UMR CNRS 6597

Institut de Recherche en Communications et Cybernétique de Nantes

Singularity analysis of cuspidal parallel robots: uniqueness domains and classification

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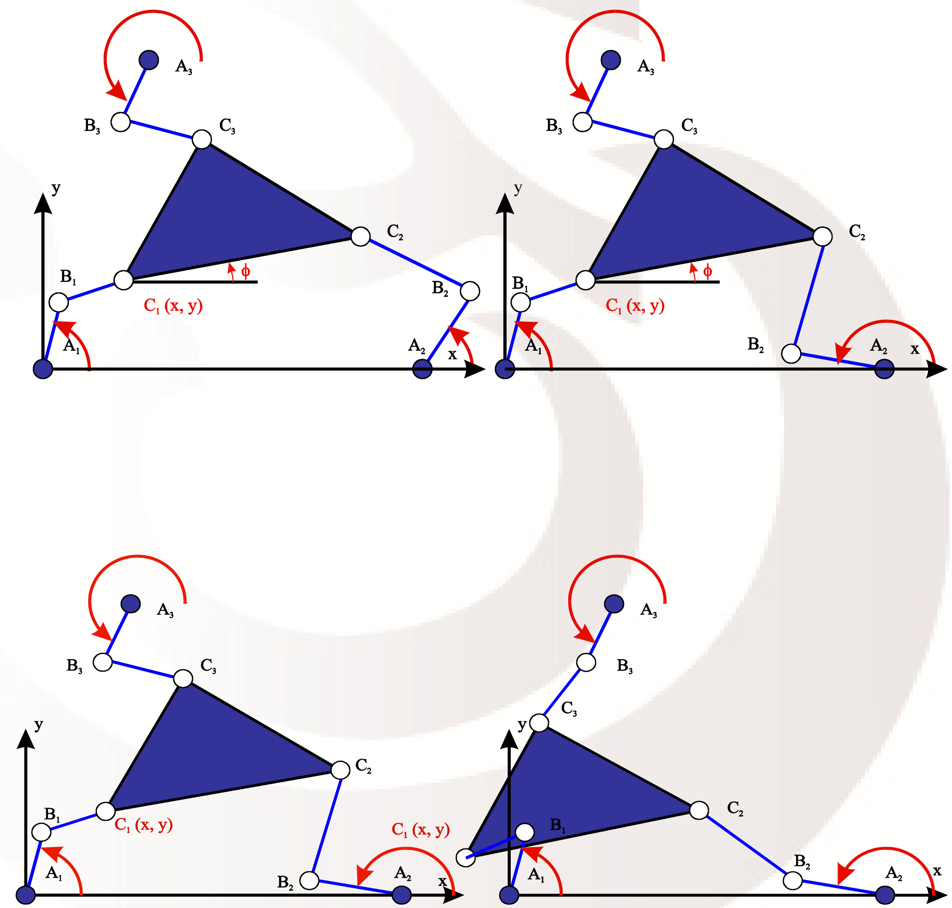
SIROPA project presentation

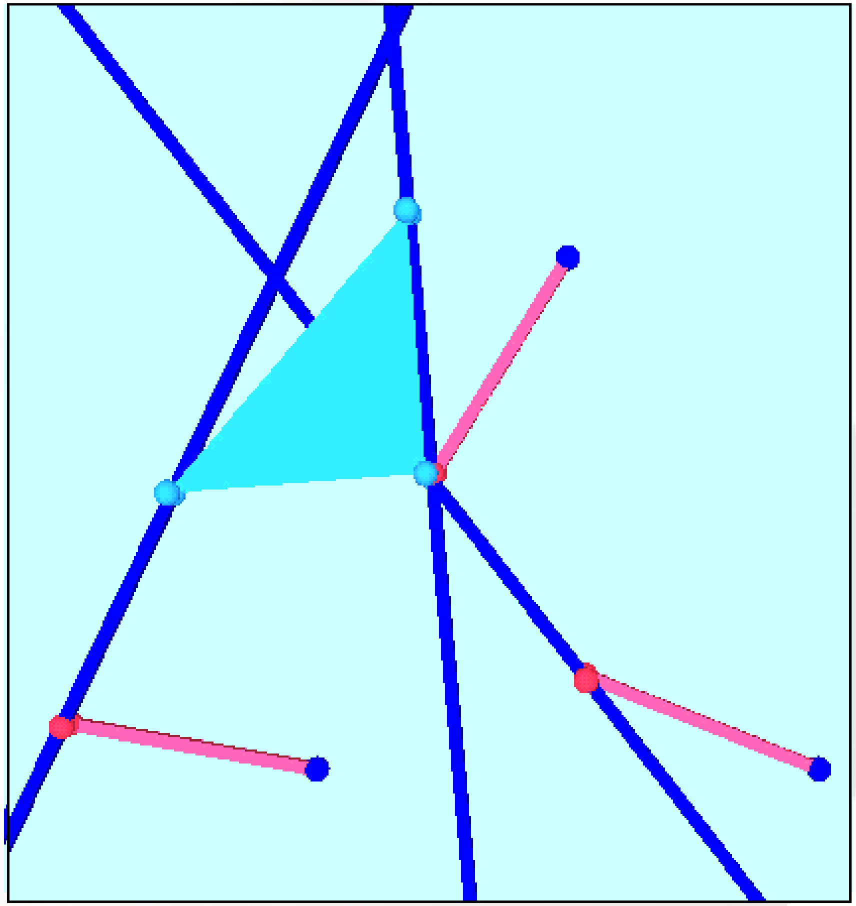
Problem statement

- Properties of parallel robots
  - Several inverse and direct kinematic solutions
- Classical definitions
  - Assembly mode
  - Working mode
- Tools for trajectory planning
  - Singularity free domains= aspect
  - Uniqueness domains
- Key issues
  - The boundaries of the domains (singularities, characteristic surfaces, joint limits)
  - The internal definition.



Cuspidal mechanism?

- For a serial mechanism:
  - Non singular changing trajectory between two inverse kinematic solutions
- For a parallel mechanism:
  - Non singular changing trajectory between two direct kinematic solutions
- The notion of aspect does not permit do define the uniqueness domain of the inverse and direct kinematics
- One common property:
  - Cusp point in the workspace or the joint space



Outline

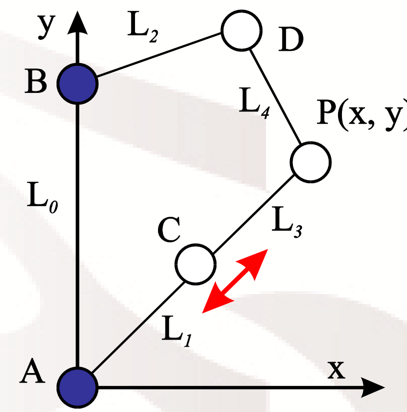
- Robotic definitions
- The aspects and uniqueness domains
- The cusp in the design parameter space
- The non singular assembly modes trajectories
- Conclusions

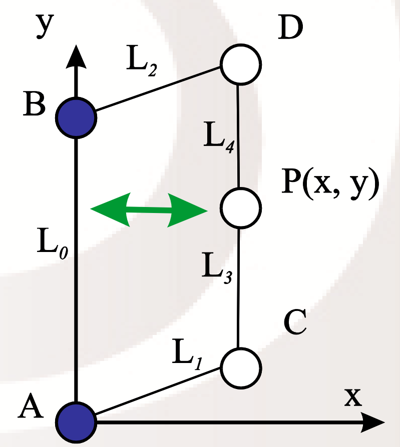
Robotic definitions (1/5)

- Kinematic model

$$F(\mathbf{q}, \mathbf{X}) = \mathbf{0} \qquad \mathbf{A}\,\mathbf{t} + \mathbf{B}\,\dot{\mathbf{q}} = 0$$

- Mapping between workspace and joint space
- Singular configurations
  - Parallel singularities from **A**
  - Serial singularities from **B**
  - Constraint singularities from joint limits
- Computation
  - Determinant of the Jacobian matrix





Robotic definitions (2/5)

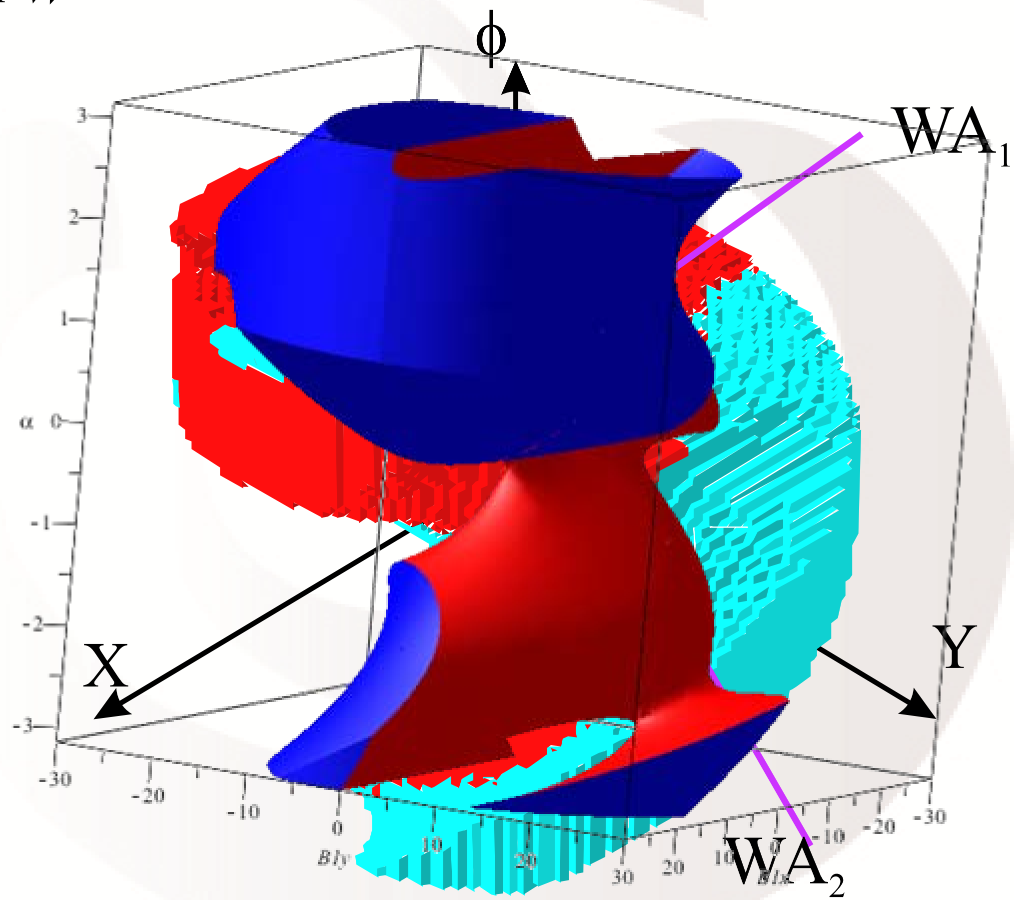
- Aspect for robot with one inverse kinematics solutions [Wenger, ICAR 1997]
  - Maximal singularity free region in $\mathcal{W}$

$$A_i \subset W$$

$$A_i \text{ is connected}$$

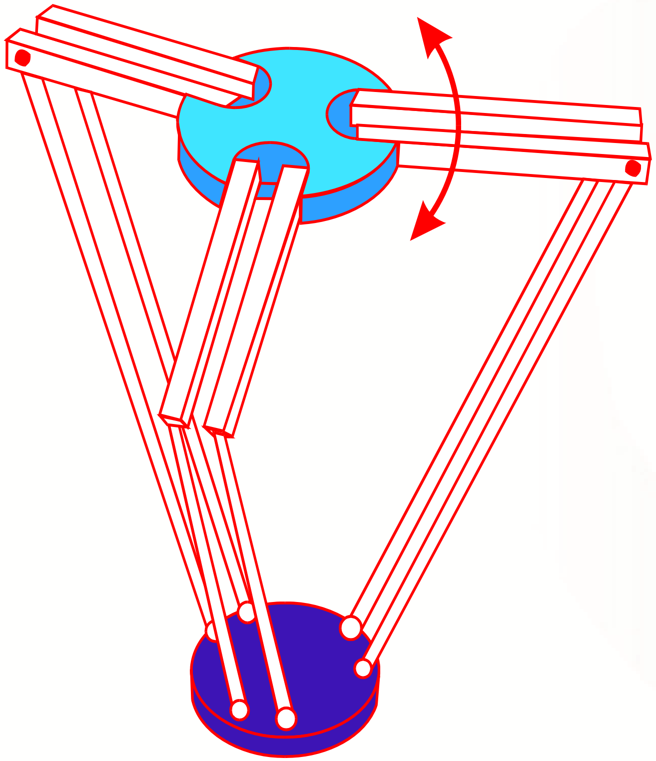
$$A_i \subset \{\forall \mathbf{X} \backslash \det(\mathbf{A}) \neq 0\}$$

Example: 3RPR



Robotic definitions (3/5)

- Working mode [Chablat, ICRA 1998]
  - A set of robot configurations for which the sign of $\mathbf{B}_{jj}$ does not change and does not vanish
  - We define Mf_i . $g_i(\mathbf{X}) = \mathbf{q}$ and $g_i^{-1}(\mathbf{q}) = \{\mathbf{X} \setminus (\mathbf{X}, \mathbf{q}) \in Mf_i\}$



$2^6=64$

Robotic definitions (4/5)

- Generalized aspect [Chablat, ICRA 1998]
  - Maximal singularity free region in $\mathcal{W}$ x Q

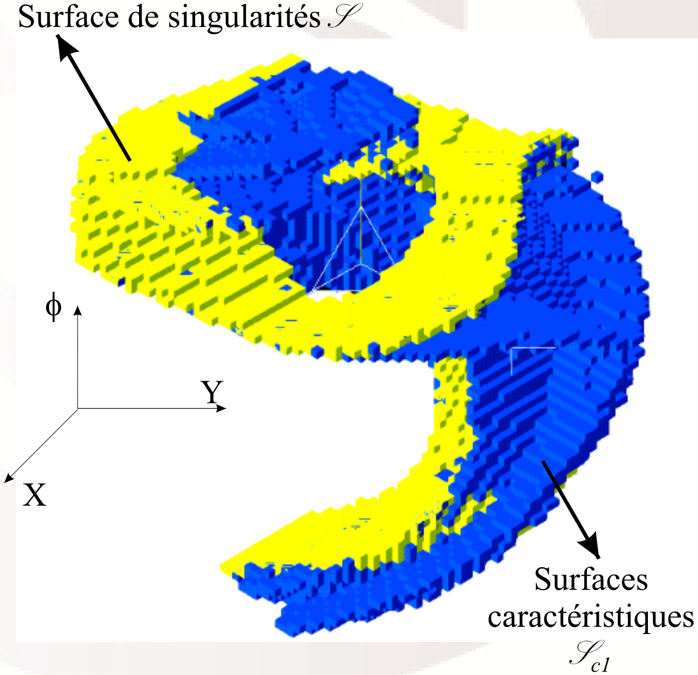
$$A_{ij} \subset W \times Q$$

$$A_{ij} \text{ is connected}$$

$$A_{ij} \subset \left\{ \left(\mathbf{X}, \mathbf{q} \right) \in Mf_i \setminus \det\left(\mathbf{A} \right) \neq 0 \right\}$$

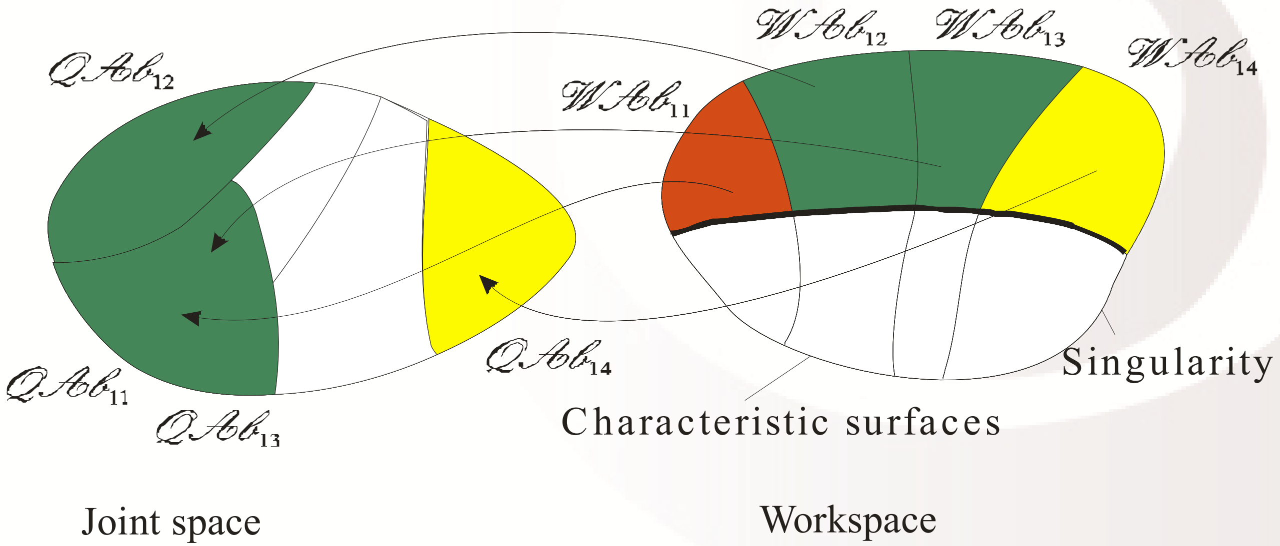
  - Projection onto workspace π_W and joint space π_Q
- Characteristic surface

$$S_C\left(WA_{ij}\right) = g_i^{-1}\left(g_i\left(\partial WA_{ij}\right)\right) \cap WA_{ij}$$



Robotic definitions (5/5)

- Basic components $WA_{ij} = \left(\cup_{k \in K} WAb_{ijk} \right) \cup S_C \left(WA_{ij} \right)$
- Basic regions $QA_{ij} = \left(\cup_{k \in K} QAb_{ijk} \right) \cup g \left(S_C \left(WA_{ij} \right) \right)$
- Uniqueness domains $Wu_{il} = \left(\cup_{k \in K'} WAb_{ijk} \cup S_C \left(WAb_{ijk} \right) \right)$



The uniqueness domains of the RPR-2PRR parallel robot

Case study: kinematic definition

- Actuators

  $\rho_1, \alpha_2, \alpha_3$

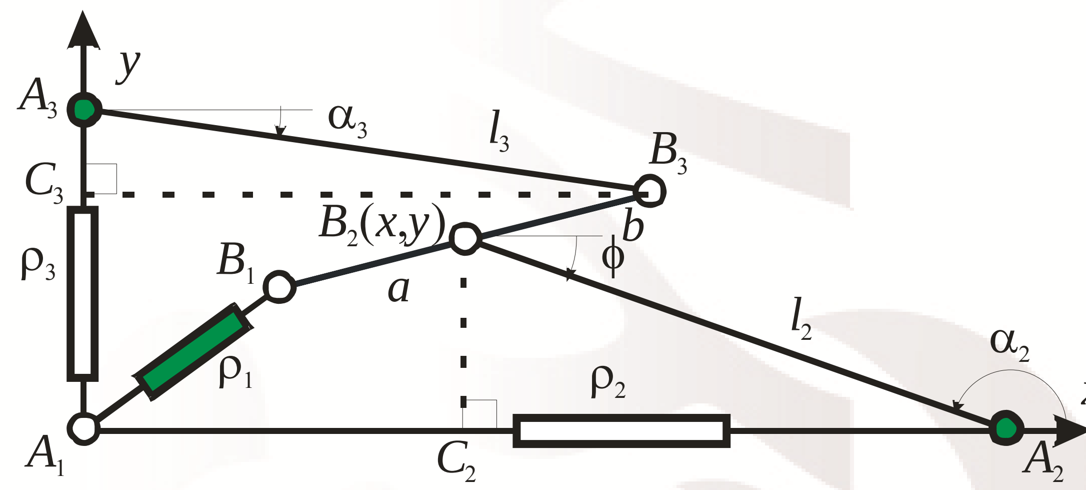
- End-effector

  x, y, α

- Passive joints

  ρ_2, ρ_3

- Parameters
  - $b = \|B_2B_1\|$
  - $a = \|B_3B_1\|$
  - $L_2 = \|A_2B_2\|$
  - $L_3 = \|A_3B_3\|$



$$\rho_2 + l_2 \cos(\alpha_2) - x = 0$$
$$l_2 \sin(\alpha_2) - y = 0$$
$$(x - a \cos(\phi))^2 + (y - a \sin(\phi))^2 - \rho_1^2 = 0$$
$$l_3 \cos(\alpha_3) - b \cos(\phi) - x = 0$$
$$\rho_3 + l_3 \sin(\alpha_3) - b \sin(\phi) - y = 0$$

- Note:
  - We can make slices if we set y, $\alpha_2 = \arcsin(y / l_2)$ or $\alpha_2 = \pi - \arcsin(y / l_2)$

Singularities

- Serial singularities

$$\mathcal{S}_S : \rho_1 l_2 l_3 \cos(\alpha_2)\sin(\alpha_3) = 0$$

- Parallel singularities

$$\mathcal{S}_P : ya\cos(\phi) - xa\sin(\phi) - b\sin(\phi)x + ab\sin(\phi)\cos(\phi) = 0$$

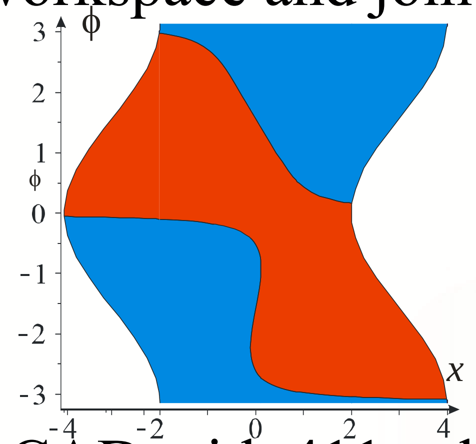
- Projections

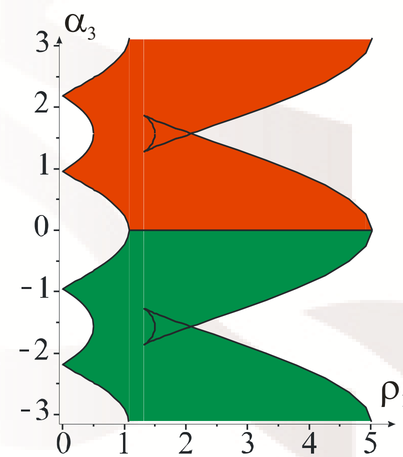
$$\pi_{\mathcal{Q}}(\mathcal{S}_P) : \rho_1^8 + (42\cos(\alpha_2)^2 - 52 - 12\cos(\alpha_3)^2)\rho_1^6 + (468\cos(\alpha_3)^2 + 960 - 1584\cos(\alpha_2)^2 - 558\cos(\alpha_3)^2$$
$$\cos(\alpha_2)^2 - 18\cos(\alpha_3)^4 + 657\cos(\alpha_2)^4)\rho_1^4 + (-2988\cos(\alpha_3)^4 - 5760\cos(\alpha_3)^2 + 4536\cos(\alpha_2)^6$$
$$+2430\cos(\alpha_3)^4\cos(\alpha_2)^2 - 7168 + 18432\cos(\alpha_2)^2 - 15840\cos(\alpha_2)^4 + 324\cos(\alpha_3)^6 + 13320\cos(\alpha_3)^2$$
$$\cos(\alpha_2)^2 - 7290\cos(\alpha_3)^2\cos(\alpha_2)^4)\rho_1^2 + (9\cos(\alpha_2)^4 - 18\cos(\alpha_3)^2\cos(\alpha_2)^2 - 24\cos(\alpha_2)^2 + 9\cos(\alpha_3)^4$$
$$+12\cos(\alpha_3)^2 + 16)(36\cos(\alpha_2)^2 - 32 - 9\cos(\alpha_3)^2)^2 = 0$$

$$\pi_{\mathcal{W}}(\mathcal{S}_S) : \left\{ \begin{array}{l} y - 3 = 0,\ y + 3 = 0, \\ (2(\cos(\phi) - 3 + x))/(\cos(\phi) + 1) = 0, \\ (2(\cos(\phi) + 3 + x))/(\cos(\phi) + 1) = 0 \end{array} \right\}$$

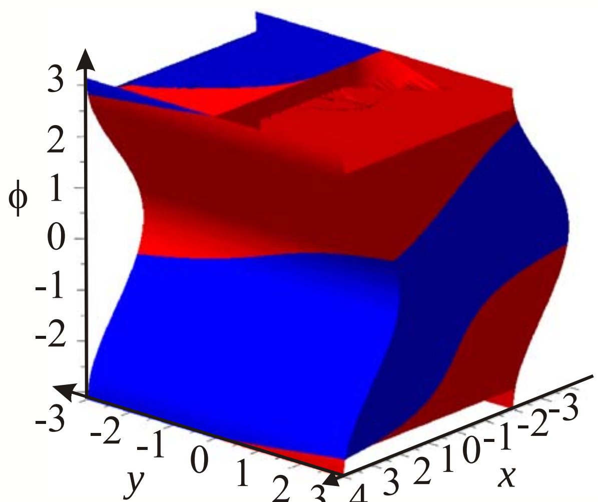
Generalized aspects

- Slices of the workspace and joint space





- W-aspect and CAD with 411 cells for $[x, y, \phi]$ and 30 for $[\phi, x, y]$



$$x_0 \text{ in } \left]\mathrm{Root}\left(4+x,1\right);\mathrm{Root}\left(28x+22x^2+8x^3+17+x^4,1\right)\right[$$

$$y_0 \text{ in } \left]\begin{array}{l}\mathrm{Root}\left(y+3,\ 1\right);\\ \mathrm{Root}\left(\begin{array}{l}9y^2+6y^2x+x^2y^2+72+198x+189x^2+\\ 72x^3+9x^4,\ 1\end{array}\right)\end{array}\right[$$

$$\phi_0 \text{ in } \left]\begin{array}{l}\mathrm{Root}\left(4+2TanHalfphi^2+x+xTanHalfphi^2,1\right);\\ \mathrm{Root}\left(4+2TanHalfphi^2+x+xTanHalfphi^2,2\right)\end{array}\right[$$

Maple

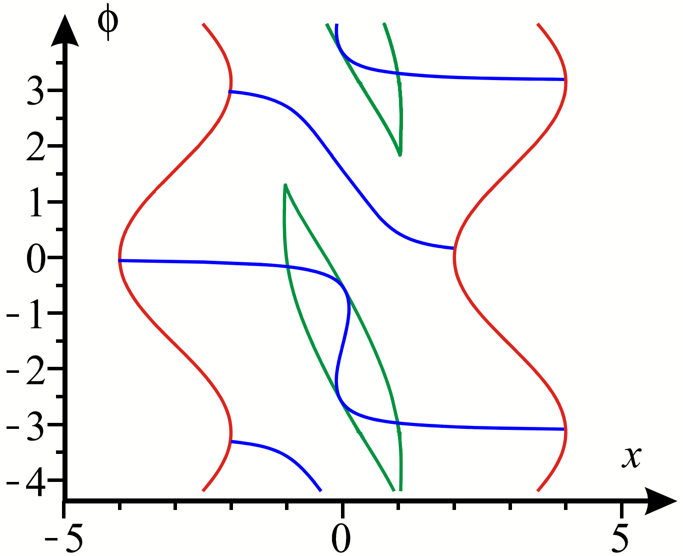
Characteristic surface

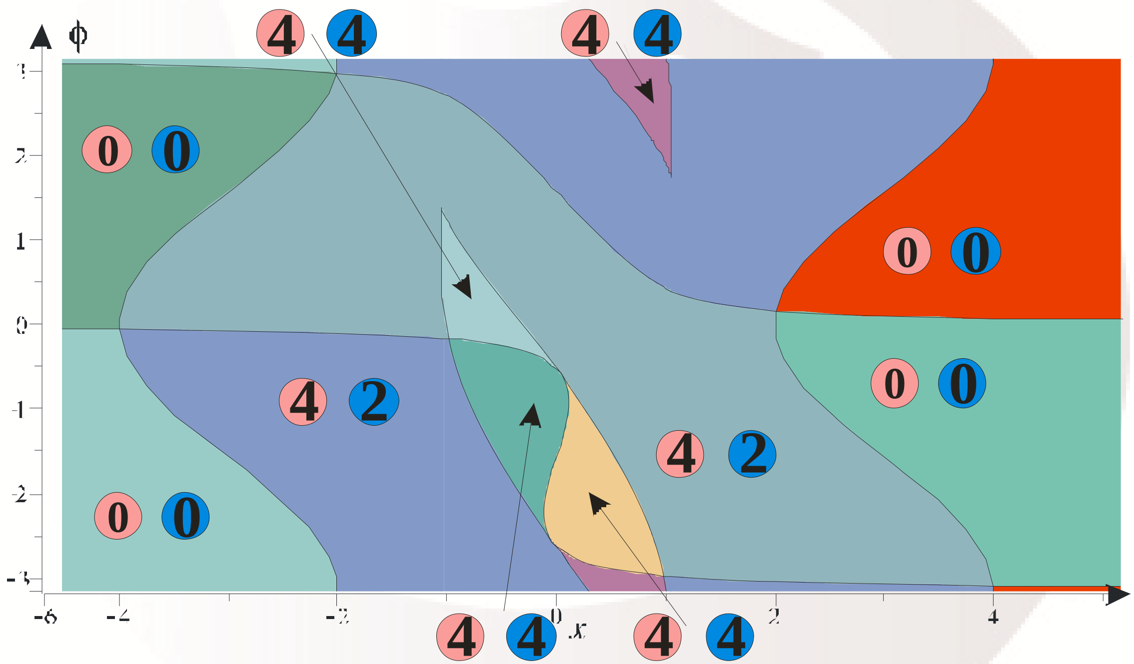
- Equation:

Maple

$$S_C: 4y^4 + 36\sin(\phi)y^3 + (32x^2 + 35\cos(\phi)^2 + 108$$
$$+184x\cos(\phi))y^2 - 6\sin(\phi)(\cos(\phi)^2 - 18 + 14x\cos(\phi)$$
$$+40x^2)y + (\cos(\phi) + 4x + 3)(\cos(\phi) + 4x - 3)$$
$$(\cos(\phi) - 2x)^2 = 0$$

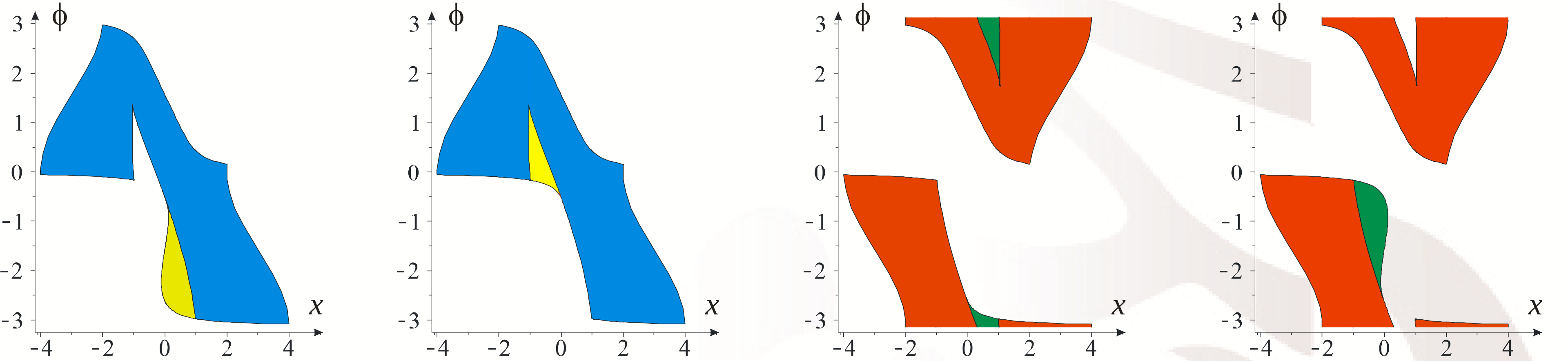
- Analysis of the workspace



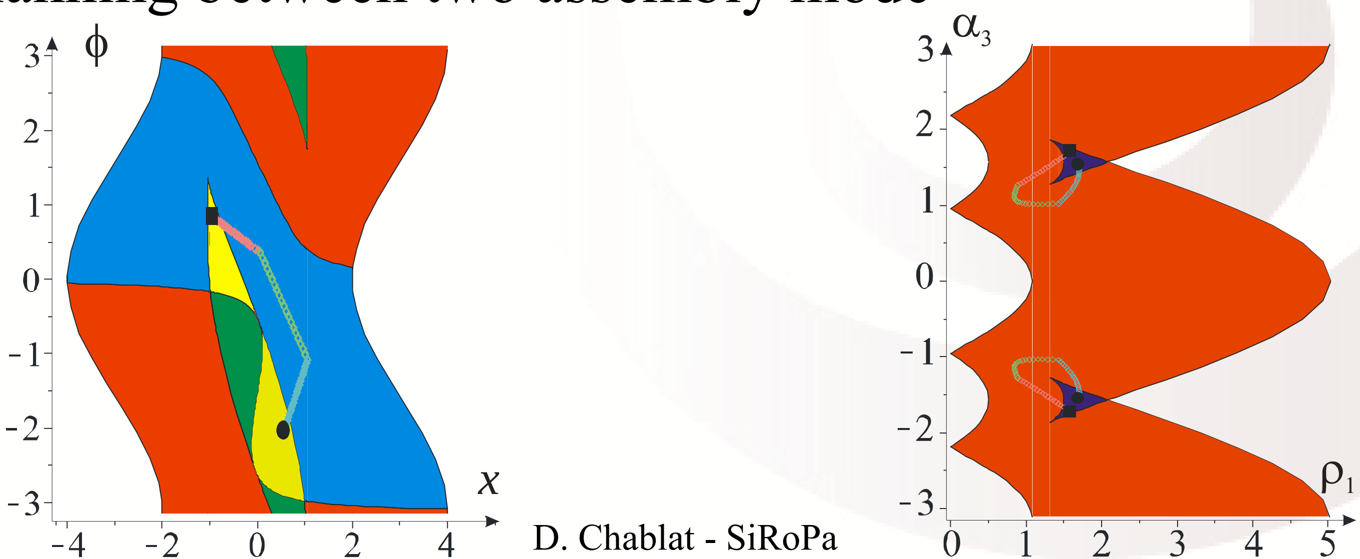


Uniqueness domains

- Characterization of the uniqueness domains



- Path planning between two assembly mode



Problem for the approach

- Computation of the projection of parallel singularities into the joint space
- Computation of the characteristic surfaces
- Computation of the CAD to obtain the basic regions an basic components
- Connectivity analysis only for 2D spaces (actually)

Cusp points in the parameter space of RPR-2PRR parallel manipulators

Case study: kinematic definition

- Actuators

  $\rho_1, \theta_2, \theta_3$

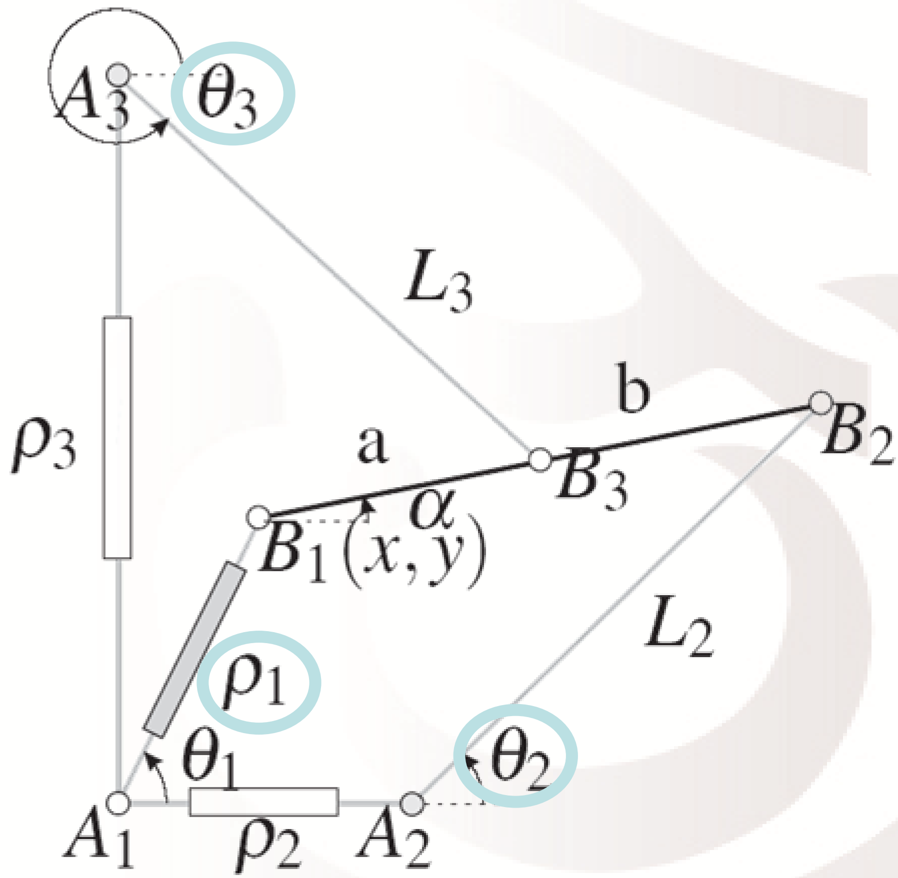
- End-effector

  x, y, α

- Passive joints

  ρ_2, ρ_3

- Parameters
  - $b = \|B_2B_1\|$
  - $a = \|B_3B_1\|$
  - $L_2 = \|A_2B_2\|$
  - $L_3 = \|A_3B_3\|$



Definition of the cusp points

- Definition based on the study of the Jacobian matrix
  - *P* is a list of polynomials,
  - *X* a list of variables,
  - $J_k(P, X)$ be the union of $P = \{p1, \ldots, p_m\}$ and of all the k x k minors of the Jacobian matrix of the p_i with respect to the X_i.
- For the analysis of the RPR-2PRR manipulator, we introduce:

$$Y := [x, y, \alpha, \rho_2, \rho_3] \qquad W := [b, L_2, L_3, \rho_1, \theta_2, \theta_3] \qquad S := \{f_1, \ldots, f_5\}.$$

- The parallel singularities

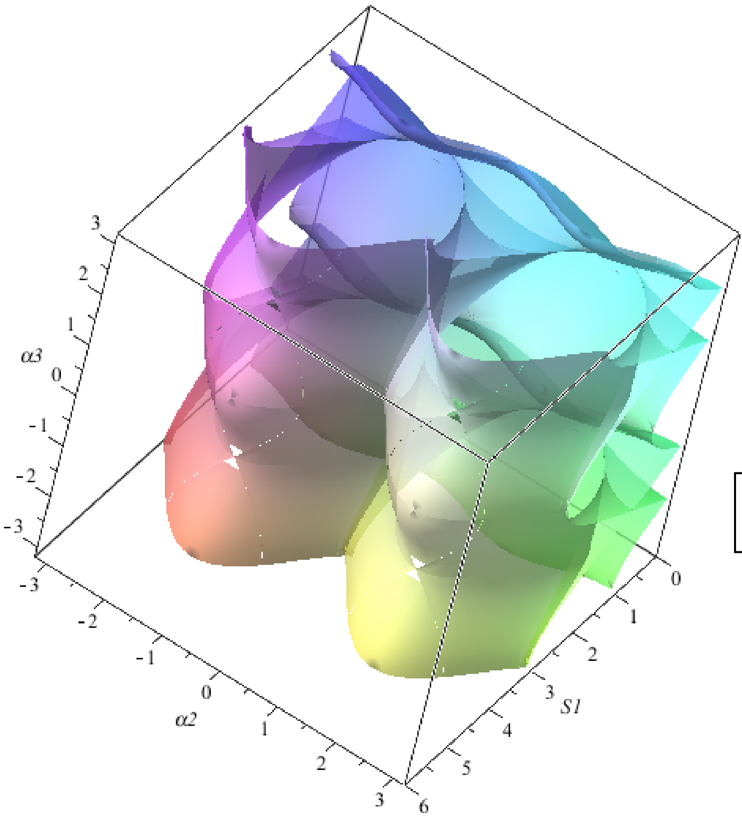
$$\{\mathbf{v} \in \mathbb{R}^{11}, p(\mathbf{v}) = 0, q(\mathbf{v}) > 0, \forall p \in J_5(S, Y), \forall q \in \{b, L_2, L_3, \rho_1\}$$

- The cusp points

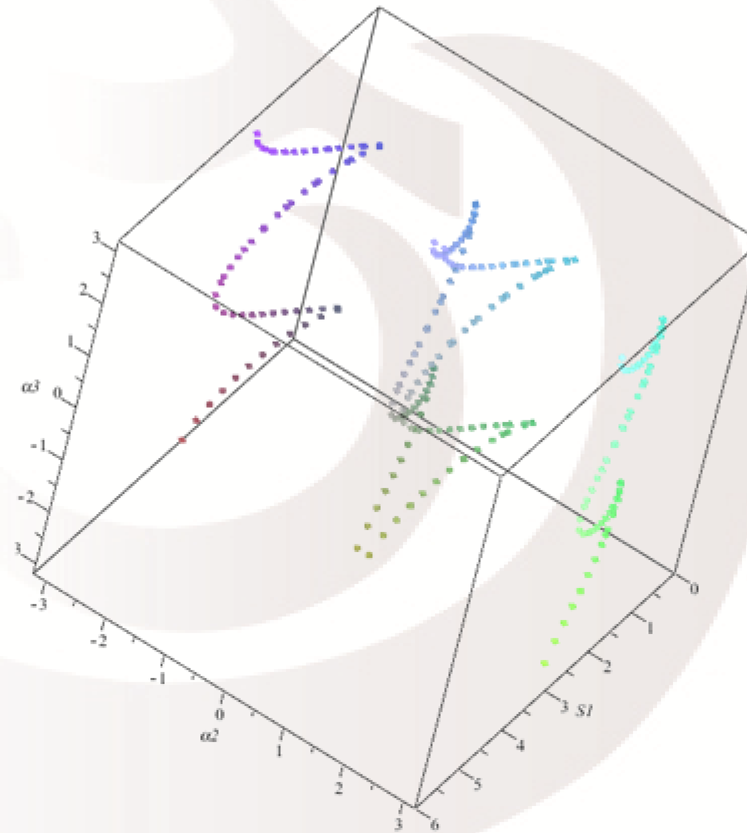
$$\mathscr{S} = \{\mathbf{v} \in \mathbb{R}^{11}, p(\mathbf{v}) = 0, q(\mathbf{v}) > 0, \forall p \in J_5(J_5(S, Y), Y), \forall q \in \{b, L_2, L_3, \rho_1\}\}$$

Joint space and cusp points

$$cusp := [x^2 + y^2 - S1^2, x - S2 - 2\cos(\alpha 2) + 2\cos(\phi), y - 2\sin(\alpha 2) + 2\sin(\phi), x - 2\cos(\alpha 3) + \cos(\phi), y - S3 - 2\sin(\alpha 3) + \sin(\phi), 4\cos(\phi)\,y - 2\sin(\phi)\,x, 16\,x\cos(\phi)^2 + 8\cos(\phi)\,y\sin(\phi) + 8\,x\,y\sin(\phi) + 4\,x^2\cos(\phi), -8\sin(\phi)\,x\cos(\phi) - 4\,y\sin(\phi)^2 - 8\,y^2\sin(\phi) - 4\,y\,x\cos(\phi), 8\cos(\phi)^2 + 2\sin(\phi)^2 + 4\,y\sin(\phi) + 2\,x\cos(\phi)]$$

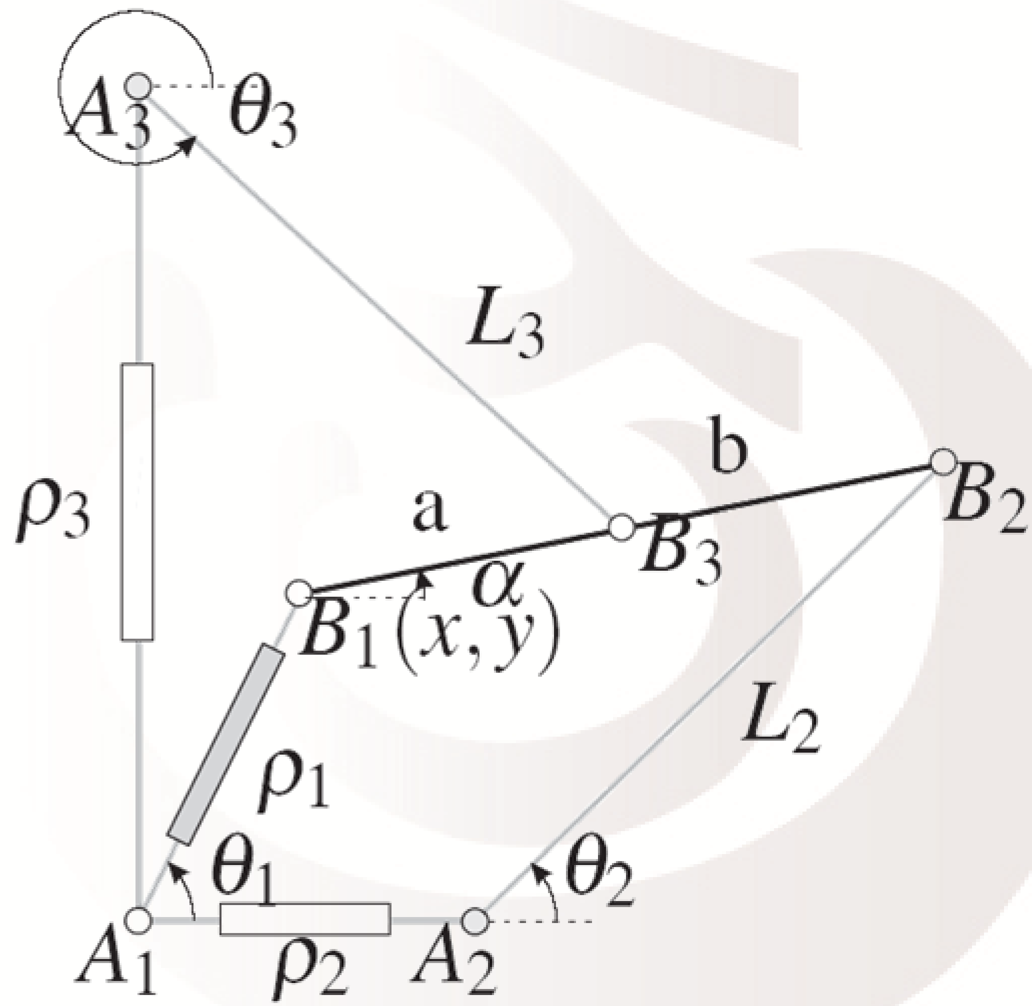


Maple



Classification on the design parameter space

- Parameters
  - $L_2 = L_3$
  - $\rho_1 = [\rho_{min}, \rho_{max}]$
  - $a = 1$
- No joint limits



Classification on the design parameter space

- Cusp definition with the design parameters

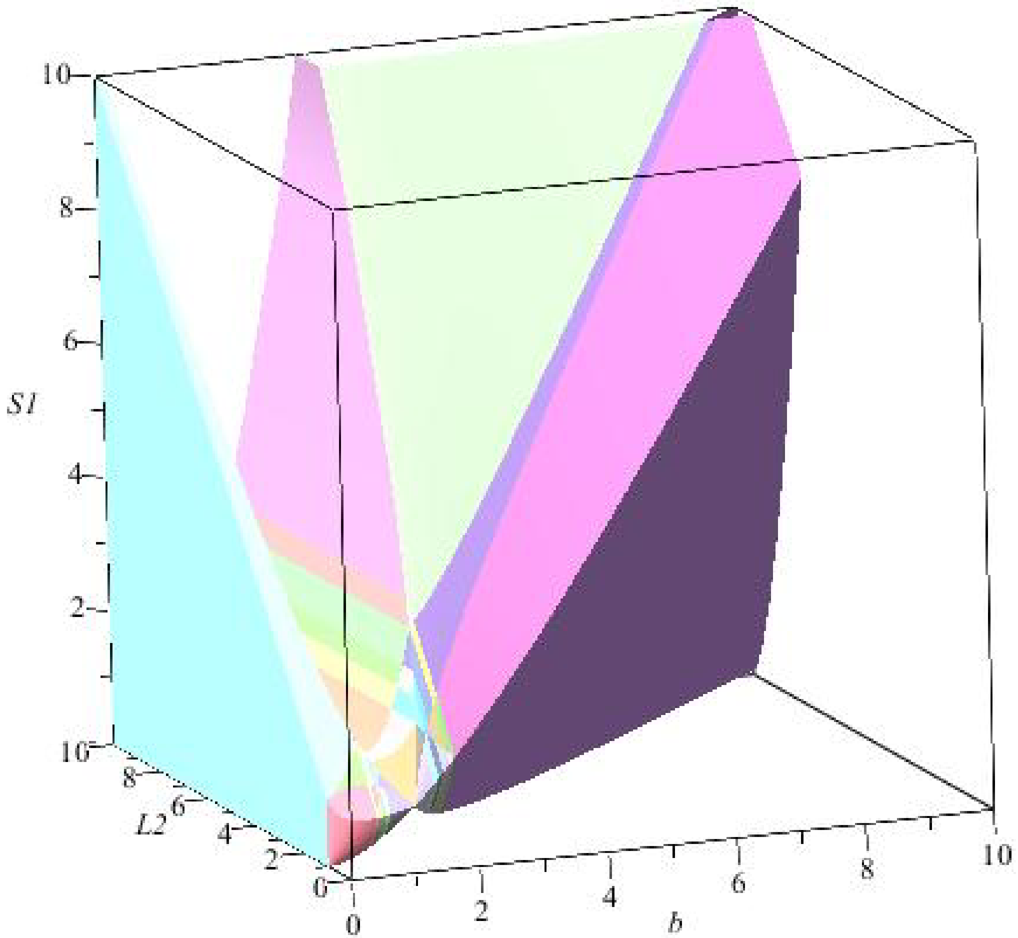
$$cusp\_bis := [x^2 + y^2 - S1^2, x - S2 - L2\cos(\alpha 2) + b\cos(\phi), y - L2\sin(\alpha 2) + b\sin(\phi),$$
$$x - L2\cos(\alpha 3) + \cos(\phi), y - S3 - L2\sin(\alpha 3) + \sin(\phi), 2\,b\cos(\phi)\,y - 2\sin(\phi)\,x,$$
$$4\,b^2\cos(\phi)^2\,x + 4\,b\cos(\phi)\,y\sin(\phi) + 4\,x\,b\sin(\phi)\,y + 4\,x^2\cos(\phi),$$
$$-4\sin(\phi)\,x\,b\cos(\phi) - 4\,y\sin(\phi)^2 - 4\,b\sin(\phi)\,y^2 - 4\,y\,x\cos(\phi),$$
$$2\,b^2\cos(\phi)^2 + 2\sin(\phi)^2 + 2\,b\sin(\phi)\,y + 2\,x\cos(\phi)]$$

- Study of this equation
  - In the projection space

$$-4\sin(\phi)\,x\,b\cos(\phi) - 4\,y\sin(\phi)^2 - 4\,b\sin(\phi)\,y^2 - 4\,y\,x\cos(\phi),$$
$$2\,b^2\cos(\phi)^2 + 2\sin(\phi)^2 + 2\,b\sin(\phi)\,y + 2\,x\cos(\phi)$$

  - With the complete system

Classification on the design parameter space



Maple

344 cells

58 cells with 16 cusp points

Notes:

- cells with 16 cusps point are well defined
- In the other cells, we do not known if an assembly mode exists

Classification on the design parameter space

$b_1 = 0, b_2 = \mathrm{Root}(8b^6 - 11b^4 + 6b^2 - 1, 2)$ $b_3 = \mathrm{Root}(4b^2 + b^6 - 3b^4 - 1, 2), b_4 = \mathrm{Root}(b^8 + 3b^6 + 3b^4 + b^2 - 1, 2)$ $b_5 = \mathrm{Root}(-2b^4 + b^6 + 3b^2 - 1, 2), b_6 = 1/\sqrt{2}, b_7 = 1, b_8 = \sqrt{(2)}$ $b_9 = \mathrm{Root}(2b^2 + b^6 - 3b^4 - 1, 2), b_{10} = \mathrm{Root}(b^8 - b^6 - 3b^4 - 3b^2 - 1, 2)$ $b_{11} = \mathrm{Root}(-4b^4 + b^6 + 3b^2 - 1, 2)$ $b_{12} = \mathrm{Root}(b^6 - 6b^4 + 11b^2 - 8, 2), b_{13} = \infty$	$L_{2_1}(b) = \mathrm{Root}\left(\left(b^2+1\right)^3 L_2^6 - 3\left(b^2+3b+1\right)\left(b^2-3b+1\right)(b-1)^2(b+1)^2 L_2^4 + 3\left(b^2+1\right)(b-1)^4(b+1)^4 L_2^2 - (b-1)^6(b+1)^6, 2\right)$ $L_{2_2}(b) = 1-b^2, L_{2_3}(b) = 1/\sqrt{1-b^2}, L_{2_4}(b) = (1-b^2)/b, L_{2_5}(b) = \infty$ $L_{2_6}(b) = b^2/\sqrt{1-b^2}, L_{2_7}(b) = 1/\sqrt{b^2-1}, L_{2_8}(b) = b^2/\sqrt{b^2-1}$ $L_{2_9}(b) = (b^2-1)/b, L_{2_{10}}(b) = b^2-1$

$\rho_{1_1}(b,L_2) = \mathrm{Root}(-\rho_1^6 b^6 + 3b^4\left(L_2^2 b^2 + 1 - L_2^2\right)\rho_1^4 - 3b^2\left(-7L_2^2 b^2 + 7L_2^2 + L_2^4 + L_2^4 b^4 - 2L_2^4 b^2 + 1\right)\rho_1^2 + \left(L_2^2 b^2 + 1 - L_2^2\right)^3, 2)$

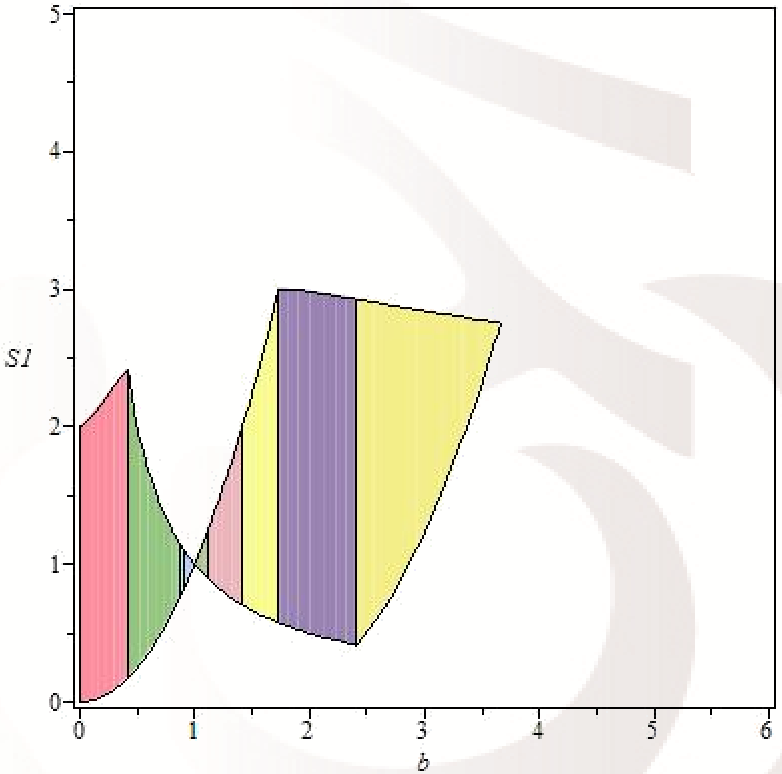
$\rho_{1_2}(b,L_2) = \mathrm{Root}(\rho_1^6 + \left(-3b^4 + 3L_2^2 b^2 - 3L_2^2\right)\rho_1^4 + \left(21L_2^2 b^6 + 3L_2^4 b^4 - 6L_2^4 b^2 + 3b^8 - 21L_2^2 b^4 + 3L_2^4\right)\rho_1^2 + \left(L_2^2 b^2 - b^4 - L_2^2\right)^3, 2)$

$\rho_{1_2}(b,L_2) = b^2, \rho_{1_3}(b,L_2) = 1/b$

$]b_1\ b_2[$	$(]L_{2_1}\ L_{2_2}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_2}\ L_{2_3}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_3}\ L_{2_4}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_4}\ L_{2_5}[,]\rho_{1_3}\ \rho_{1_4}[)$
$]b_2\ b_3[$	$(]L_{2_1}\ L_{2_6}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_6}\ L_{2_2}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_2}\ L_{2_3}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_3}\ L_{2_4}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_4}\ L_{2_5}[,]\rho_{1_3}\ \rho_{1_4}[)$
$]b_3\ b_4[$	$(]L_{2_1}\ L_{2_6}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_6}\ L_{2_2}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_2}\ L_{2_4}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_4}\ L_{2_3}[,]\rho_{1_3}\ \rho_{1_4}[), (]L_{2_3}\ L_{2_5}[,]\rho_{1_3}\ \rho_{1_4}[)$
$]b_4\ b_5[$	$(]L_{2_1}\ L_{2_6}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_6}\ L_{2_2}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_2}\ L_{2_4}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_4}\ L_{2_3}[,]\rho_{1_3}\ \rho_{1_4}[), (]L_{2_3}\ L_{2_5}[,]\rho_{1_3}\ \rho_{1_4}[)$
$]b_5\ b_6[$	$(]L_{2_1}\ L_{2_2}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_2}\ L_{2_6}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_6}\ L_{2_4}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_4}\ L_{2_3}[,]\rho_{1_3}\ \rho_{1_4}[), (]L_{2_3}\ L_{2_5}[,]\rho_{1_3}\ \rho_{1_4}[)$
$]b_6\ b_7[$	$(]L_{2_1}\ L_{2_2}[,]\rho_{1_1}\ \rho_{1_2}[), (]L_{2_2}\ L_{2_4}[,]\rho_{1_3}\ \rho_{1_2}[), (]L_{2_4}\ L_{2_6}[,]\rho_{1_3}\ \rho_{1_4}[), (]L_{2_6}\ L_{2_3}[,]\rho_{1_3}\ \rho_{1_4}[), (]L_{2_3}\ L_{2_5}[,]\rho_{1_3}\ \rho_{1_4}[)$
$]b_7\ b_8[$	$(]L_{2_1}\ L_{2_9}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_9}\ L_{2_{10}}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_{10}}\ L_{2_7}[,]\rho_{1_4}\ \rho_{1_3}[), (]L_{2_7}\ L_{2_8}[,]\rho_{1_4}\ \rho_{1_3}[), (]L_{2_8}\ L_{2_5}[,]\rho_{1_4}\ \rho_{1_3}[)$
$]b_8\ b_9[$	$(]L_{2_1}\ L_{2_9}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_9}\ L_{2_7}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_7}\ L_{2_{10}}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_{10}}\ L_{2_8}[,]\rho_{1_4}\ \rho_{1_3}[), (]L_{2_8}\ L_{2_5}[,]\rho_{1_4}\ \rho_{1_3}[)$
$]b_9\ b_{10}[$	$(]L_{2_1}\ L_{2_7}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_7}\ L_{2_9}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_9}\ L_{2_{10}}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_{10}}\ L_{2_8}[,]\rho_{1_4}\ \rho_{1_3}[), (]L_{2_8}\ L_{2_5}[,]\rho_{1_4}\ \rho_{1_3}[)$
$]b_{10}\ b_{11}[$	$(]L_{2_1}\ L_{2_7}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_7}\ L_{2_9}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_9}\ L_{2_{10}}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_{10}}\ L_{2_8}[,]\rho_{1_4}\ \rho_{1_3}[), (]L_{2_8}\ L_{2_5}[,]\rho_{1_4}\ \rho_{1_3}[)$
$]b_{11}\ b_{12}[$	$(]L_{2_1}\ L_{2_7}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_7}\ L_{2_9}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_9}\ L_{2_8}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_8}\ L_{2_{10}}[,]\rho_{1_4}\ \rho_{1_1}[, (]L_{2_{10}}\ L_{2_5}[,]\rho_{1_4}\ \rho_{1_3}[)$
$]b_{12}\ b_{13}[$	$(]L_{2_1}\ L_{2_9}[,]\rho_{1_2}\ \rho_{1_1}[), (]L_{2_9}\ L_{2_8}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_8}\ L_{2_{10}}[,]\rho_{1_4}\ \rho_{1_1}[), (]L_{2_{10}}\ L_{2_5}[,]\rho_{1_4}\ \rho_{1_3}[)$

Notes

- We can study when we fix L_2



- We can study when $L_2 \neq L_3$
  - 688 cells
  - 130 with 16 cusps

Problems for this mechanism

- Study of the mechanism properties for more than 3 parameters
- One joint value is added to the design parameters
- Difficult to achieve for mechanisms without any simplification
- The number of cells depends on the parameterization of the mechanism

Non singular assembly mode changing trajectory

Aspects

- For a parallel robot: how many aspects?
  - 3-R$\underline{\text{P}}$R:
    - 2 aspects in general case
    - 4 aspects for similar base and mobile platform
  - R$\underline{\text{P}}$R-2P$\underline{\text{R}}$R:
    - 2 aspects
  - 3-$\underline{\text{PP}}$PS:
    - 2 aspects
- How many postures in the same aspect?
  - Evaluate the maximum number of assembly modes
  - Distribute the assembly modes in each aspect.

Case study: kinematic definition

- Actuators

  $x_1, x_3, y_1, y_2, z_2, z_3$

- End-effector

  $cx, cy, cz, \theta, \sigma, \psi$ (T&T)

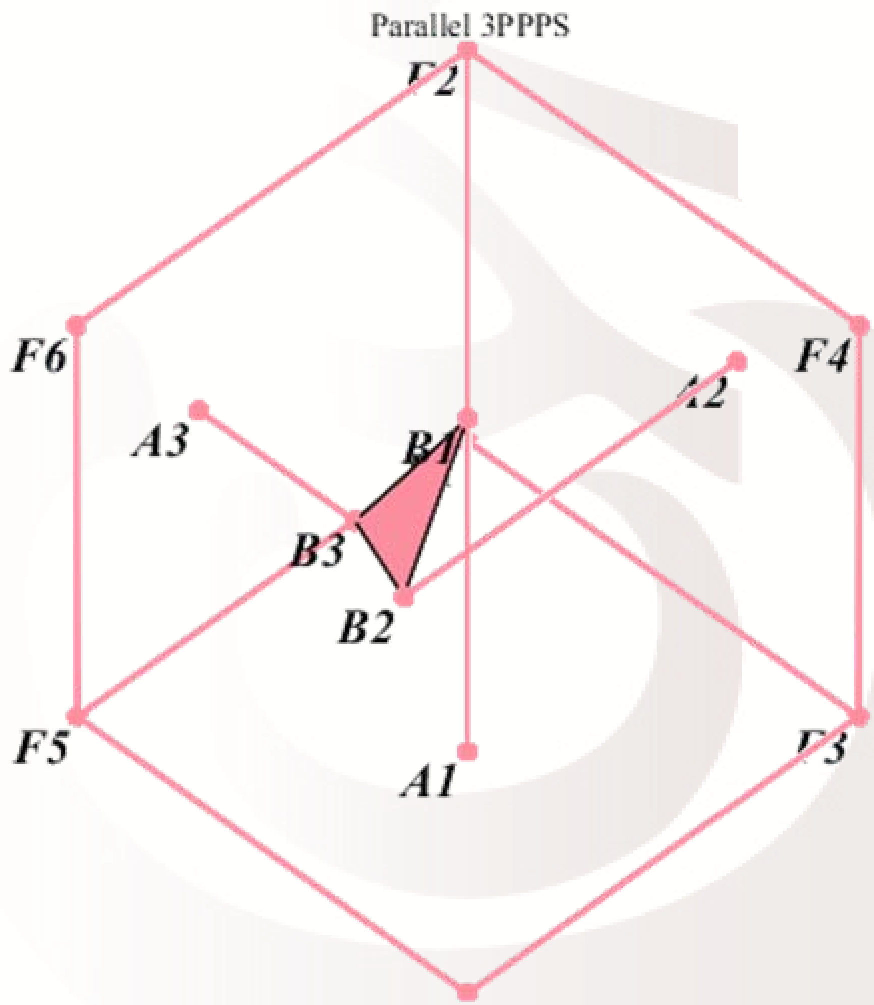
  $cx, cy, cz, q_1, q_2, q_3, q_4$

- Passive joints

  X_2, Y_3, Z_1

- Parameters
  - **Ax= 0, Ay= 0, Az= 0,**
  - **Bx=-1, By= 0, Bz= 1,**
  - **Cx= 0, Cy=-1, Cz= 1.**

- Application: haptic interface



Main properties

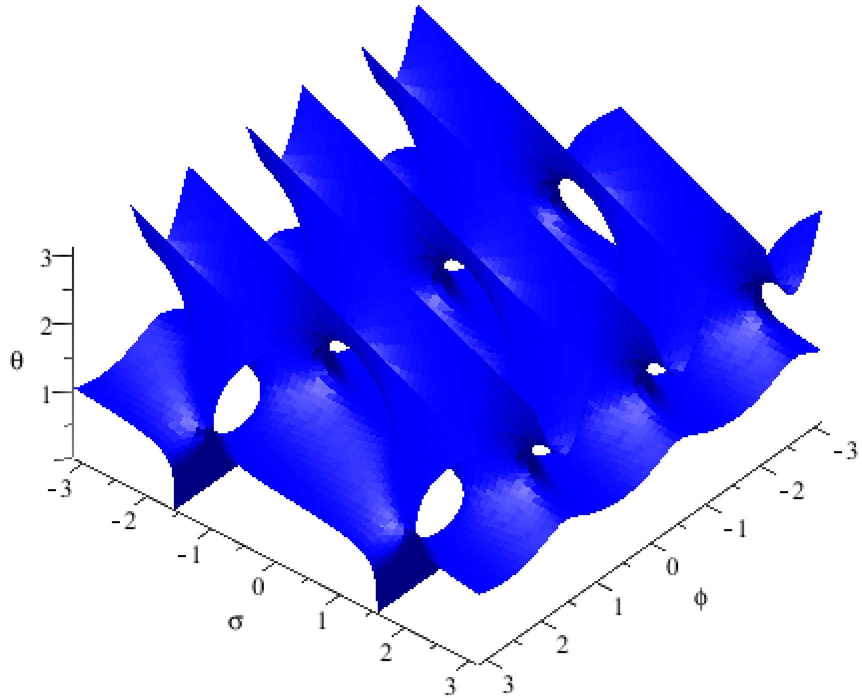
- 6 degrees of freedom
- But
  - The singular configurations does not depend on the position.
  - The joint space can be studied only in a three dimension space by a simple variable changing.

$$x_1 = X_1 - X_3/2, x_3 = X_1 + X_3/2$$
$$y_1 = Y_2 - Y_1/2, \quad y_2 = Y_2 + Y_1/2$$
$$z_3 = Z_3 - Z_2/2, \quad z_2 = Z_3 + Z_2/2$$

Parallel Singularities

- After simplification and factorization

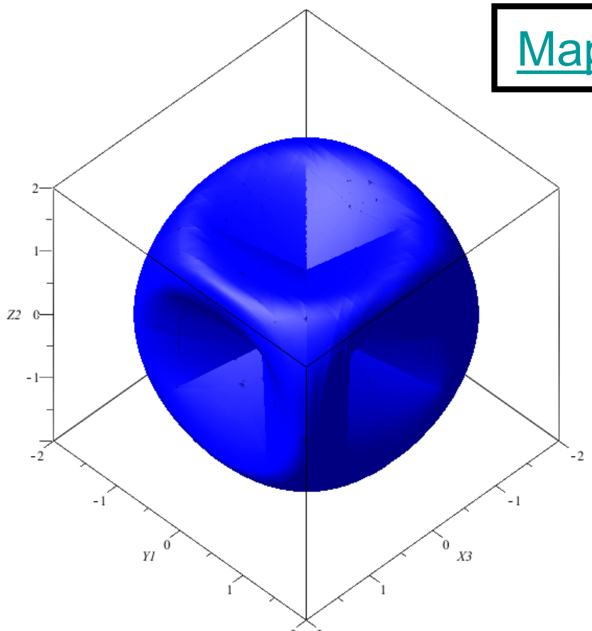
$$4\sin\left(\frac{1}{2}\theta\right)\cos\left(\frac{1}{2}\theta\right)^2\left(-\sqrt{2}\sin(3\phi-\sigma)\sin\left(\frac{1}{2}\theta\right)^3+\cos(\sigma)\cos\left(\frac{3}{2}\theta\right)\right)$$



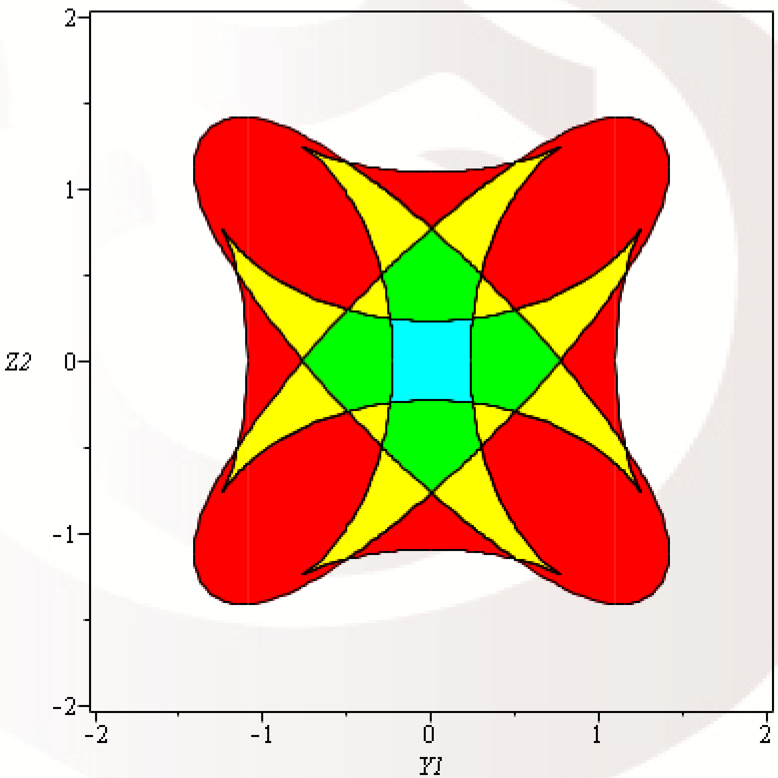
Maple

Number of assembly modes?

- With T&T parameters
- Projection of the constraints equations
  - 8 direct kinematic solutions for cx, cy, cz
  - 16 direct kinematics solutions for θ, σ, ψ
  - Keep only $\theta>0$
- Slice of the joint space



Maple



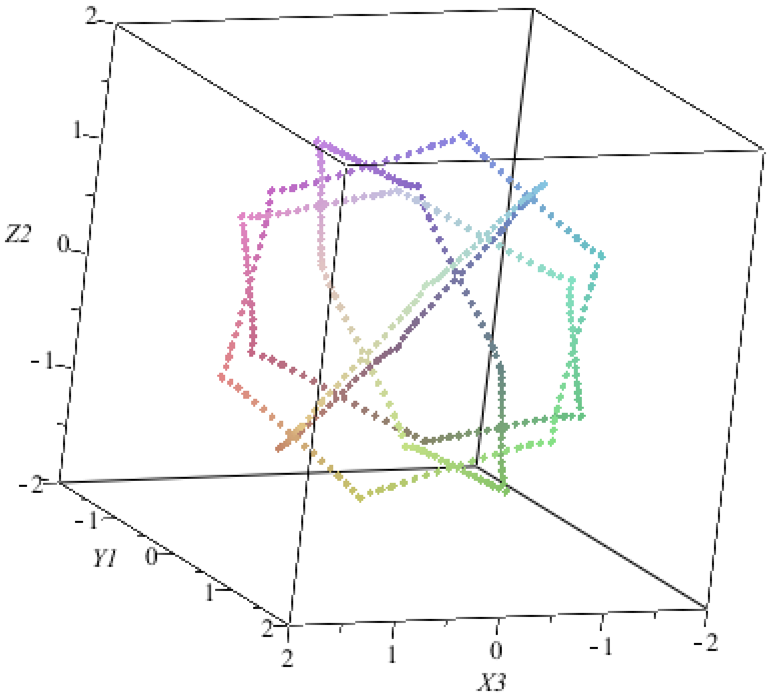
Cusp points?

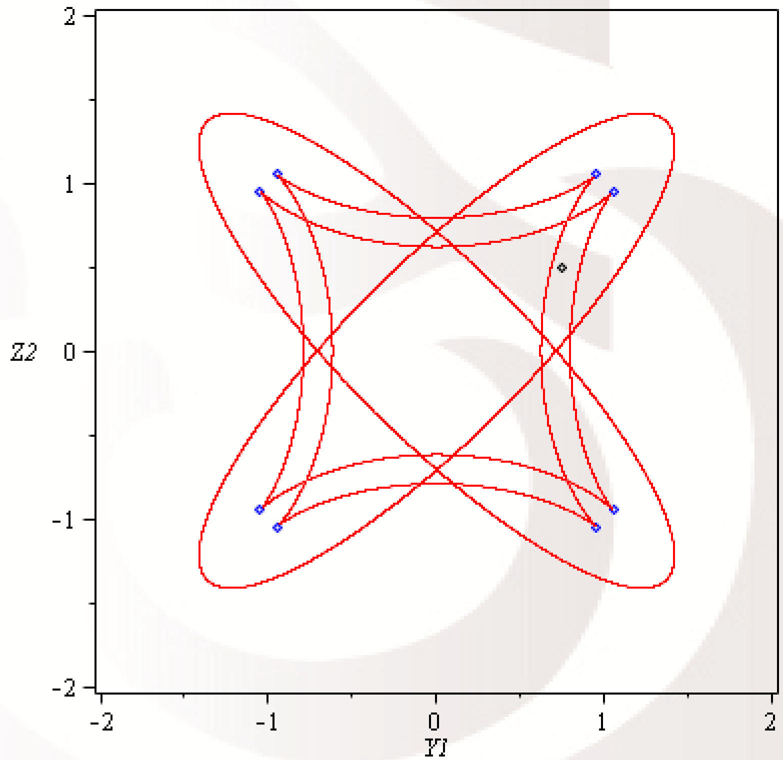
- Depends on X_3, Y_1, Z_2

$$
\begin{aligned}
[&24\, Z2^4\, Y1^2 - 704 + 24\, Z2^2\, Y1^4 + 24\, Z2^4\, X3^2 - 144\, Z2^2\, Y1^2\, X3^2 + 24\, Y1^4\, X3^2 + 24\, Z2^2\, X3^4 + 24\, Y1^2\, X3^4 - 215\, Z2^4 - 82\, Z2^2\, Y1^2 \\
&- 215\, Y1^4 - 82\, Z2^2\, X3^2 - 82\, Y1^2\, X3^2 - 215\, X3^4 + 584\, Z2^2 + 584\, Y1^2 + 584\, X3^2,\ 8\, Z2^6 - 32 + 8\, Y1^6 - 24\, Z2^2\, Y1^2\, X3^2 \\
&+ 8\, X3^6 - 37\, Z2^4 + 10\, Z2^2\, Y1^2 - 37\, Y1^4 + 10\, Z2^2\, X3^2 + 10\, Y1^2\, X3^2 - 37\, X3^4 + 40\, Z2^2 + 40\, Y1^2 + 40\, X3^2,\ -151136 \\
&+ 25512\, Z2^2\, X3^4 + 576\, Z2^2\, Y1^6 - 4440\, Z2^2\, Y1^2\, X3^2 + 97496\, Y1^2 - 9792\, Z2^2\, Y1^4\, X3^2 + 22464\, Z2^2\, Y1^2\, X3^4 + 28296\, X3^6 \\
&+ 576\, X3^8 - 12984\, Y1^6 + 576\, Y1^8 - 135706\, Z2^2\, X3^2 - 23578\, Z2^2\, Y1^2 + 9768\, Z2^2\, Y1^4 - 4608\, Z2^4\, X3^4 + 232664\, X3^2 \\
&- 11699\, Y1^4 - 104794\, Y1^2\, X3^2 - 42611\, Z2^4 + 133592\, Z2^2 - 123827\, X3^4 - 3456\, Y1^4\, X3^4 - 3456\, Y1^2\, X3^6 + 38064\, Y1^4\, X3^2 \\
&+ 12528\, Y1^2\, X3^4 + 1152\, Y1^6\, X3^2 - 4032\, Z2^2\, X3^6 + 41280\, Z2^4\, X3^2,\ 138768 - 119722\, Z2^2\, X3^4 - 3072\, Z2^4\, X3^6 - 2688\, Z2^2\, X3^8 \\
&- 26278\, Z2^2\, Y1^2\, X3^2 - 87480\, Y1^2 - 5760\, Z2^2\, Y1^4\, X3^4 + 14592\, Z2^2\, Y1^2\, X3^6 + 18096\, Z2^2\, Y1^4\, X3^2 - 24360\, Z2^2\, Y1^2\, X3^4 \\
&- 109115\, X3^6 + 17712\, X3^8 + 384\, X3^{10} + 15648\, Y1^6 - 1152\, Y1^8 + 236984\, Z2^2\, X3^2 + 33264\, Z2^2\, Y1^2 - 13152\, Z2^2\, Y1^4 \\
&+ 32232\, Z2^4\, X3^4 - 326456\, X3^2 + 3096\, Y1^4 + 183992\, Y1^2\, X3^2 + 36888\, Z2^4 - 125880\, Z2^2 + 272864\, X3^4 + 29832\, Y1^4\, X3^4 \\
&+ 12744\, Y1^2\, X3^6 - 51227\, Y1^4\, X3^2 - 87178\, Y1^2\, X3^4 + 576\, Y1^8\, X3^2 + 576\, Y1^6\, X3^4 - 2496\, Y1^4\, X3^6 - 2112\, Y1^2\, X3^8 \\
&- 8976\, Y1^6\, X3^2 + 21720\, Z2^2\, X3^6 - 70619\, Z2^4\, X3^2]
\end{aligned}
$$

Cusp points?

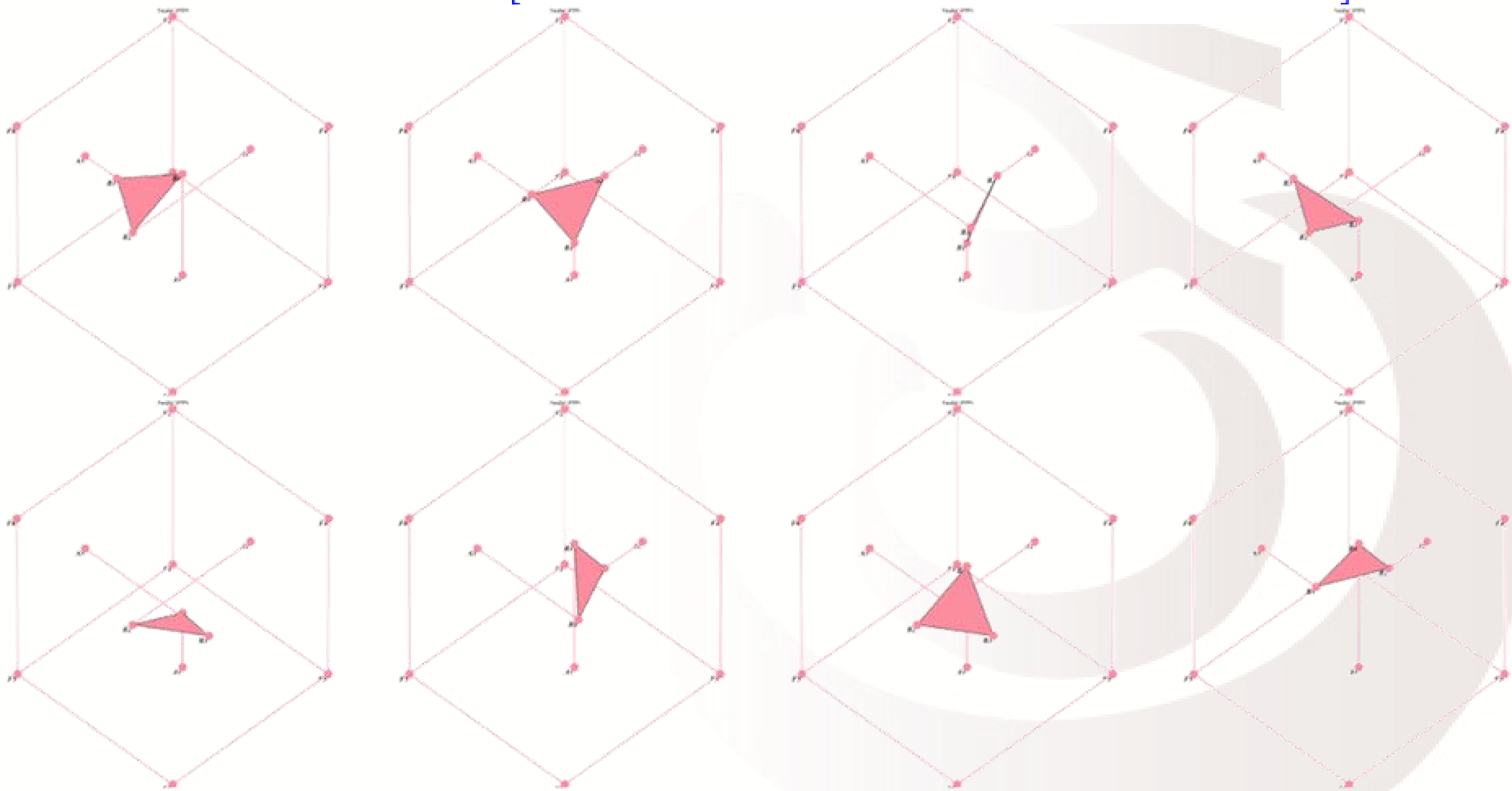
- From the intersection of the surfaces





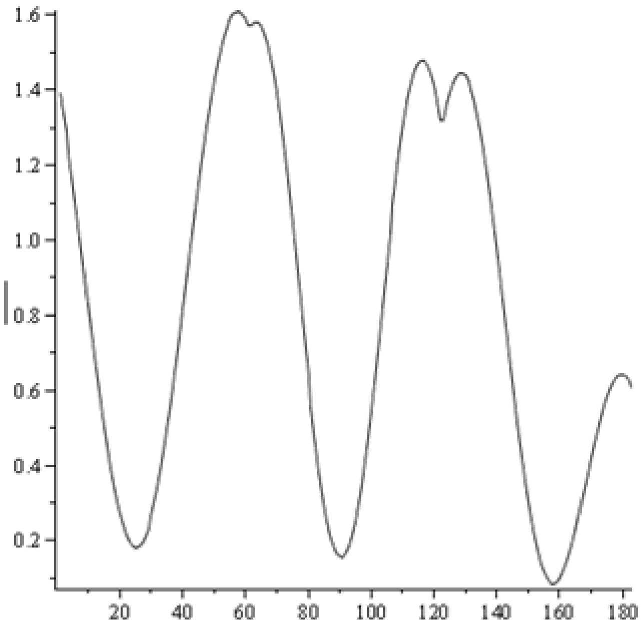
Define the trajectory (1/4)

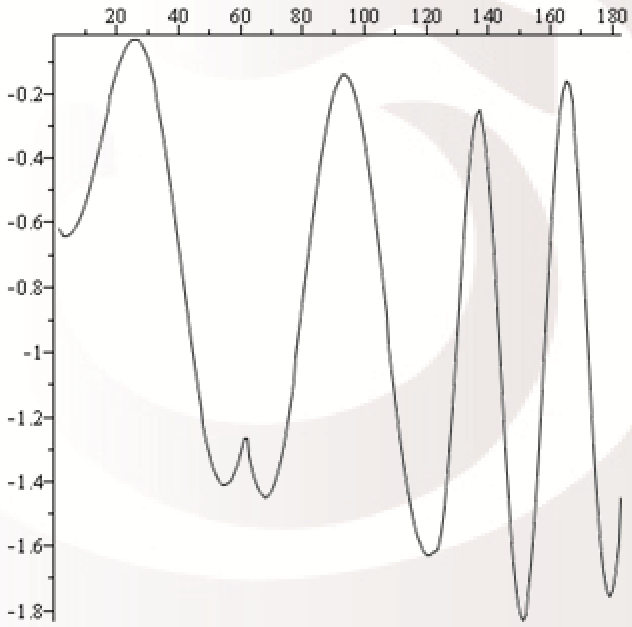
- Select a joint value $\left[Y1 = 0,\ X3 = \frac{9}{20},\ Z2 = \frac{1}{40},\ X1 = 0,\ Y2 = 0,\ Z3 = 0\right]$



Define the trajectory (2/4)

- Compute the direct kinematic solutions
- Evaluate the determinant of the Jacobian matrix
- Define a path connected 4 assembly modes in the same aspect
- Evaluate the determinant of the Jacobian matrix along the path

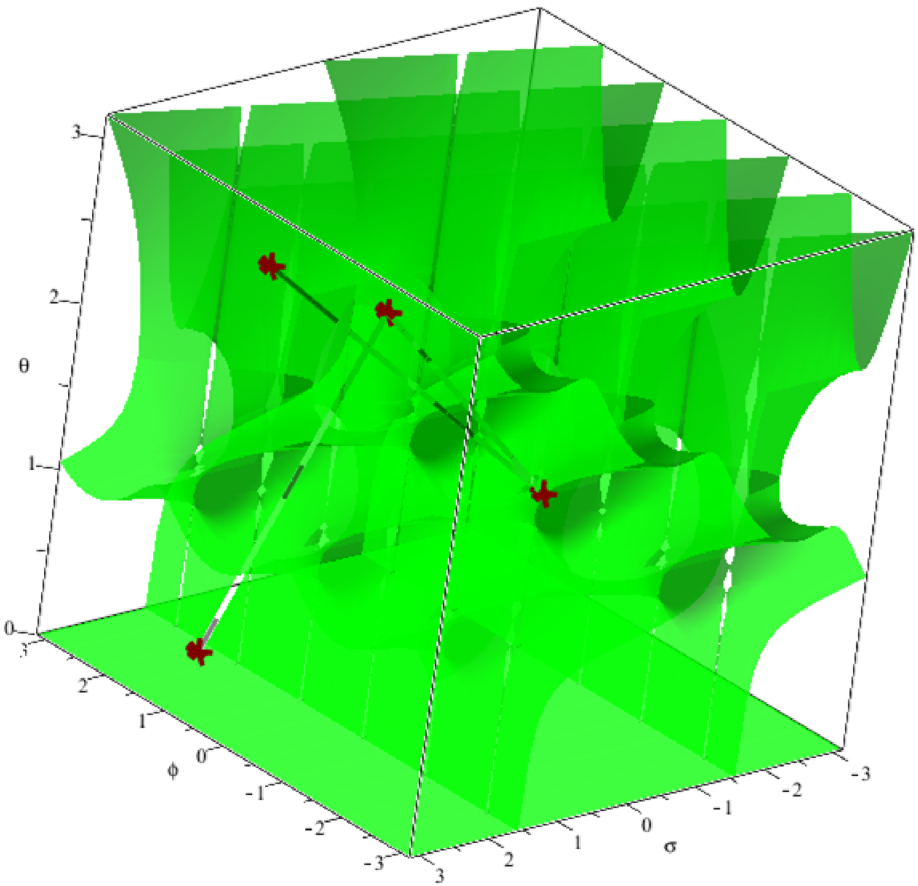


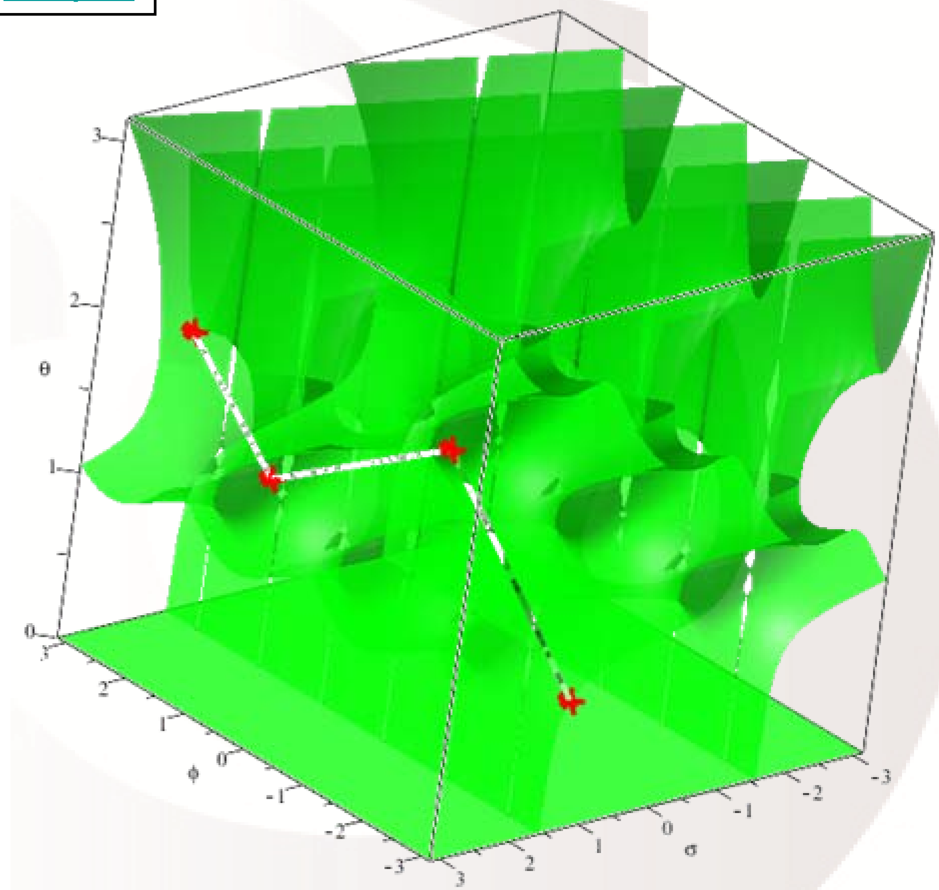


Define the trajectory (3/4)

- Path in the workspace

Maple

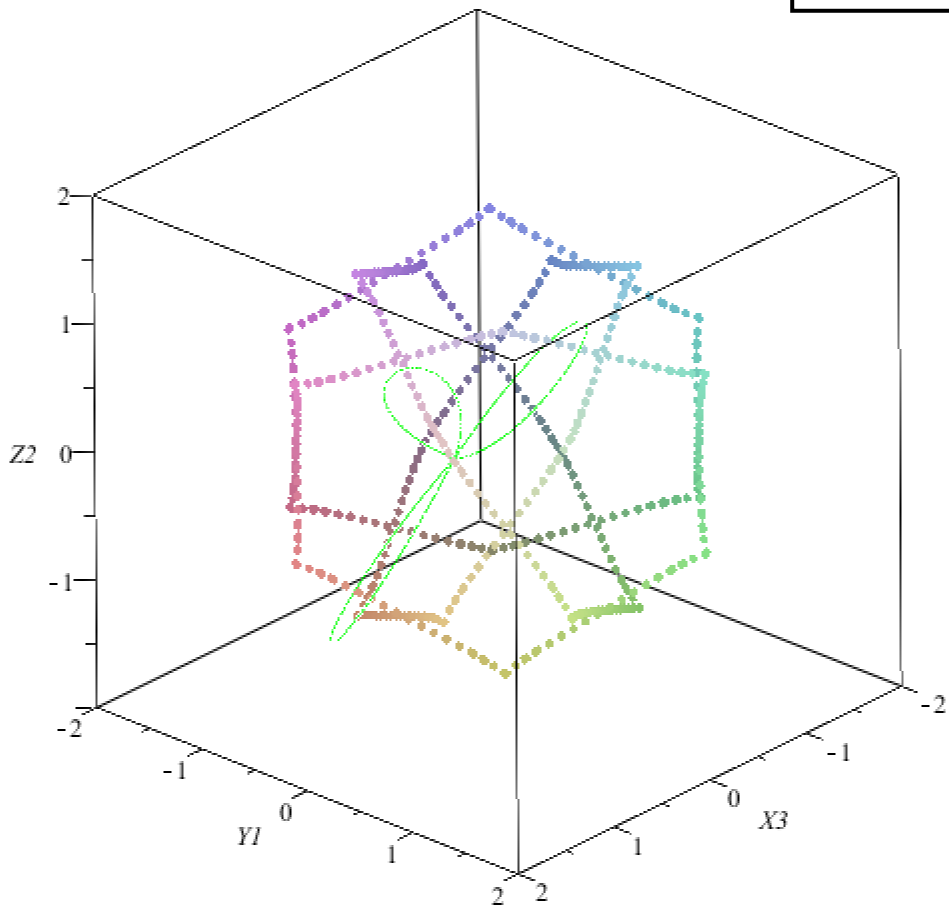


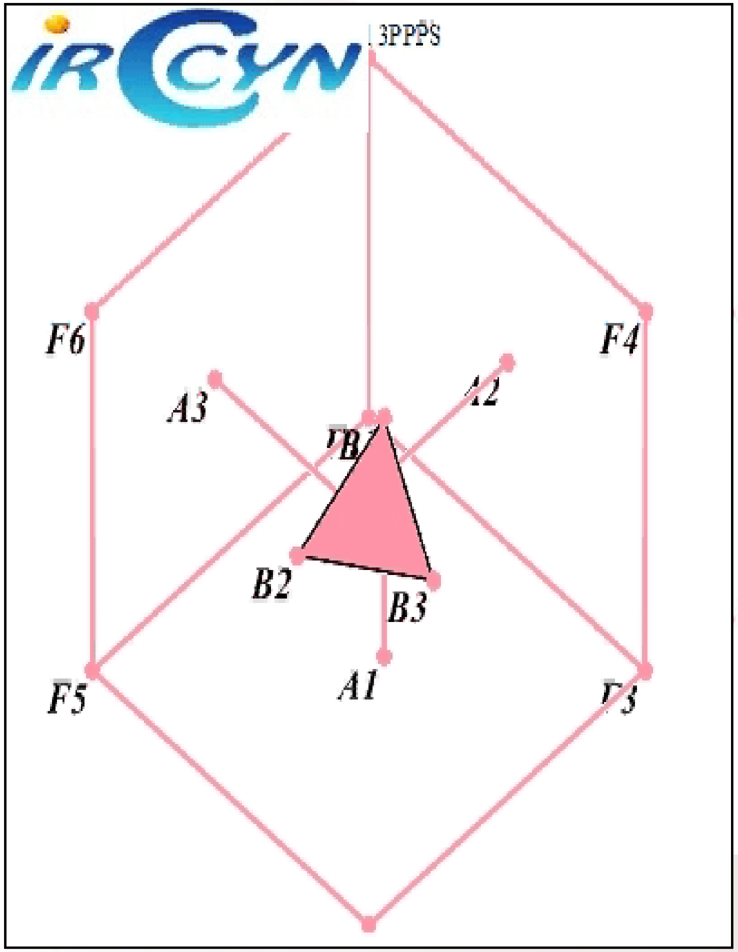


Define the trajectory (4/4)

- Path in the joint space

Maple





Problems for this mechanism

- Definition of the characteristic surface
- Computation of the basic regions and basic components
- Too many cells to depicts
- No tools to make connectivity analysis for 3D cells

Conclusions

- A set of problem was solved by using algebraic tools
- Numerical approaches (discretization, …) were replaced by using implicit definition of the curves/surfaces
- Main tools used:
  - Gröbner based elimination to eliminate variables
  - Cylindrical algebraic decomposition to represent surfaces and volumes
  - RootFinding to solve parametric systems of polynomial equations and inequalities

Future works

- Integrate tools to mix numerical approaches with algebraic one.
- Couple the cell decomposition with interval analysis based method
- Implement new approaches for mechanism with several inverse kinematic solution
- Main limitations:
  - Gröbner base elimination
  - Representation of the cells in2D/3D

Thank you for your kind attention

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