_{k \geq 0}^{\tau_{0,l}} &\sim \{W_{\nu_{\gamma,l},l} + S_{k,l}\}_{k \geq 0}^{\tau_{0,l}} \\ &\sim \left\{W_{\nu_{\gamma,l},l} + \sigma_S\sqrt{N}\zeta\left(\frac{k}{N}\right) + k\mu\right\}_{k \geq 0}^{\tau_{0,l}}\end{aligned}$$

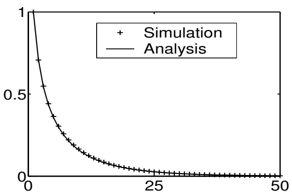

Fig. 2. Complementary CDF of the batches

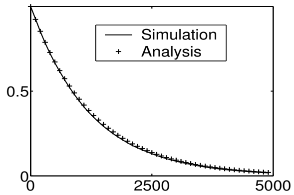

Fig. 3. Complementary CDF of time to reach γ.

where $\zeta(t), t \geq 0$ is a standard Brownian motion (BM), $\mu = E\xi_{1,1}$ and $\sigma_S = var(\xi_{1,1})$. Also, we approximate $W_{\nu_{\gamma,l},l}$ by its mean (which equals $\gamma+$ the mean of the overshoot for the corresponding random walk ([2])). Then, $\tau_{0,l}$ is approximated by the time taken by the above BM to reach 0 starting with $\gamma_{ov} = E\left(W_{\nu_{\gamma,l},l}\right)$. This is given by ([3]):

$$P\{\tau_{0,l} > i\} = \Phi\left(\frac{\gamma_{ov} - \mu i}{\sigma_S\sqrt{i}}\right) - e^{\frac{2\mu\gamma_{ov}}{\sigma_S^2}}\Phi\left(\frac{-\gamma_{ov} - \mu i}{\sigma_S\sqrt{i}}\right), \quad (7)$$

where Φ denotes the CDF of the standard Gaussian distribution.

We obtain the batch distribution using occupation measure, above γ, of the BM till time $\tau_{0,l}$ ([12]). Choose time t_B such that for some small enough $\epsilon > 0$, $P(\tau_{0,l} \leq t_B) > 1 - \epsilon$ and $P(\nu_{\gamma,l} \geq t_B) > 1 - \epsilon$. This is possible if, $P[\tau_{0,l} << \nu_{\gamma,l}]$ is close to 1, which is true for small P_{FA} (and hence large γ).

Define $\delta = E\left(W_{\nu_{\gamma,l},l}\right)/(\sigma_S\sqrt{t_B})$, $m = \mu\sqrt{t_B}/\sigma_S$ and

$$\eta = \#\{k : W_{k,l} \geq \gamma; \nu_{\gamma,l} \leq k \leq \nu_{\gamma,l} + \tau_{0,l}\}. \quad (8)$$

The batch size distribution is approximated using [12]

$$\begin{aligned} P(\{\eta > j\}) \quad &= \quad 2\int_0^j \left[\frac{\varphi(m\sqrt{1-u})}{\sqrt{1-u}} + m\Phi(m\sqrt{1-u})\right] \\ &\left[\varphi\left(\frac{\delta - mu}{\sqrt{u}}\right)\frac{1}{\sqrt{u}} - me^{2m\delta}\Phi\left(\frac{-\delta - m}{\sqrt{u}}\right)\right]du, \quad (9) \end{aligned}$$

where, φ represents the standard Gaussian pdf.

We plot these approximations in Figs 2 and 3 which show a good match. This approximation is used in the next section to obtain P_{FA}.

3.2. False Alarm Analysis

In the DualCUSUM algorithm, transmissions happen only within the batches. Hence, the probability of a FA within a batch is a crucial element in the computation of P_{FA}. However, FA (at the fusion center) can also occur because of the Gaussian $\{Z_{MAC,k}\}$ alone. Our approach is to compute the two FAs separately and then combine them in an appropriate way to obtain the P_{FA}.

3.2.1. Global False Alarm

We assume that $P(\eta << T) \approx 1$ (valid for small P_{FA}). By independence of $\{W_{k,l}\}$ and $\left\{W_{k,l'}\right\}, l \neq l'$, a batch occurs (at the fusion center) with exponential rate $L\lambda_\gamma$. Let ψ represent the number of batches before change. Then, $P(\psi = j|T = i) = \frac{(L\lambda_\gamma i)^j e^{L\lambda_\gamma i}}{j!}$ by independence of η and T. Hence,

$$P_{FA} = \sum_{i=1,j=0}^{\infty} P(FA|T = i; \psi = j)P(\psi = j|T = i)P(T = i). \quad (10)$$

Let $\tilde{p}$ denote the FA probability inside a batch. In Section 3.2.3 given below, we will show that the time to FA outside a batch is exponentially distributed with parameter λ_0. Hence, by the assumptions made in this section and by independence of observations,

$$\begin{aligned} P(FA|T = i; \psi = j) \quad &\approx \quad 1 - P(\{\text{No FA in } j \text{ batches }\} \quad (11) \\ &\cap\{\text{No FA in } i \text{ observations}\}|T = i; \psi = j) \\ &\approx \quad 1 - (1 - \tilde{p})^j e^{-\lambda_0 i}. \end{aligned}$$

In the above, i (minus negligible amount due to batches) observations, with no transmission from any sensor, occur in $j + 1$ batches. But by exponentiality of the distribution, P_{FA} outside the batches is approximately $e^{-\lambda_0 i}$.

If $T \sim \text{Geom}(\rho)$, from (10) and (11) we get:

$$P_{FA} = 1 - \frac{e^{-(\lambda_0 + \lambda_\gamma L\tilde{p})}\rho}{1 - e^{-(\lambda_0 + \lambda_\gamma L\tilde{p})}(1 - \rho)}. \quad (12)$$

3.2.2. False Alarm within a Batch

The false alarm probability within a batch, $\tilde{p}$ can be computed as,

$$\tilde{p} \approx \sum_{i=1}^{\infty} P(\eta = i)P(\text{FA } |\eta = i),$$

where η is the batch size from (8) and $P(\text{FA } |\eta = i)$ represents the probability of FA (CUSUM at the fusion center crossing β) in i transmissions when, $Y_k = b + \sum_{l=1}^{L-1} Y_{k,l} + Z_{MAC,k}$. As the batch sizes are small, one can directly calculate it by,

$$P(\text{FA } |\eta = i) = 1 - E\left[\Pi_{k=1}^{i} I_{\{F_k \leq \beta\}}\right]. \quad (13)$$

This integral can be computed using Monte Carlo (MC) methods.

3.2.3. False Alarm outside a Batch

In the absence of any transmission from the sensors, $Y_k \sim N(0, \sigma_M^2)$. Hence, F_k is in negative drift. Thus the time to reach β, i.e. time till FA approximately is exponentially distributed with parameter λ_0 which can be obtained from [13] as we have done in (6).

3.2.4. Comparison of Analysis and Simulation for P_{FA}

In this section we tabulate the results obtained for the P_{FA} by analysis and by simulation. We take f_0, f_1, $Z_{MAC,k}$ Gaussian and T Geometric with $b = 1$, $m_0 = 0$, $m_1 = 1$, $\sigma = \sigma_M = 1$ and provide P_{FA} values for different values of ρ, γ. β, L and I in Table 1.

L	I	γ	β	ρ	P_{FA} Simulation	P_{FA} Analysis
3	1	9	17	0.005	3.66e-5	3.07e-5
3	3	8	15	0.005	3.64e-5	3.95e-5
4	1	10	18	0.0005	1.58e-4	1.14e-4
4	2	9	14	0.005	1.12e-4	1.23e-4
4	4	9	14	0.005	2.21e-5	2.38e-5
4	4	10	17	0.005	1.40e-6	1.29e-6

Table 1. Comparison of P_{FA} obtained via analysis and Simulation: number of sample paths used 10,000,000

3.3. Delay Analysis

The mean detection delay can be written as

$$E_{DD} = E_T\left[\sum_{k=T}^{\infty}(k - T)P(\{F_k > \beta\} \cap_{n=1}^{k-1} \{F_n < \beta\})\right], \quad (14)$$

where, E_T is the expectation w.r.t the change time T. Because of the mostly positive drift of F_k after the occurrence of change, the time to detection is small. Hence, the above integral can be computed using Monte Carlo methods by setting F_{T-1} and $W_{T-1,l}$ to the corresponding stationary means.

This way of computing E_{DD} takes negligible computing time as compared to system simulations which run for a long time. We can then use our FA analysis and this computation to obtain optimal parameters for DualCUSUM in Section 4.

We are presently working towards getting good approximations or tighter upper bounds for the mean detection delay.

4. OPTIMIZATION

In the following although our optimization algorithm works for general distributions, we limit ourselves to Gaussian distribution and Geometric T. For our choice of parameters we have observed that $I = 2$ gives the best result. Hence forth, we fix $I = 2$ and perform optimization only w.r.t. (β, γ, b). Having obtained the analytical expressions (12), (14) for P_{FA} and E_{DD} respectively and with average energy given by,

$$\mathcal{E}_{avg} = b^2 \left[E\left(\tau - \inf_{n \geq T} \{W_{n,1} > \gamma\} \right) + \frac{\lambda_\gamma L E(\eta)}{\rho} \right], \qquad (15)$$

one can use appropriate optimization technique to solve (5). The term $E\left(\tau - \inf_{n \geq T}\{W_{n,1} > \gamma\}\right)$ is estimated in a way similar to that of E_{DD}. To begin with, for each value of β, we obtain (γ, b) as a fixed point of the two dimensional function (constant C is calculated from (6)),

$$h^\beta(\gamma, b) := \left[\; log\left(\frac{CL\tilde{p}}{log\left(1+\rho\alpha(1-\alpha)^{-1}\right) - \lambda_0} \right), \quad \sqrt{\mathcal{E}_0 b^2 \mathcal{E}_{avg}^{-1}} \; \right]^T,$$

constructed using the P_{FA}, $\mathcal{E}_{avg}$ constraints (which will be satisfied with equality). We now have a single parameter β and almost unconstrained (of course we will still have positivity constraints) optimization problem. We initially use grid method (getting optimal point by exhaustive search over discretized space) to obtain a coarse optimal point, which is improved upon using a steepest decent algorithm.

We used the above algorithm to obtain optimal parameters in Table 2 for two different values of L. We set $L\mathcal{E}_0 = 20$. The FA constraint used to obtain the optimal parameters (β^*, γ^*, b^*) is given in the first column. Table 2 also provides P_{FA}, $\mathcal{E}_{avg}$ of DualCUSUM with parameters (β^*, γ^*, b^*). We see that the theory is matching well with the simulations for low values of P_{FA} which are of practical concern. This shows the accuracy of approximations in (10) and (14) and of the optimization algorithm.

			Analysis	Simulation
α	L	(β^*, γ^*, b^*)	E_{DD}^*	$(P_{FA},\ E_{DD}^*,\ \mathcal{E}_{avg})$
5e-5	2	(15.17, 7.88, .69)	47.59	(4.7e-5, 47.06, 10.05)
1e-5	2	(16.54, 8.13, .60)	55.60	(1.1e-5, 55.04, 10.01)
5e-5	4	(21.00, 6.76, .71)	30.0	(4.0e-5, 29.80, 05.02)
1e-5	4	(22.00, 7.32, .65)	34.3	(9.1e-5, 34.02, 05.01)

Table 2. Performance of Optimal DualCUSUM : Comparison of Simulation with Analysis for $m_0 = 0$, $m_1 = 0.75$, $\sigma_S = 1$, $\sigma_M = 1$, $\rho = 0.005$, $I = 2$ and $L\mathcal{E}_0 = 20$, .

5. CONCLUSIONS AND FUTURE WORK

We have proposed a Page's CUSUM based energy efficient scheme which uses the physical layer fusion technique and CUSUM at the sensors as well as at the fusion center. We have analyzed the FA performance of the scheme and computed the approximate mean delay using Monte Carlo techniques. The analytical results obtained give us a good approximation which can be used to choose the optimal parameters. We have also compared our scheme with the scheme proposed in [16] for a fixed energy constraint. The comparisons show that our scheme saves a lot more energy for small values of FA and uses it to improve on the detection delay.

At present we are working towards good approximations for E_{DD}, and some variants of DualCUSUM, and some analytical way of solving for the optimal choice of parameters.

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