

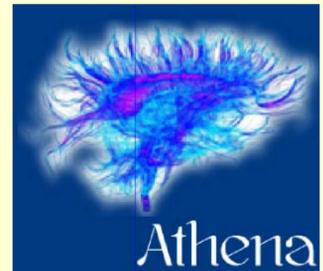


Fast, model-free, analytical diffusion PDF profile estimation from the DWI signals

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Introduction

How to estimate the diffusion Probability Density Function (PDF) from the q-space DWI signals is an open problem in diffusion MRI field. The diffusion PDF and the diffusion attenuation of dMRI signal is related by 3 dimensional Fourier transform. Diffusion Orientation Transform (DOT) [2] was proposed based on the assumption of the mono-exponential decay. Thanks to this assumption, PDF profile at a given radius R_0 could be calculated analytically. However the estimated PDF is a low-pass filtered PDF. And this kind of low-pass filtering comes from the intrinsic modeling error [2], which is not a well defined filtering. Although DOT has been extended to multi-exponential model [2] and can reduce the modeling error dramatically, a nonlinear fitting needs to be performed for every voxel, which will limit its usefulness. Here we propose a model-free, linear, robust, analytical PDF profile reconstruction method which avoids the exponential or multi-exponential assumption.

Methods

• Spherical Polar Fourier Expression (SPFE) [2]

$$E(\mathbf{q}\mathbf{u}) = \sum_{n=0}^N \sum_{l=0}^L \sum_{m=-l}^l a_{n,l,m} R_n(q) Y_l^m(\mathbf{u})$$

$$R_n(q) = \kappa(\zeta) \exp\left(-\frac{q^2}{2\zeta}\right) L_n^{0.5}\left(\frac{q^2}{\zeta}\right) \quad \kappa(\zeta) = \left(\frac{2}{\zeta^{3/2}} \frac{n!}{\Gamma(n+1.5)}\right)^{0.5}$$

• Numerical PDF profile estimation [2]:

$$P(\mathbf{r}R_0) = \sum_{n,l,m} a_{n,l,m} h_{n,l,m}(\mathbf{r}) \quad H(\mathbf{q}) \xrightarrow{iFFT} h(\mathbf{r}) \xrightarrow{SPFE} \{h_{n,l,m}(\mathbf{r})\}$$

• Analytical PDF profile estimation :

$$P(\mathbf{R}) = \int_{R^3} E(\mathbf{q}) \exp(-2\pi i \mathbf{q}^T \mathbf{R}) d\mathbf{q}$$

$$P(\mathbf{r}R_0) = \sum_{l,m} c_{l,m} Y_l^m(\mathbf{r})$$

$$c_{l,m} = 4(-1)^{l/2} \frac{\zeta^{0.5l+1.5} \pi^{l+1.5} R_0^l}{\Gamma(l+1.5)} \sum_{n=0}^N f_{n,l,m}(\zeta, R_0) a_{n,l,m}$$

$$f_{n,l,m}(\zeta, R_0) = \kappa_n(\zeta) \sum_{i=0}^n (-1)^i \binom{n+0.5}{n-i} \frac{2^{0.5l+i-0.5}}{i!} \Gamma(0.5l+i+1.5) {}_1F_1\left(\frac{2i+l+3}{2}; l+1.5; -2\pi^2 R_0^2 \zeta\right)$$

where ${}_1F_1$ is the confluent hypergeometric function of the first kind.

$$E(\mathbf{q}) \rightarrow \{a_{n,l,m}\} \rightarrow \{c_{l,m}\}$$

Analytical linear transformation

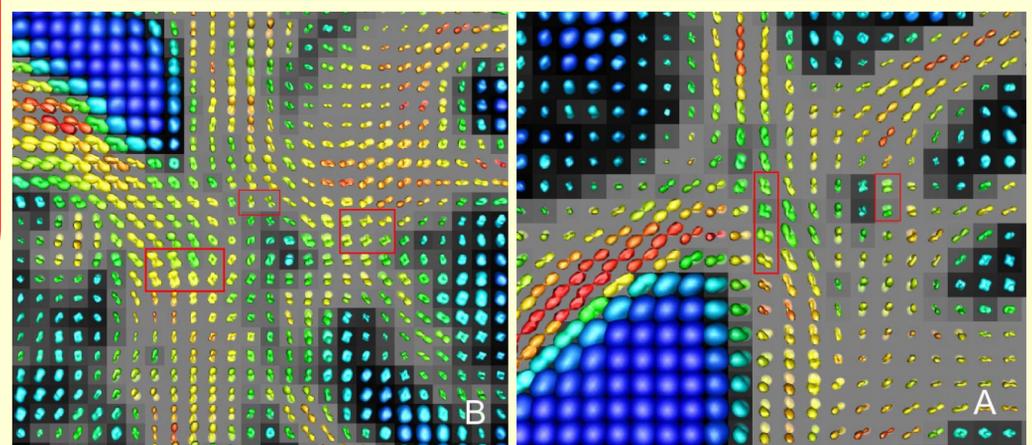
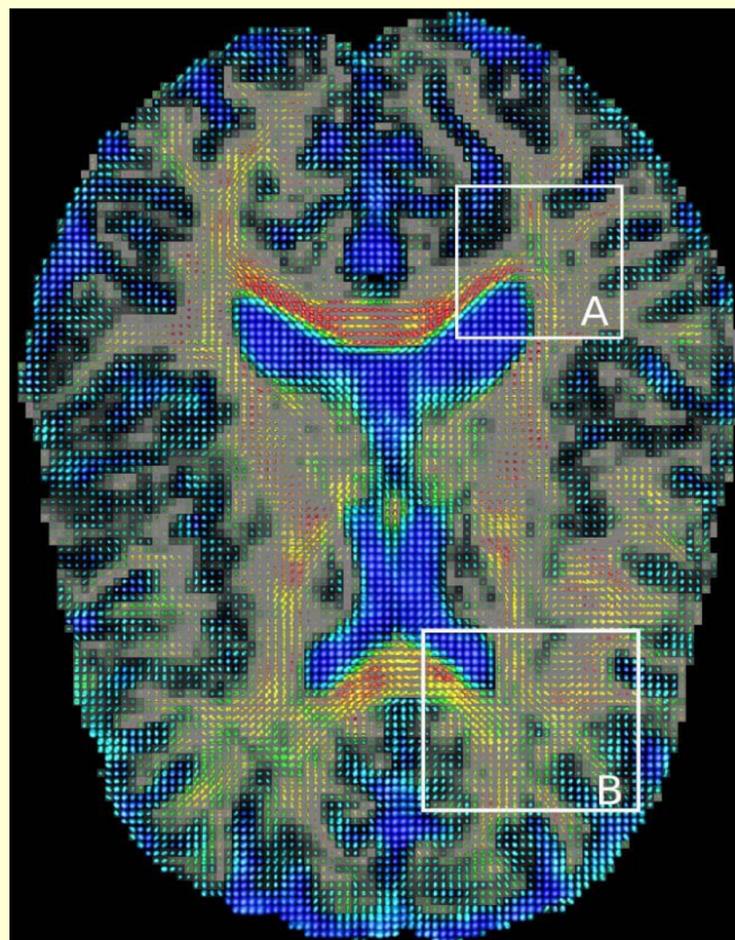
Conclusion

We proposed a fast analytical PDF profile reconstruction method based on spherical polar Fourier expression of the signal in Q-space. It can avoid the modeling error from mono-exponential model or multi-exponential model. The coefficients of the PDF profile under spherical harmonics expression could be linearly and analytically calculated from the coefficients of the signal. It is a linear transformation that is independent with data. This transformation matrix is just needed to be calculated only once for a whole data set, which makes the method very fast. The experiments validate our method which could provide both the radial information and the direction information and could detect the fiber crossing even for the data with small b-values.

Results

• Real human data (multiple b values)

- Total 110 DWIs: 50 for $b=1126 \text{ s/mm}^2$, 30 for $b=820 \text{ s/mm}^2$, 10 for b-value of 100, 307, 512 s/mm^2 .
- The sampling scheme is not appropriate for a good estimation of the coefficients of the signal, since the b-values here are too small. Even though this is not a good sampling, the result is still impressive.
- Visualize the PDF profile in $R_0 = 17\text{ms}$ for one slice. Two areas were enlarged for good visualization. The glyphs were colored by GFA calculated from the estimated PDF profiles. Some crossings were found.
- **No normalization** here, e.g. min-max normalization, because of two reasons.
 - The PDF profiles in white matter seem sharper enough and the profiles in CSF and gray matter are almost isotropic.
 - We will lose the radial information if we do some normalization.



Reference

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