Line Drawing Interpretation in a Multi-View Context
(Supplementary Material)

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Details on the computation of $\mathcal{M}(l)$

We detail here the resolution of the quadratic minimization problem under linear constraints formulated in Section 4.3. We want to minimize the function $\epsilon$

$$
\epsilon = \sum_{i \in V} \|RP_i - SP_ip_i\|^2
$$

under the following linear constraints

$$(\forall i \in F) \sum_{j \in E} c_{ij} \lambda_j v_j = 0$$

Each vertex $P_i$ is defined in homogeneous coordinates using

$$
P_i = P_0 + \sum_{j \in E} \delta_{ij} \lambda_j v_j
$$

where $P_0$ is the reference vertex expressed in the camera frame as

$$
P_0 = \left( \alpha \hat{n} + \beta \hat{u} + \gamma \hat{v} \right) + \left( \begin{array}{c} 0_1 \\ 0_1 \end{array} \right)
$$

where $\hat{n}$ is the camera direction. $P_0$ is rewritten in a more compact form as

$$
P_0 = \omega + \sum_{k=1}^{2} \eta_k \hat{n}_k
$$

with

$$
\eta_1 \hat{n}_1 = \left( \begin{array}{c} \beta \hat{u} \\ 0 \end{array} \right)
$$

$$
\eta_2 \hat{n}_2 = \left( \begin{array}{c} \gamma \hat{v} \\ 0 \end{array} \right)
$$

$$
\omega = \left( \alpha \hat{n} \\ 1 \right)
$$

By first replacing the expression of $P_0$ from Eq. 5 into Eq. 3, and then replacing the expression of $P_i$ into Eq. 1, we can formulate $\epsilon$ as

$$
\epsilon = \sum_{i \in V} \|RP - SP_ip_i\|^2 + \sum_{k=1}^{2} \eta_k (R\hat{n}_k - S\hat{n}_kp_i) + \sum_{j \in E} \delta_{ij} \lambda_j (Rv_j - Sv_j p_i)\|^2
$$

This can also be formulated as

$$
\epsilon = \lambda^T A_0 \lambda + \lambda^T B_2 \eta + \eta^T J \eta + K^T \eta + C + B_1^T \lambda
$$
where $A_0$, $B_1$, $B_2$, $J$, $K$ and $C$ are matrices defined by

$$
(\forall (j, k) \in \mathbb{[1, |E|]^2}) A_{jk}^0 = \sum_{i \in V} (<\delta_{ij}(Rv_j - Sv_j p_i), \delta_{ik}(Rv_k - Sv_k p_i)>) > \\
(\forall (j, k) \in \mathbb{[1, |E|] \times [1, 2]^2}) B_{jk}^2 = \sum_{i \in V} 2<\delta_{ij} <Rv_j - Sv_j p_i, R\tilde{v}_k - S\tilde{v}_k p_i> \\
(\forall j \in \mathbb{[1, 2]^2}) J_{jk} = \sum_{i \in V} <R\tilde{v}_j - S\tilde{v}_j p_i, R\tilde{v}_k - S\tilde{v}_k p_i> \\
(\forall j \in \mathbb{[1, |E|]}) B_1^j = \sum_{i \in V} \delta_{ij} <Rv_j - Sv_j p_i, R\tilde{v}_j - S\tilde{v}_j p_i> \\
(\forall j \in \mathbb{[1, |E|]}) B_2^j = \sum_{i \in V} 2<\delta_{ij} <Rv_j - Sv_j p_i, R\tilde{v}_j - S\tilde{v}_j p_i> \\
(\forall j \in \mathbb{[1, |E|]}) B_3^j = \sum_{i \in V} \delta_{ij} <Rv_j - Sv_j p_i, R\omega - S\omega p_i> \\
(\forall j \in \mathbb{[1, |E|]}) B_4^j = \sum_{i \in V} 2<\delta_{ij} <Rv_j - Sv_j p_i, R\omega - S\omega p_i> \\
$$

Eq. 7 can be rewritten as

$$
\epsilon = (\lambda^T \eta^T) \left( \begin{array}{c}
A_0 \\
B_3^T \\
J
\end{array} \right) \left( \begin{array}{c}
\lambda \\
\eta
\end{array} \right) + (B_1^T K^T) \left( \begin{array}{c}
\lambda \\
\eta
\end{array} \right) + C \\
$$

where $B_3 = \frac{1}{2} B_2$. We have indeed $\lambda^T B_2 \eta = \lambda^T B_3 \eta + \eta^T B_3^T \lambda$.

This is also equivalent to

$$
\epsilon = X^T AX + BX + C \\
$$

where $X, A$ and $B$ are matrices give by

$$
X = \left( \begin{array}{c}
\lambda \\
\eta
\end{array} \right), A = \left( \begin{array}{c}
A_0 \\
B_3^T \\
J
\end{array} \right), B = \left( \begin{array}{c}
B_1^T \\
K^T
\end{array} \right)
$$

The equation (12) is the constraints defined in (2) rewriting in matrix mode using (13) and (14).

$$
DX = 0 \\
D = \left( \begin{array}{c c}
D_1 \\
0
\end{array} \begin{array}{c c}
D_2 \\
0
\end{array} \begin{array}{c c}
D_3 \\
0
\end{array} \\
\end{array} \right)
$$

$$
\left\{ \begin{array}{l}
(\forall (i, j) \in \mathbb{[1, |F|] \times [1, |E|]} ) D_{ij}^y = c_{ij} v_j^y \\
(\forall (i, j) \in \mathbb{[1, |F|] \times [1, |E|]} ) D_{ij}^y = c_{ij} v_j^y \\
(\forall (i, j) \in \mathbb{[1, |F|] \times [1, |E|]} ) D_{ij}^y = c_{ij} v_j^y \\
\end{array} \right.
$$

The new formulation of the problem is to find $\hat{X}$ such that the equation (15) is respected.

$$
\hat{X} = \arg\min_{X \in \text{Ker}(D)} (X^T AX + BX + C) \\
$$

We solve Eq. (15) by Lagrange multipliers.