Efficient Gathering in Radio Grids with Interference†

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We study the problem of gathering information from the nodes of a radio network into a central destination node. A transmission can be received by a node if it is sent from a distance of at most d_T and there is no interference from other transmissions. One transmission interferes with the reception of another transmission if the sender of the first transmission is at distance d_I or less from the receiver of the second transmission. In this paper we study the case $d_T = 1$ and $d_I > 1$ for two-dimensional grid networks with unit time transmissions. We prove lower bounds on the number of rounds required for any two-dimensional grid and we describe protocols for $n \times n$ grids with n odd that are optimal for odd d_I and near-optimal for even d_I.

Keywords: radio communication, interference, grids, gathering

1 Introduction

In this paper, we study a problem suggested by France Telecom concerning the design of efficient strategies to provide Internet access using wireless devices. Typically, several houses in a village wish to access a gateway (a satellite antenna) to transmit and recieve data on the Internet. To reduce the cost of the transceivers, multi-hop wireless relay routing is used. Information can be transmitted from a node to any node within distance d_T. However, a transmission can *interfere* with reception at nodes at distances up to $d_I > d_T$ from the transmitter. If two calls are mutually non-interfering, we say that they are *compatible*. The goal is to enable access to the gateway by the users with the constraint that the number of different transmissions that can be done is limited by the number of channels and time slots available. This depends on the technology used (see [**?**, **?**, **?**], for example). We will use the term *round* to mean a time slot on a given channel during which we can have only compatible transmissions or *calls*. We are interested in schedules that minimize the number of rounds.

We consider the case in which the nodes are at the points of the 2-dimensional Euclidean plane. This is the situation when the streets in the village form a grid pattern. We assume that $d_T = 1$, so a node at position (x,y) on the plane can transmit to nodes $(x,y\pm1)$ and $(x\pm1,y)$. This choice of d_T minimizes the cost of the transceivers. We study the problem of *gathering* one piece of information from each node into a central gateway node v_0 for transmission over the Internet. The inverse problem of gathering, in which each node receives a personalized message from the central node, is called *distribution* or *scattering* and can be solved by reversing the order and directions of the transmissions in a gathering protocol. We assume that all pieces of information are of the same size, such as the size of a packet, so all transmissions take one time unit. The gathering problem then becomes one of organizing the transmissions into rounds of compatible calls so that the number of rounds is minimized.

We represent the wireless network as a graph $G = (V,E)$ in which the vertices represent wireless network nodes and there is an edge between each pair of vertices that are at distance d_T or less from each other.

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When $d_T = 1$, the graph is a 2-dimensional grid. We assume that G is a square grid with $N = n^2$ vertices. We will concentrate on the case when $n = 2p+1$ is odd and the vertices are arranged symmetrically around the central vertex $v_0 = (0,0)$ with p columns of vertices on either side of the vertical axis through v_0 and p rows above and below the horizontal axis through v_0. If n is even, v_0 will be slightly off-centre. Minor modifications of the techniques described in this paper will work for n even.

In a radio network, a transmission can be received by all nodes at distance d_T or less, but in the gathering problem, only one node will forward the message towards the gateway, so we can assume that each call involves a pair (s,r) where s is the sender and r the receiver of the message. When the distance $d(s_i,r_j)$ between the sender of call (s_i,r_i) and the receiver of call (s_j,r_j) is such that $d_T < d(s_i,r_j) \leq d_I$, then the call (s_i,r_i) is too weak to be received by r_j, but it is strong enough to interfere with the reception of call (s_j,r_j) by r_j. In this paper, we use rectilinear distance (i.e., distance on the grid) as an approximation to Euclidean distance, mainly because it simplifies the analysis. Rectilinear distance is a good approximation to Euclidean distance when the ratio between d_I and d_T is small, and this is usually the case in practice.

Several examples are shown in Figure 1. In Figure 1(a), the calls (s_1,r_1) and (s_3,r_3) are compatible when $d_I = 3$ and so are the calls (s_3,r_3) and (s_4,r_4). All other pairs of calls are incompatible. For example, the call (s_1,r_1) does not interfere with reception at r_2, but (s_2,r_2) interferes with reception at r_1, so these calls are incompatible. Figure 1(b) shows the calls around v_0, which is represented by a large circle, when $d_I = 3$. None of the calls in the shaded *interference zone* are compatible with each other, so at most one of these calls can be done at any given time. The situation is slightly different when d_I is even. In Figure 1(c), all of the calls in the shaded interference zone interfere with each other as in the odd case, but the ways in which information can enter vertices on the boundary of the interference zone are more restricted. The only subset of four compatible calls from vertices on the dashed square to vertices on the boundary of the interference zone is shown with solid arrows. All other such calls can only be done two or three at a time.

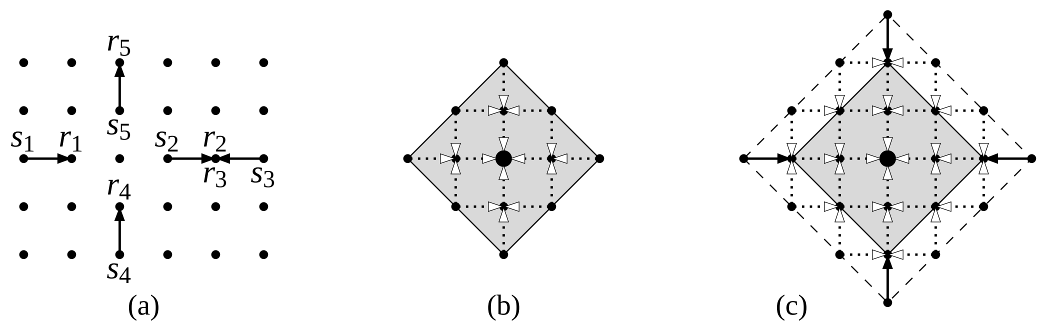


Fig. 1: Interference for $d_T = 1$ and (a) $d_I = 3$, (b) $d_I = 3$, (c) $d_I = 4$.

The one-dimensional version of the problem studied in this paper, that is, gathering into a designated node of a path, is solved in [**?**]. In [**?**], the case where there is a continuing demand is considered and systolic algorithms are given. In [**?**], general lower bounds and protocols are given for $d_T \geq 1$ for various networks such as trees and stars. Note that the case $d_T = d_I = 1$ is more constrained than the *half-duplex* 1-*port* model which has been widely studied in the literature (e.g., [**?**]). The broadcasting and gossiping problems in radio networks with $d_T = d_I = 1$ are studied in [**?**] and [**?**] respectively.

2 Lower Bounds

We say that the calls entering v_0 are *level* 1 calls, the calls entering neighbours of v_0 are level 2 calls, and so on. A call (s,r) is a level ℓ call if $d(s,v_0) = \ell$ and $d(r,v_0) = \ell - 1$. Note that if $\ell + \ell' \leq d_I + 1$ then a level ℓ call interferes with a level ℓ' call. In particular, when $d_I = 2k-1$ is odd, all calls at level k or less interfere with each other (see Figure 1(b)). When $d_I = 2k$ is even, all calls at level k or less interfere with each other and the subsets of compatible level $k+1$ calls are restricted. In particular, there is only one subset of four compatible calls at level $k+1$ (see Figure 1(c)), and this subset is incompatible with all calls in the interference zone.

Note that information at a vertex at distance d from v_0 will use at least one level ℓ call for each $1 \leq \ell \leq d$ to reach v_0. We will use N_d to denote the number of vertices of the square grid (with $N = n^2$ vertices, $n = 2p+1$) that are at distance exactly d from v_0. We have that $N_0 = 1$, $N_d = 4d$ for $d \leq p$, and $N_d = 4(2p+1-d)$ for $d > p$.

Theorem 1 *The number of rounds to gather in the $n \times n$ grid is at least $\sum_{i=1}^{k} iN_i + k(N - \sum_{i=0}^{k} N_i)$ if $d_I = 2k-1$ is odd, and $\sum_{i=1}^{k} iN_i + (k+\frac{1}{4})(N - \sum_{i=0}^{k} N_i)$ if $d_I = 2k$ is even.*

Proof Outline:

Case $d_I = 2k-1$ is odd: Information from vertices at distance $d > k$ from v_0 must use k calls inside the interference zone, all of them pairwise interfering, to reach v_0. Information from vertices at distance $d \leq k$ from v_0 must use d calls inside the interference zone. So, the total number of rounds is at least $\sum_{i=1}^{k} iN_i + k(N - \sum_{i=0}^{k} N_i) = k(N-1) - \sum_{i=1}^{k}(k-i)N_i$. For example, for $d_I = 3$ (and $p \geq 2$), we need $2N-6$ rounds.

Case $d_I = 2k$ is even: The lower bound on the number of calls inside the interference zone is the same as for the odd case: $\sum_{i=1}^{k} iN_i + k(N - \sum_{i=0}^{k} N_i)$. The difference from the odd case is that the level $k+1$ calls are also restricted, so that the information from vertices at levels $k+1$ and greater requires $\frac{1}{4}(N - \sum_{i=0}^{k} N_i)$ additional rounds for a total of $\sum_{i=1}^{k} iN_i + (k+\frac{1}{4})(N - \sum_{i=0}^{k} N_i)$ rounds. □

3 Gathering Protocol

In this section, we describe protocols that achieve or come close to the lower bounds of Theorem 1. We say that a vertex is *active* if it has information that needs to be sent or forwarded to v_0. A vertex that is not active and is not on any path that will be used to forward information in future rounds is *dormant*.

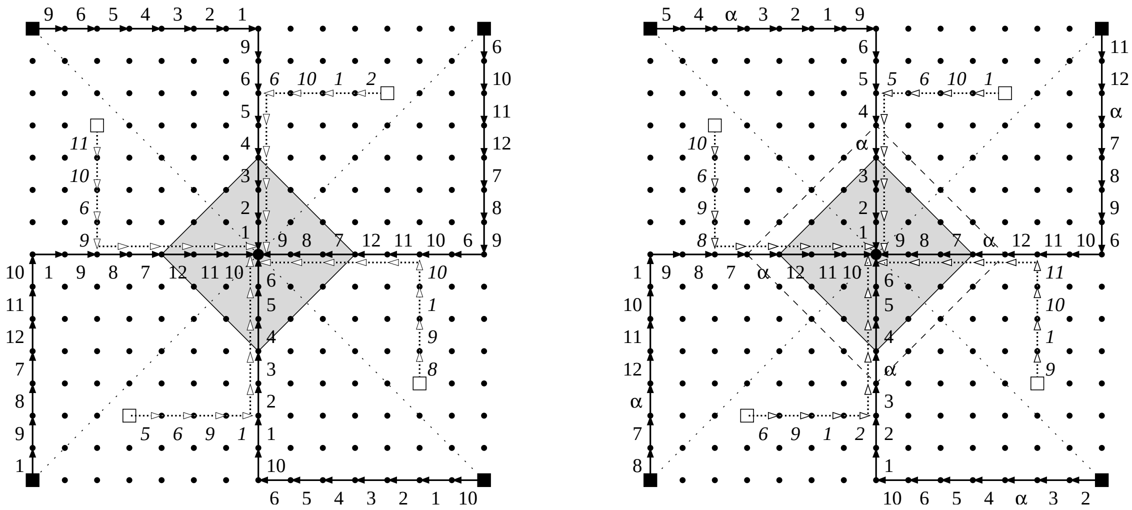
Theorem 2 *Suppose that $n = 2p+1$ and $d_I = 2k-1$ are odd. Then gathering in the $n \times n$ grid can be completed in $\sum_{i=1}^{k} iN_i + k(N - \sum_{i=0}^{k} N_i)$ rounds and this is optimal.*

Proof Outline: The general idea is to organize the calls into stages of $4k$ rounds. In each stage, four active vertices that are symmetrically arranged around v_0 are chosen. Information is forwarded along paths from each of these vertices to v_0 for $4k$ rounds. At the end of the stage, the four vertices become dormant, all other vertices on the four paths have sent one piece of information and recieved another, and v_0 has received four more pieces of information. The paths are chosen in such a way that the calls in each round are compatible. We iterate this procedure until the only remaining active vertices are inside the interference zone around v_0. It takes $\sum_{i=1}^{k} iN_i$ sequential calls inside the interference zone to move the remaining information into v_0. □

Figures 2(a) shows two examples of stages, one with solid arrows and the other with dotted arrows. The numbers indicate the rounds during which the calls are made. Stages are executed sequentially, so that at any given time, only one set of paths is being used. It is not hard to verify that the calls on the solid paths are compatible in each round. Similarly, the calls of the dotted paths are compatible. In general, paths in the upper right quadrant go towards the closest axis and then along the axis to v_0. The paths in the three other quadrants are obtained by rotation.

Theorem 3 *Suppose that $n = 2p+1$ and $d_I = 2k$ is even. Then gathering in the $n \times n$ grid can be completed in $\sum_{i=1}^{k} iN_i + (k+\frac{1}{4})(N - \sum_{i=0}^{k} N_i) + k - 1$ rounds.*

Proof Outline: If d_I is even, the paths are similar to the odd case except for the level $k+1$ calls which enter vertices on the boundary of the shaded interference zone from vertices on the dashed square (see Figure 1(c)). These calls require an extra round in each stage. The most efficient way to do the level $k+1$ calls is to use the four calls labelled α in the example in Figure 2(b). This works for all paths starting at level $k+2$ or greater. Most of the paths that start at the $4k+4$ vertices on the dashed box can only be done two at a time without interference. Careful use of the calls labelled α allows us to do twelve of these calls three at a time for a total of $k(4k+4)+2k$ rounds to move the information of these $4k+4$ vertices to v_0. □



(a) (b)

Fig. 2: Gathering stages for $d_T = 1$ and (a) $d_I = 5$, (b) $d_I = 6$.

Addendum

We have recently improved the lower bound for the case d_I even to match the upper bound of Theorem 3. The proof of this lower bound and also the complete proofs of Theorems 2 and 3 are considerably more complicated than the proof outlines presented here. All details will appear in the full version of this paper.

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