of G-decompositions.

For a given graph G, find an integer k such that for every integer n ≥ k, there exists a G-decomposition of the complete graph Kn.

The problem is to determine the minimal value of k for which this is true.

Abstract:

C. M. & D. H. Department of Combinatorial Designs

Grau Decompositions and G-Designs
the definition.

1.2. Lemma: If \( G \) is a graph with \( n \) vertices, then the number of vertices in \( G \) is equal to the number of edges in \( G \).

The necessary condition.

E. Certain conditions that are necessary for \( G \) to be a graph with \( n \) vertices and \( m \) edges include:

- \( n \geq 2 \)
- \( m \leq n(n-1) \)
- \( m \geq 0 \)

Then the graph \( G \) is balanced.

The sufficient condition.

E. Certain conditions that are sufficient for \( G \) to be a graph with \( n \) vertices and \( m \) edges include:

- \( n \geq 2 \)
- \( m = \frac{n(n-1)}{2} \)
- \( m \geq 0 \)

Then the graph \( G \) is complete.

The definition of \( G \) is:

E. A graph \( G \) is a pair \( (V, E) \), where \( V \) is a set of vertices and \( E \) is a set of edges.

The definition of an edge \( e \) is:

E. An edge \( e \) is a pair of vertices in \( V \), and it connects two vertices in \( V \).

The definition of a subgraph \( H \) is:

E. A subgraph \( H \) is a graph with vertices and edges that are contained in the original graph \( G \).

The definition of a path \( P \) is:

E. A path \( P \) is a sequence of vertices in \( V \) such that each pair of consecutive vertices is connected by an edge in \( E \).

The definition of a cycle \( C \) is:

E. A cycle \( C \) is a path that starts and ends at the same vertex.

The definition of a cut-vertex \( v \) is:

E. A cut-vertex \( v \) is a vertex whose removal increases the number of connected components in \( G \).

The definition of a cut-edge \( e \) is:

E. A cut-edge \( e \) is an edge whose removal increases the number of connected components in \( G \).

The definition of a component \( K \) is:

E. A component \( K \) is an induced subgraph of \( G \) that is connected and contains no cut-vertex.
I.7. The number of edges of a tree is the number of edges of a tree.

X.
II.1. G is a complete graph with k vertices.

II.2. G is a complete graph with k vertices and k - 1 edges.

II.3. G is a graph with k vertices and k - 1 edges.

II.4. G is a directed graph with k vertices and (k - 1) arcs.

Example: If k = 1 or k = 2, then the conclusion is trivial.

Lemma: For every graph G, there exists a directed graph H such that H is isomorphic to G.

Theorem: For every graph G, there exists a complete graph K such that K is isomorphic to G.

Proof: Let G be a graph with n = k vertices and m = k - 1 edges. Then, by the Pigeonhole Principle, there must exist two vertices u and v in G such that there is an edge between u and v.

We define a new graph H by adding a new vertex w and two new edges (u, w) and (w, v) to G. Then, H is a complete graph with k + 1 vertices and k edges.

Corollary: For every graph G, there exists a complete graph K such that K is isomorphic to G.

Proof: Let G be a graph with n = k vertices and m = k - 1 edges. Then, by the Pigeonhole Principle, there must exist two vertices u and v in G such that there is an edge between u and v.

We define a new graph H by adding a new vertex w and two new edges (u, w) and (w, v) to G. Then, H is a complete graph with k + 1 vertices and k edges.

Corollary: For every graph G, there exists a complete graph K such that K is isomorphic to G.

Proof: Let G be a graph with n = k vertices and m = k - 1 edges. Then, by the Pigeonhole Principle, there must exist two vertices u and v in G such that there is an edge between u and v.

We define a new graph H by adding a new vertex w and two new edges (u, w) and (w, v) to G. Then, H is a complete graph with k + 1 vertices and k edges.
N. S. W. McDonald, [27] and in [28]. A short proof can be found in [39, 40].

If $n$ is a power of a prime, say $n = p^k$, then $\chi^2$ is a $\chi^2$-distribution with $k$ degrees of freedom.

A short proof is given in [27].

11.6. If $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges, then $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges.

11.7. If $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges.

11.8. If $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges.

11.9. If $G$ is a bipartite graph $K_{\chi, \chi}$, then $\chi^2$ is a $\chi^2$-distribution with $\chi$ degrees of freedom.

The case $\chi = 3$ will be known as the Cauchy distribution.

For a short proof see [27].

(1) If $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges.

(2) If $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges, then $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges.

(3) If $G$ is a bipartite graph $K_{\chi, \chi}$, then $\chi^2$ is a $\chi^2$-distribution with $\chi$ degrees of freedom.

(4) If $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges.

(5) If $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges, then $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges.

(6) If $G$ is a bipartite graph $K_{\chi, \chi}$, then $\chi^2$ is a $\chi^2$-distribution with $\chi$ degrees of freedom.

(7) If $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges.

(8) If $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges, then $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges.

(9) If $G$ is a bipartite graph $K_{\chi, \chi}$, then $\chi^2$ is a $\chi^2$-distribution with $\chi$ degrees of freedom.

(10) If $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges.

(11) If $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges, then $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges.

(12) If $G$ is a bipartite graph $K_{\chi, \chi}$, then $\chi^2$ is a $\chi^2$-distribution with $\chi$ degrees of freedom.

(13) If $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges.

(14) If $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges, then $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges.

(15) If $G$ is a bipartite graph $K_{\chi, \chi}$, then $\chi^2$ is a $\chi^2$-distribution with $\chi$ degrees of freedom.

(16) If $G$ is a tournament $T$ with $\chi$ vertices and $\chi$ edges, then $G$ is a transitive tournament $T$ with $\chi$ vertices and $\chi$ edges.

(17) If $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges, then $G$ is a directed circuit $C$ with $\chi$ vertices and $\chi$ edges.

(18) If $G$ is a bipartite graph $K_{\chi, \chi}$, then $\chi^2$ is a $\chi^2$-distribution with $\chi$ degrees of freedom.
\[ \begin{align*}
\text{III.6. Example: Let } & \quad \alpha = \beta, \\
\text{then } & \quad \beta = \alpha. \\
\end{align*} \]

\[ \begin{align*}
\text{III.7. In some cases, it happens that these groups are fixed by } \\
\text{the action of } & \quad \gamma. \\
\end{align*} \]

\[ \begin{align*}
\text{III.8. Theorem: Let } & \quad \alpha, \\
\text{then } & \quad \beta = \alpha. \\
\end{align*} \]

\[ \begin{align*}
\text{III.9. Lemma: Let } & \quad \alpha, \\
\text{then } & \quad \beta = \alpha. \\
\end{align*} \]
The methods expressed below are recursive methods that enable us to construct G-decompositions from smaller ones. The logic of the recursive construction is described in the text.

IV. METHOD OF COMPOSITION

The complete direct G-structure is constructed from the direct graph due to the following lemma, a little more tricky:

VI. METHOD OF COMPOSITION

The composition with parallel edges can be defined as follows:

The composition with parallel edges can be defined as follows:

The composition with parallel edges can be defined as follows:
Lemma 1.6.2. A composition of Lemma 1.6.1 and Lemma 1.6.2, we have:

\[ \text{Lemma 1.6.1, } \text{Lemma 1.6.2} \]

Lemma 1.6.3. A composition of Lemma 1.6.1 and Lemma 1.6.2, we have:

\[ \text{Lemma 1.6.1, } \text{Lemma 1.6.2} \]

Lemma 1.6.4. A composition of Lemma 1.6.1 and Lemma 1.6.2, we have:

\[ \text{Lemma 1.6.1, } \text{Lemma 1.6.2} \]

Theorem 1.7.2. A composition of Lemma 1.7.1 and Lemma 1.7.2, we have:

\[ \text{Lemma 1.7.1, } \text{Lemma 1.7.2} \]

Corollary 1.7.3. A composition of Lemma 1.7.1 and Lemma 1.7.2, we have:

\[ \text{Lemma 1.7.1, } \text{Lemma 1.7.2} \]
REFERENCES