# Fast, Model-Free, Analytical ODF Reconstruction from the Q-Space Signal

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**Authors:**

Jian Cheng[1], Aurobrata Ghosh[2], Tianzi Jiang[1], Rachid Deriche[2]

**Institutions:**

[1]Institute Of Automation, Chinese Academy Of Sciences, Beijing, China, [2]INRIA Sophia Antipolis, Sophia Antipolis, France

## Introduction:

The orientation distribution function (ODF) is very important in diffusion MRI. There are two types of ODFs. One is proposed using radial projection in Q-ball imaging [7]. Another one is the marginal pdf proposed in diffusion spectrum imaging (DSI) [8]. Since the marginal pdf is much sharper and mathematically correct, it could be more useful. Recently some reconstruction methods were proposed for this kind of ODF [1, 6]. They are both based on mono-exponential model, globally or locally, which has some intrinsic modeling error [4]. Although the authors in [1] extended the mono-exponential model to multi-exponential model, this multi-exponential model needs to be estimated non-linearly for every voxel and only in some special sampling scheme it has a analytical solution. Here we give a model-free analytical reconstruction method based on the Spherical Polar Fourier expression of the signal. It can estimate the ODF fast and analytically from the signal.

## Methods:

Spherical Polar Fourier expression is a kind of orthonormal basis representation and it has been shown in [2] that it can represent the diffusion weighted image signal sparsely. See formula (1), where R_n(q) is the nth Gaussian-Laguerre polynomial and Y_l^m is the l order m degree spherical harmonics (SH). After we estimate the coefficients of the signal under the basis from a least square fitting or a nonlinear robust estimation [2], some PDF features could be calculated through an inner product of the coefficients $a_{n,l,m}$ and a kernel $b_{n,l,m}$. The problem in [2] is that the kernel needs to be calculated numerically from FFT for every direction or calculated for one direction then rotated for other direction. That is inefficient and may bring some numerical error. In [1] the authors showed that the ODF could be represented in the formula (2), where ∇_b^2 is the Laplacian-Beltrami operator. Based on the spherical Polar Fourier expression, we proved that the ODF coefficients can be represented analytically by the same order of SH. Please see formula (3), where P_l(0) is the Legendre polynomial of order l at 0. This formula tells us that we need to use at least order 1 of R_n to represent the anisotropic ODF. That is true because if we just use the radial basis of order zero, it is easily seen that the estimated signal is just an isotropic one, which means the estimated ODF is isotropic. Our method here is very fast since it is just a linear transformation on the coefficients, which is actually independent with data. Moreover, the coefficients of the signal could be estimated using least square with regularization term. Thus everything here is analytical and linear.

## Results:

To estimate the coefficients using at least order 1 of radial basis, it means that we need to use at least two shells to get the reasonable results. Here we show a result from a public phantom data with 3 shells in q-space, where b-value is 650,1500,2000 s/mm^2 respectively. This data has been used in the fiber cup contest in MICCAI 2009 to evaluate tracking methods [5]. It could be downloaded from the website http://www.lnao.fr/spip.php?article112 . We believe that it is complex enough to evaluate different reconstruction methods and tracking methods. We compare our reconstruction method using 3 shells with the method in [1] using one shell (b=2000), since the result of b=2000 is better than the results of b=650 and 1500 for the method in [1]. We choose N=3, L=4 in our method and use least square fitting with regularization term 5e-8 in the spherical part and 1e-9 in the radial part [2] for our method. To perform a fair comparison. we choose L=4 and tune the Laplacian regularization term lambda from 0.006 (suggested in [3]) to 0.02, 0.03 and 0.04 for the method in [1]. Two crossing areas were chosen for visualization and all the pictures were visualized using min-max normalization [7].The results were shown below and the first column is the result from our method. It shows that the method using 3 shells is better. The bad performance of the method in [1] probably comes from the error of the mono-exponential model. It can always give sharper ODFs even if there is no obvious fiber.

## Conclusions:

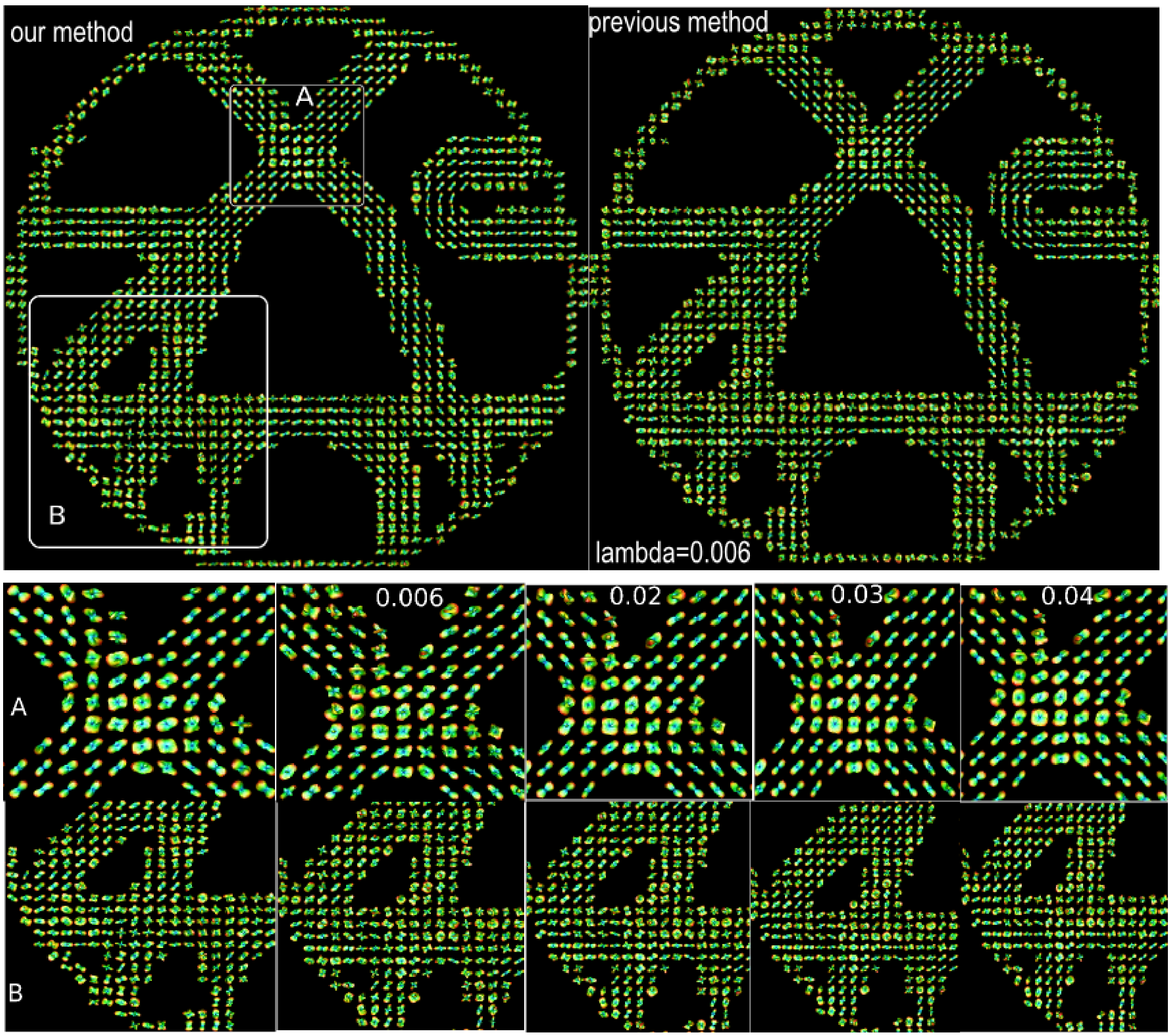
We proposed a fast analytical ODF reconstruction method based on spherical polar Fourier expression of the signal in Q-space. The coefficients of the ODF under Spherical Harmonics expression could be linearly and analytically calculated from the coefficients of the signal. It is a linear transformation that is independent with data. This transformation matrix is just needed to calculated only once for a whole data set, which makes the method very fast.

$$E(\mathbf{q}) = \sum_{n=0}^{N} \sum_{l=0}^{L} \sum_{m=-l}^{l} a_{n,l,m} R_n(\|\mathbf{q}\|) Y_l^m(\mathbf{u}) \tag{1}$$

$$ODF(\mathbf{u}) = \frac{1}{4\pi} - \frac{1}{8\pi^2} \int_0^{2\pi} \int_0^{\infty} \frac{1}{q} \triangle_b E(\mathbf{q}) dq d\phi \tag{2}$$

$$ODF(\mathbf{u}) = \sum_{l=0}^{L} \sum_{m=-l}^{l} f_{l,m} Y_l^m(\mathbf{u}) \tag{3}$$

$$f_{l,m} = \frac{1}{\sqrt{4\pi}} \delta(l)\delta(m) - \frac{1}{8\pi} \sum_{n=1}^{N} \sum_{i=1}^{n} (-1)^i \binom{n+0.5}{n-i} \frac{2^i}{i} P_l(0)(-l)(l+1) a_{n,l,m}$$



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