Fast, model-free, analytical diffusion PDF profile estimation from the DWI signals

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Introduction:

How to estimate the diffusion Probability Density Function (PDF) from the q-space DWI signals is an open problem in diffusion MRI field. The diffusion PDF and the diffusion attenuation of dMRI signal is related by 3 dimensional Fourier transform. Diffusion Orientation Transform (DOT) [2] was proposed based on the assumption of the mono-exponential decay. Thanks to this assumption, PDF profile at a given radius R0 could be calculated analytically. However the estimated PDF is a low-pass filtered PDF. And this kind of low-pass filtering comes from the intrinsic modeling error [2], which is not a well defined filtering. Although DOT has been extended to multi-exponential model [2] and can reduce the modeling error dramatically, a nonlinear fitting needs to be performed for every voxel, which will limit its usefulness. Here we propose a model-free, linear, robust, analytical PDF profile reconstruction method which avoids the exponential or multi-exponential assumption.

Methods:

The diffusion Probability Density Function and the diffusional attenuation of dMRI signal is related by 3D Fourier transform. See formula (1). where $E(q) = S(q)/S_0$ is the normalized DWI signal measured on q, and S_0 is the baseline image acquired without diffusion gradients. Spherical Polar Fourier expression is a kind of orthonormal basis representation and it has been shown in [1] that it can represent the diffusion weighted image signal sparsely. See formula (2). $R_n(||q||)$ is the Gaussian-Laguerre polynomial in formula (3), where ζ is the scale parameter and Y_l^m is the l order m degree spherical harmonics (SH). After we estimate the coefficients of the signal under the basis from a least square fit or nonlinear robust estimation [1], we proved that there is a linear, analytical solution to get the coefficients of the PDF profile at a given radius R0 under spherical harmonics representation. To get this result, firstly consider the plane wave equation in spherical coordinates in formula (4). Then put formula (2) and (4) into (1), we have the expression of the PDF profile in formula (5), where 1F1 is the confluent hypergeometric function. Please note that $f_{n,l}$ is independent with data and the summation over n actually is just a linear transformation which could be implemented as an matrix multiplication. Since the coefficients $a_{n,l,m}$ also could be estimated through a linear matrix multiplication [1]. Thus calculating the coefficients for PDF profile from the DWI signal here is very fast. The main computation complex is in the estimation of $a_{n,l,m}$. But it is still very fast if least square fitting is used. Our suggestion here is to store $a_{n,l,m}$ once it is estimated. Then estimating the coefficients for PDF profile at several given R0 could be performed on the stored $a_{n,l,m}$. That means we just need to calculate $a_{n,l,m}$ once and could re-calculate different PDF profile in different radii very fast.

Results:

We apply our method to a real human data. This data has 110 DWIs, 50 diffusion encoding gradients with a b-value of 1126 s/mm^2, 30 gradients with b-value of 820 s/mm^2, 10 gradients respectively with b-value of 100, 307, 512 s/mm^2, twice-refocused spin-echo EPI sequence, TE = 100 ms, 1.5x1.5x1.5 mm voxel resolution, three repetitions, corrected for subject motion. It should be noted that this sampling scheme is not appropriate for a good estimation of the coefficients of the signal, since the b-values here are too small. For a good representation of the signal, we should keep the b-values range from small values to large values. A discussion about evaluating the reconstruction error in dierent sampling schemes could be found in [1]. However, even though this is not a good sampling, the result is still impressive. We visualize the PDF profile in R0 = 17ms for one slice. Two areas were enlarged for good visualization. The glyphs were colored by GFA calculated from the estimated PDF profiles. Some crossings were found. Please note that we did not do any normalization here, e.g. min-max normalization [3]. That is because of two reasons. 1) the PDF profiles in white matter seem sharper enough and the profiles in CSF and gray matter are almost isotropic. 2) we will lost the radial information if we do some normalization. Please note that the radial information in PDF is also important compared with its peaks. It tell us how much the probability P(R0) is. From the result, we can see that the CSF has the largest probability (glyph size) compared with white matter and gray matter, just like the visualization of tensors in DTI. Tensors can not tell us crossing fibers, while PDF profiles can.

Conclusions:

We proposed a fast analytical PDF profile reconstruction method based on spherical polar Fourier expression of the signal in Q-space. It can avoid the modeling error from mono-exponential model or multi-exponential model. The coefficients of the PDF profile under spherical harmonics expression could be linearly and analytically calculated from the coefficients of the signal. It is a linear transformation that is independent with data. This transformation matrix is just needed to calculated only once for a whole data set, which makes the method very fast. The experiments validate our method which could provide both the radial information and the direction information and could detect the fiber crossing even for the data with small b-values.

$$P(\mathbf{R}) = \int E(\mathbf{q}) e^{-2\pi i \mathbf{q}\cdot\mathbf{R}} d\mathbf{q} \quad q = |\mathbf{q}|,\ R_0 = |\mathbf{R}|,\ \mathbf{q} = q\mathbf{u},\ \mathbf{R} = R_0\mathbf{r} \tag{1}$$

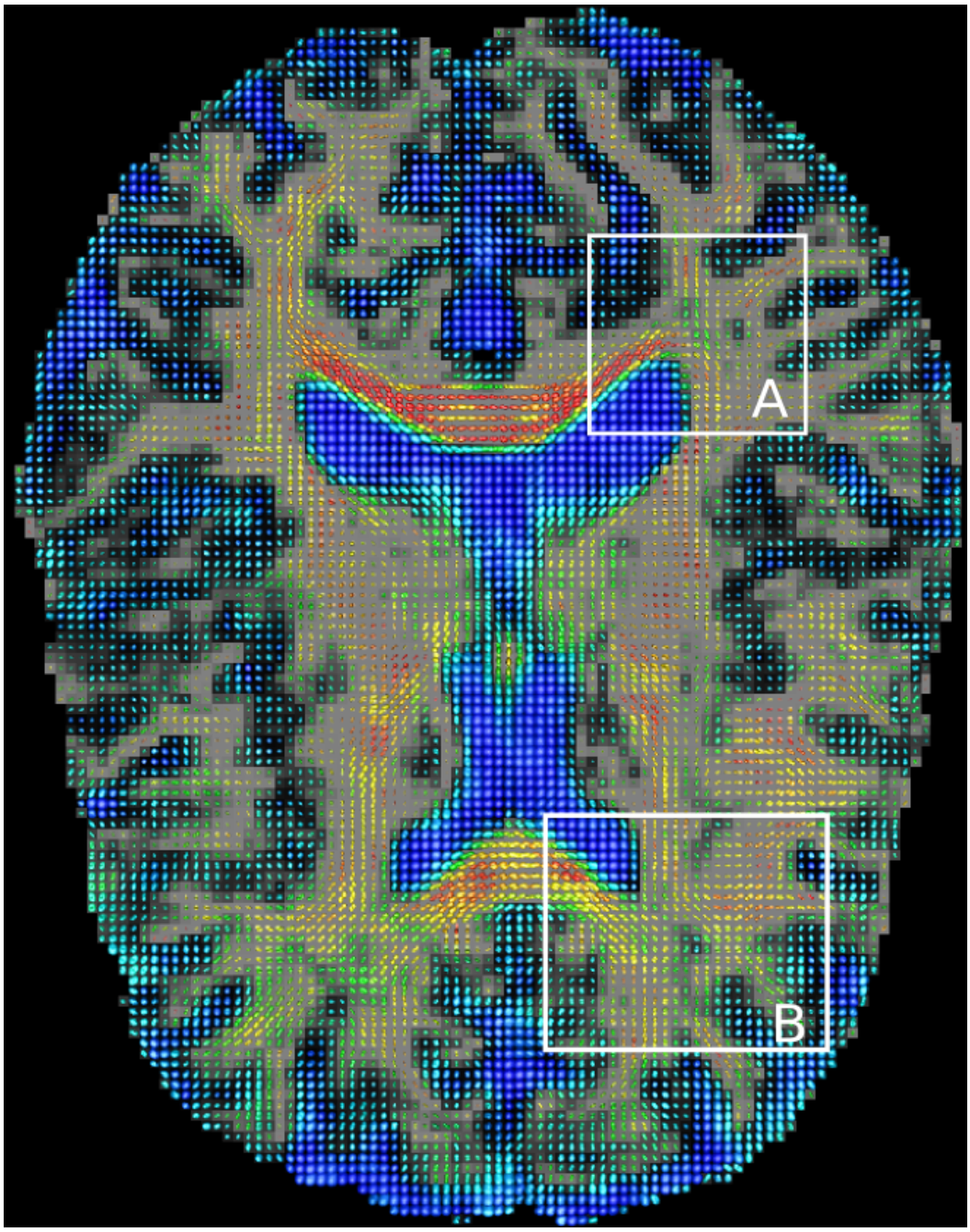
$$E(\mathbf{q}) = \sum_{n=0}^{N} \sum_{l=0}^{L} \sum_{m=-l}^{l} a_{n,l,m} R_n(\|\mathbf{q}\|) Y_l^m(\mathbf{u}) \tag{2}$$

$$R_n(\|\mathbf{q}\|) = \left[\frac{2}{\zeta^{3/2}} \frac{n!}{\Gamma(n+3/2)}\right]^{1/2} \exp(-\frac{\|\mathbf{q}\|^2}{2\zeta}) L_n^{1/2}(\frac{\|\mathbf{q}\|^2}{\zeta}) \tag{3}$$

$$e^{\pm 2\pi i \mathbf{q}\cdot\mathbf{R}} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (\pm i)^l j_l(2\pi q R_0) Y_l^m(\mathbf{u}) Y_l^m(\mathbf{r}) \tag{4}$$

$$\begin{aligned} P(\mathbf{R}) &= 4\pi \sum_{l=0}^{L} \sum_{m=-l}^{l} (-1)^{l/2} \sum_{n=0}^{N} \left\{ a_{n,l,m} \int_0^{\infty} j_l(2\pi q R_0) R_n(\|\mathbf{q}\|) q^2 dq \right\} Y_l^m(\mathbf{r}) \\ &= \sum_{l=0}^{L} \sum_{m=-l}^{l} \left\{ 4\pi (-1)^{l/2} \zeta^{l/2+3/2} R_0^l \sum_{n=0}^{N} a_{n,l,m} f_{n,l} \right\} Y_l^m(\mathbf{r}) \end{aligned} \tag{5}$$

$$f_{n,l} = \left[\frac{2}{\zeta^{3/2}} \frac{n!}{\Gamma(n+3/2)}\right]^{1/2} \sum_{i=0}^{n} \frac{(-1)^i \binom{n+0.5}{n-i} 2^{l/2+i-1/2} \pi^{l+1/2} \Gamma(l/2+i+3/2)}{i! \Gamma(l+3/2)} 1F1(i+l/2+3/2, l+3/2, -2\pi^2 R_0^2 \zeta)$$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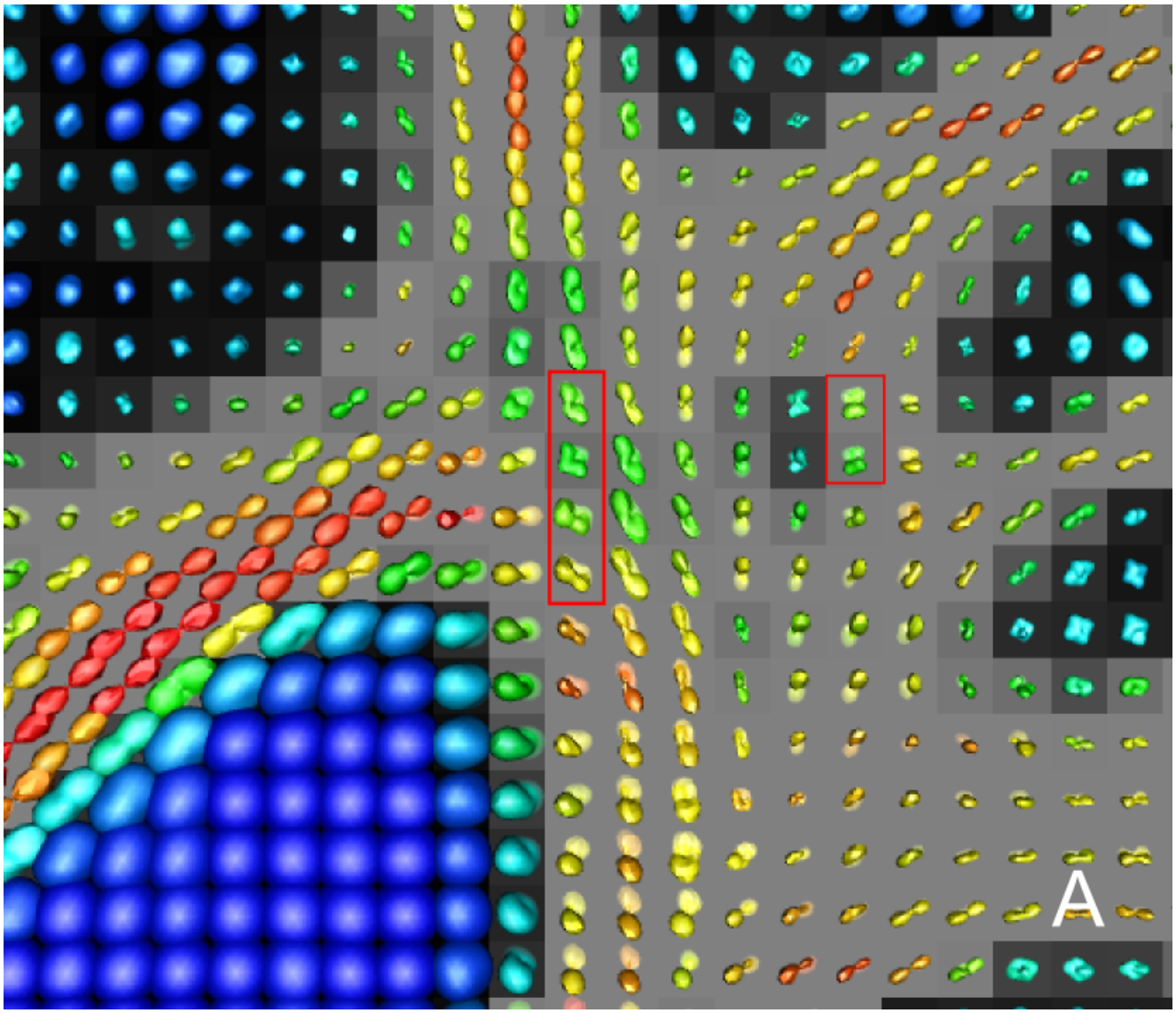
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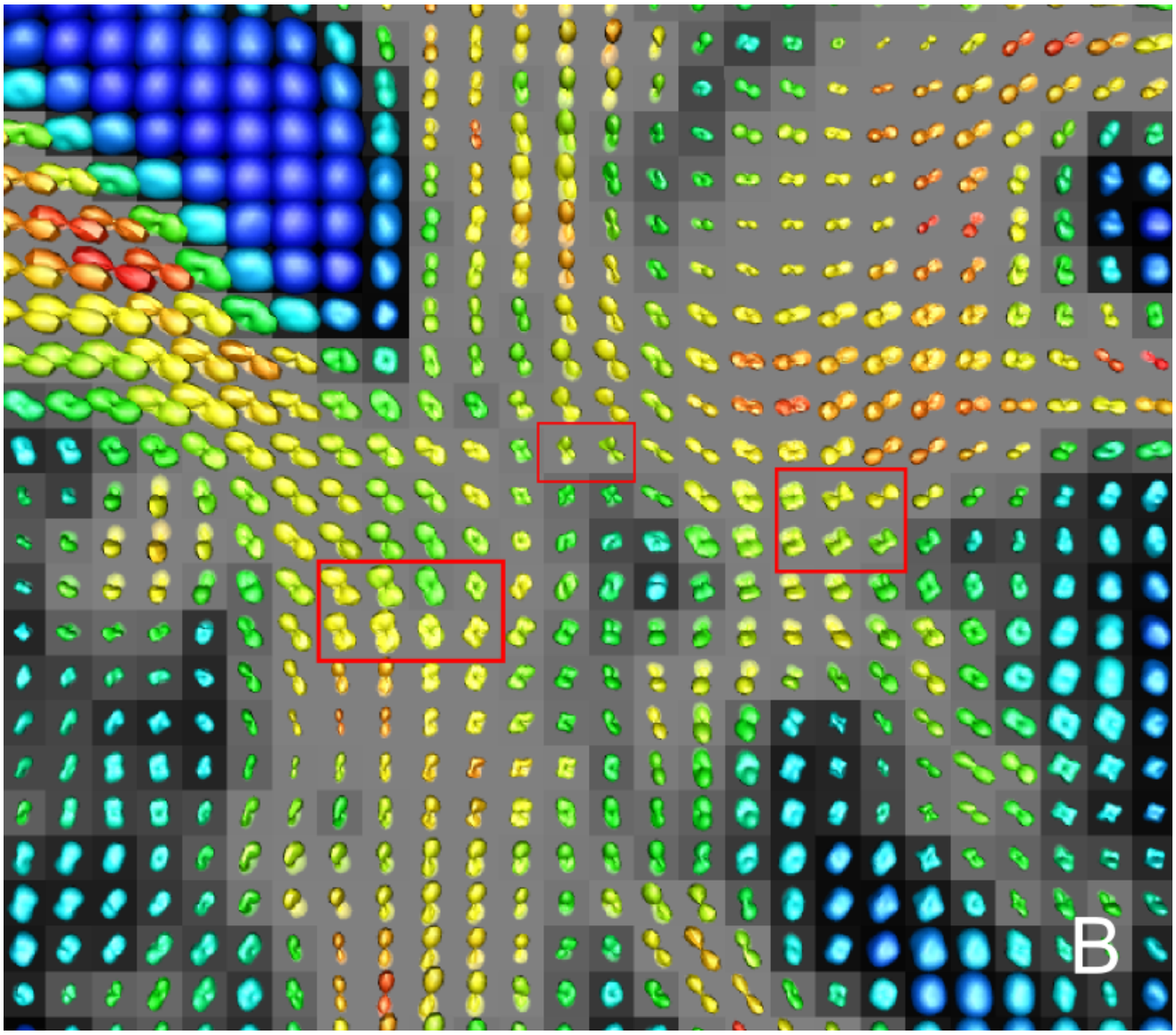

References:

Assemla, H. (2009), 'Efficient and robust computation of pdf features from diusion mr signal', *Medical Image Analysis*, vol. 13, pp. 715-729.

Ozarslan, E. (2006), 'Resolution of complex tissue microarchitecture using the diusion orientation transform (dot)', *NeuroImage*, vol. 31, pp. 1086-1103.

Tuch, D. (2004), 'Q-ball Imaging', *Magnetic Resonance in Medicine*, vol. 52, pp. 1358-1372.

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