7414

Compressive Sensing Ensemble Average Propagator Estimation via L1 Sphericl Polar Fourier Imaging

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Introduction. Since Diffusion Tensor Imaging (DTI) cannot detect the fiber crossing, many new works beyond DTI has been proposed to explore the q-space. Most works, known as single shell High Angular Resolution Imaging (sHARDI), focus on single shell sampling and reconstruct the Orientation Distribution Function (ODF). The ODF, which has no radial information at all, is just one of features of Ensemble Average Propagator (EAP). Diffusion Spectrum Imaging (DSI) is a standard method to estimate EAP via numerical Fourier Transform (FT), which needs lots of samples and is impractical for clinical study. Spherical Polar Fourier Imaging (SPFI) [1,2] was proposed to represent the signal using SPF basis, then the EAP and the ODF have analytical closed forms. So the estimation of the coefficients under SPF basis is very important. In [1,2], the coefficients are estimated based on a standard Least Square (LS) with L2 norm regularization (L2-L2). In this paper, we propose to estimate the coefficients using LS with L1 norm regularization (L2-L1), also named as Least Absolute Selection and Shrinkage Operator (LASSO). And we prove that the L2-L1 estimation of the coefficients is actually the well known Compressive Sensing (CS) method to estimate EAP, which brings lots of Mathematical tools and possibility to improve the sampling scheme in q-space.

Theory: SPFI, LS, LASSO. In SPFI, the signal is represented using SPF basis, $E(\boldsymbol{q})=\sum_{n,l,m} a_{n,l,m} R_n(q) Y_l^m(\boldsymbol{u})$, where $R_n(q)$ is the n order Gaussian-Laguerre polynomial and $Y_l^m(\boldsymbol{u})$ is the l order m degree Spherical Harmonics. Then it was proved that the EAP could be represented analytically by the Fourier dual SPF (dSPF) basis, $P(\boldsymbol{R})=\sum_{n,l,m} a_{n,l,m} D_{n,l}(R) Y_l^m(\boldsymbol{r})$, where $D_{n,l}(R)$ is the is the l order spherical Hankel transform of $R_n(q)$. Based on Parseval's theorem, $\{D_{n,l}(R)Y_l^m(\boldsymbol{r})\}$ is actually form an orthonormal basis in r-space. So theoretically speaking, SPFI actually provides two orthonormal bases which are dual under Fourier transform. In [1,2], $\{a_{n,l,m}\}$ is estimated from L2-L2 optimization problem, $\{a_{n,l,m}\} \square A=\arg\min\|B_{RY}(\boldsymbol{q}_s)A-E(\boldsymbol{q}_s)\|_2^2+\lambda_1\|LA\|_2^2+\lambda_2\|NA\|_2^2$, where $L=[l(l+1)]$ and $N=[n(n+1)]$. That L2-L2 problem has a closed form which could be implemented as matrix multiplication. Although it is very fast and can work well even for the data with low SNR, low anisotropy and non-exponential decay [1,2], L2-L2 estimation has its natural limitations. It cannot give sparse solution, so normally we need a truncation to choose low order to be used in L2-L2 problem. And it is hard to evaluate the sampling scheme, although the authors in [3] tried it using condition number. It is well known that L2-L1, known as LASSO, could be used to solve this underdetermined linear system. In that case, the condition number will not be a very big issue. So here we propose to use L2-L1 method to estimate $\{a_{n,l,m}\}$, $A=\arg\min\|B_{RY}(\boldsymbol{q})A-E(\boldsymbol{q})\|_2^2+\lambda_1\|LA\|_1+\lambda_2\|NA\|_1$.

Theory: Compressive Sensing (CS). CS theory is a very hot topic nowadays and has many applications. It was proved useful to recover signals from only a small set of samples. Assume $y=M\Psi x+v$, where v is noise, Ψ is a given orthonormal basis which could sparsely represent the signal and x is the unknown coefficient vector of the signal under Ψ. y is the sample vector got from measurement matrix M. In the Fourier transform reconstruction problem, $M=UF$, F is denoted as Fourier transform matrix and $\boldsymbol{U}$ is the undersampling matrix for given samples. Then based on the CS theory, $\boldsymbol{x}$ could be recovered via solving L2-L1 problem $f(\boldsymbol{x})=|\|UF\Psi\boldsymbol{x}-\boldsymbol{y}\|_2^2+\lambda\|\boldsymbol{x}\|_1$, where λ was set based on the noise level. This formulation has been proposed to recover the MR image from k-space samples, i.e. sparse MRI [4]. In our EAP reconstruction problem, we could represent the EAP under the dSPF basis, $P(\boldsymbol{R})=\sum_{n,l,m} a_{n,l,m} D_{n,l}(R) Y_l^m(\boldsymbol{r})$ then the signal will be $E(\boldsymbol{q})=\sum_{n,l,m} a_{n,l,m} R_n(q) Y_l^m(\boldsymbol{u})$ and the measurements will be samples of $E(\boldsymbol{q}_s)$ So the CS minimizes $f(\boldsymbol{x})=\|B_{RY}(\boldsymbol{q}_s)A-E(\boldsymbol{q}_s)\|_2^2+\lambda\|A\|_1$. So we find that using L2-L1 to solve the coefficients is actually equivalent to do CS reconstruction. And the most interesting thing is that we do not need to do numerical FFT in each iterative step as many papers proposed to do in sparse MRI, thanks to analytical SPFI [1,2]. It will significantly accelerate the speed of reconstruction algorithm. While in [5], the authors did not consider the sparse transform and proposed to recover EAP samples with numerical FFT in each step, which is very slow. In MRI image recover problem, many people add Total Variation (TV) part as another sparse term to enhance the sparsity of the solution. However, since in our case, EAP is not sparse under Finite Difference and is a Gaussian-like smooth function, so we do not choose TV. We use weighted LASSO and optimize $\|B_{RY}(\boldsymbol{q})A-E(\boldsymbol{q})\|_2^2+\lambda_1\|LA\|_1+\lambda_2\|NA\|_1$, The advantage of this setting over the standard L2-L1 setting is analogous to the advantages of Laplacian-Beltrami regularization over Tikhonov regularization in QBI.

Theory: CS Sampling. As pointed out in CS theory, incoherence sampling is very important for robust signal recover. However, most papers about the sampling scheme in q-space for sHARDI, not for whole q-space, and they suggest to evenly spread points in the shell. In sparse MRI [5], the authors suggested to sample more around the original point. And [3,4] also suggest this. However, because the prior of q-space is that $E(0)=1$, so $E(\boldsymbol{q})$ around the original point is always very close to 1. That means we do not need to sample more around the original points. Instead, we can take $E(0)=1$ as a constraint in the part of model like mono-exponential model or we can consider E(0)=1 as a shell to add into the estimation process like [1,2] did in SPFI. So we suggest to sample just in several shells and no sample around the original point. Each shell has the same number of directions which are randomly chosen. Because we do randomly sampling, we need to do Monte Carlo test to choose a good sampling scheme [4].

Experiments. We compare this L2-L1 estimation method with previous L2-L1 method in [1,2]. We use random sampling in 3 shells b=500,1500,3000*1e-3mm^2/s , 20 directions for each shell. We generate the DWI signal using tensor mixture model with Rician noise. The SNR is defined $1/d$, d is the standard deviation for complex Gaussian noise. The eigenvalues of tensors were fixed as [1.7,0.3,0.3]*1e-3. First we test the L2-L2 (L=10, N=4, lambda_1=lambda_2=1e-10) and L2-L1(lambda_1=1lambda_2=1e-6) for the data without noise with order Then we compare these two methods for the data with SNR=20. For noisy data, we will get bad results for L2 method with large basis, we do truncation and choose L=4, N=1, lambda_1=1e-7, lambda_2=1e-8 [1,2] for L2, and we still use L=10,N=4, lambda_1=3e-5, lambda_2=1e-5 for L1 method. From the results, L2 method cannot get very sharp EAP profile because of truncation, while L1 method works well.

Conclusion. We propose to use L1 norm regularization (LASSO) to estimate the coefficients in analytical SPFI, which could be seen as a Compressive Sensing reconstruction method, but do not need numerical Fourier Transform in each iteration step. It opens a door to evaluate the sample scheme in q-space. The experiments showed that proposed method works better than the previous L2 method and it just needs a small number of samples.

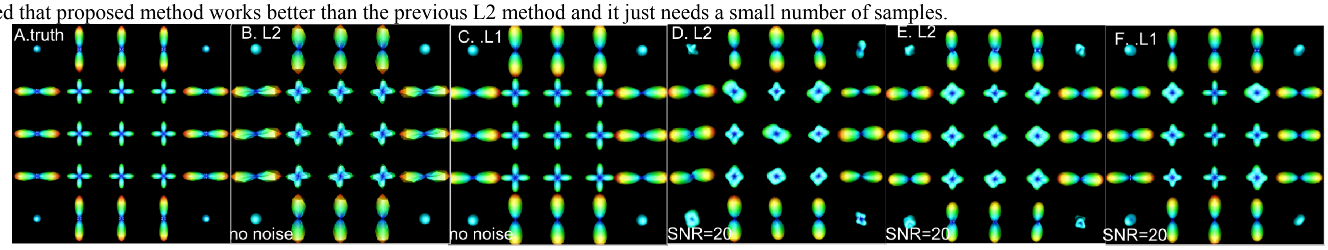

Fig. (A). ground truth of EAP at 15um, (B).L2 without noise. (c). L1 without noise. (D). L2 without truncation. (E). L2 with truncation (F) L1(SNR=20)

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