Inverse EEG source problems and approximation

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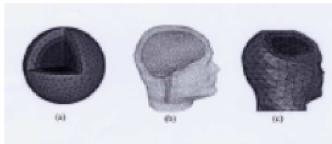
Inverse EEG (electroencephalography) Problem:

From measurements by electrodes of the electric potential u on the scalp, recover a distribution of m pointwise dipolar current sources C_k with moments p_k located in the brain (modeling the presence of epileptic foci).

Model: The head Ω is modeled as a set of 3 spherical or ellipsoidal nested regions $\Omega_i \subset \mathbb{R}^3$, $i = 0, 1, 2$ (brain, skull, scalp), separated by interfaces S_i (with $S_2 = \partial\Omega$) and with piecewise constant conductivity σ, $\sigma_{|\Omega_i} = \sigma_i > 0$.



Macroscopic model + quasi-static approximation of Maxwell equations → Spatial behavior of u in Ω [1] : (P)

$$\begin{cases} \operatorname{div}(\sigma \nabla u) = \sum_{k=1}^{m} p_k . \nabla \delta_{C_k} & \text{in } \Omega \\ u = g \text{ and } \partial_n u = \phi & \text{on } \partial\Omega \end{cases}$$

where g and ϕ denote the given potential and current flux on the scalp (or approximate interpolation of these quantities).

The resolution of this inverse problem can be divided into 3 main steps:

1. Data propagation (Cortical mapping step):

[u at electrodes, $\partial_n u = 0$ on S_2] —Cauchy problem→ [u, $\partial_n u$ on S_1] —Cauchy problem→ [u, $\partial_n u$ on S_0]

Since $C_k \in \Omega_0$, the function u is harmonic in the outer layers Ω_1 and Ω_2, where boundary conditions are given by the continuity relations

$$[u]_i = [\sigma\, \partial_n u]_i = 0 \text{ on } S_i \,,$$

$[\ ]_i$ denotes the jump across the surface S_i.

Based on these boundary conditions, data propagation can be achieved by using boundary element methods [2] , or by using robust harmonic approximation techniques and expansions on appropriate basis [3].

2. Anti-harmonic projection (Signal space separation):

From these data on S_0, the solution u to equation (P) in Ω_0:

$$\begin{cases} \triangle u = \frac{1}{\sigma_0} \sum_{k=1}^{m} p_k . \nabla \delta_{C_k} & \text{in } \Omega_0 \\ u \text{ and } \partial_n u & \text{given on } S_0 \end{cases}$$

assumes the form: $u(x) = h(x) + \sum_{k=1}^{m} \frac{< p_k,\ x - C_k >}{4\pi \|x - C_k\|^3} = h(x) + u_a(x)$, h harmonic function in Ω_0.

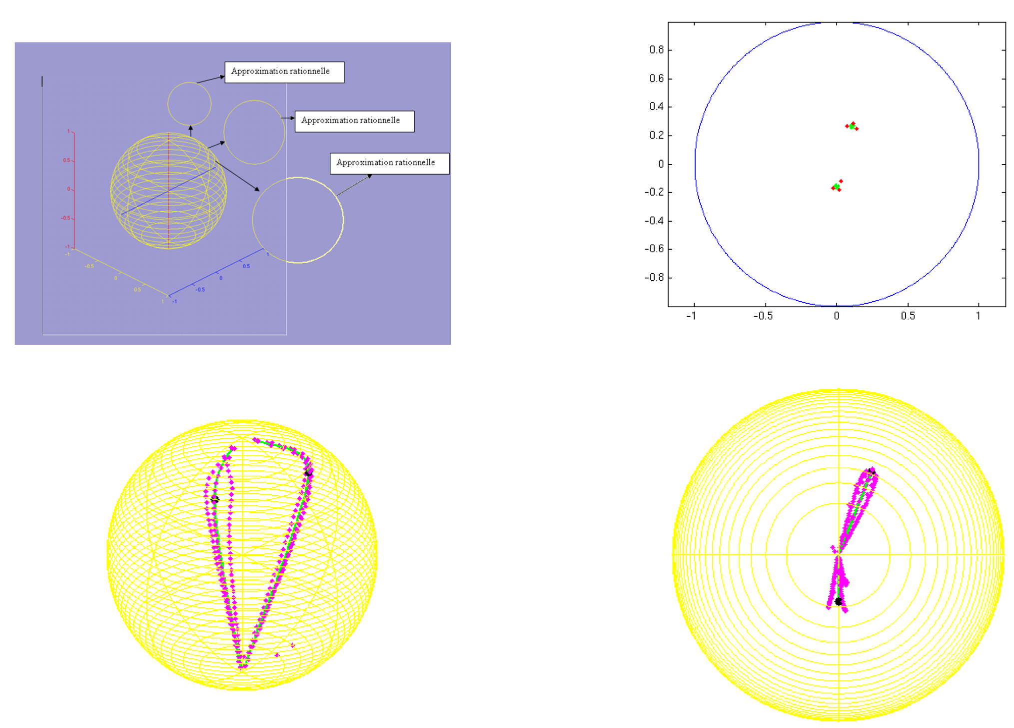
In order to recover the C_k inside Ω_0, the knowledge of the singular function u_a is required on S_0. This can be deduced from available boundary data by expanding u on bases of spherical or ellipsoidal harmonics [3, 4].

3. Best rational approximation on planar sections (Source localization):

[(pointwise values of) u_a on S_0] —$2D$ best approximation schemes on planar sections of the boundary→ [localisation of sources C_k]

- Slice Ω_0 along a family of planes Π_p: $\Pi_p \cap S_0 = \Gamma_p$ (circles or ellipses).
- From pointwise values of the singular part u_a on Γ_p, approximate $f_p = (P_{-}u_a)^2$ the square of the anti-analytic projection on Γ_p [3, 5].
Ellipses may then preliminary be mapped by a conformal rational transformation onto circles.
- f_p is a meromorphic function, analytic outside $D_p = \Pi_p \cap \Omega_0$ with singularities $\zeta_{k,\,p}$ inside D_p which are strongly and explicitely linked with the sources C_k.
- Approximate the $\zeta_{k,\,p}$ by $\widetilde{\zeta}_{k,\,p}$ the poles of the best L^2 or L^∞ rational approximation to f_p on Γ_p (degree $\geq m$ for a sphere, $2m$ for an ellipsoid), the poles $\widetilde{\zeta}_{k,\,p}$ accumulate to the singularities $\zeta_{k,p}$, [5].
- Varying p, this allows us to approximately locate the m sources C_k in Ω_0.

Illustrations (Spherical model):



2 sources   above view

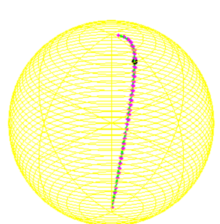
The error estimate is around 10^{-2}.

Numerical results:

Spherical model:

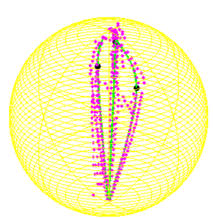
In this case the relation between $\zeta_{k,p}$ and C_k when the slice p varies is the following:

-$(\zeta_{k,\,p})$ are aligned together and also with the complex coordinates ζ_k of C_k.
-$|\zeta_{k,\,p}|$ is maximum at $\zeta_{k,\,p} = \zeta_k$ (the k^{th} source's section).

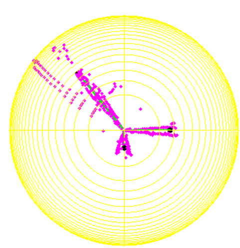


1 source

above view



3 sources



above view

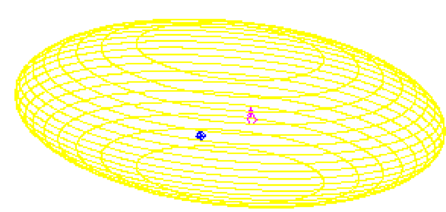
The error estimate is around 10^{-4} for the 1 source case and around 10^{-2} for the 3 sources case.

Ellipsoidal model:

In each slice p we construct a real valued polynomial $q_{k,\,p}(\zeta)$ whose coefficients are related to the conformal rational transformation from ellipse to circle, $q_{k,\,p}(\zeta)$ is minimum for $\zeta = \zeta_k$, [3].

These numerical results represent exact sources ⋇ and approximated ones ◊.

1 source



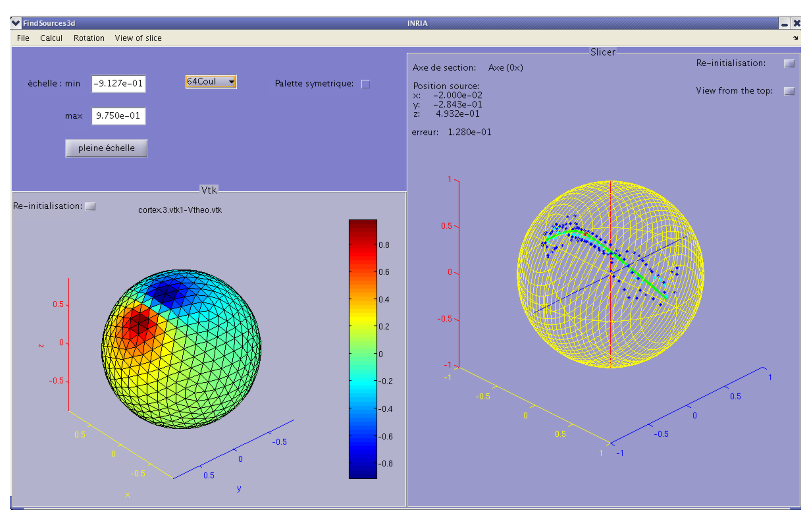
2 sources

The error estimate is around 10^{-4} for the 1 source case and around 10^{-1} for the 2 sources case .

Perspectives:

- Recover the moments p_k, improve sources estimation (slicing along different directions).
- Software development (FindSources3D).
- More realistic geometries.
- The inverse MEG source problem.

FindSources3D :



(view of u on S_2 then of C_1 into Ω_0)

References:

[1]: M. Hämäläinen, R. Hari, J. Ilmoniemi, J. Knuutila, O. V. Lounasmaa: *Magnetoencephalography theory instrumentation, and applications to noninvasive studies of the working humain brain*, Rev. Modern Physi., 65, pp. 413-497(1993).

[2]: J. Kybic, M. Clerc, T. Abboud, O. Faugeras, R. Keriven, T. Papadopoulo: *A common formalism for the integral formulations of the forward EEG problem*, IEEE Trans. Medical Imaging, vol 24, pp. 12-28 (2005).

[3]: J. Leblond, C. Paduret, S. Rigat, M. Zghal, *Sources localisation in ellipsoids by best meromorphic approximation in planar sections*, Inverse Problems 24, 035017 (2008) .

[4]: S. Taulu, J. Simola, M. Kajola, *Applications of the Signal Space Separation Method*, IEEE Trans. Signal Proces., 53, pp. 3359- 3372 (2005).

[5]: L. Baratchart, J. Leblond, J-P. Marmorat, *Inverse source problem in a 3D ball from best meromorphic approximation on 2D slices*, Elec. Trans. Numerical Analysis (ETNA), 25, pp. 41-53 (2006).