The CRC Handbook of Combinatorial Designs

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1 Grooming

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1.1 Definitions and Examples

1.1 **Remark** Traffic grooming in networks refers to group low rate traffic into higher speed streams (containers) so as to minimize the equipment cost [11, 7, 13, 12, 8, 9]. There are many variants according to the type of network considered, the constraints used and the parameters one wants to optimize which give rise to a lot of interesting design problems (graph decompositions).

To fix ideas, suppose that we have an optical network represented by a directed graph G (in many cases a symmetric one) on n vertices, for example a unidirectional ring $\vec{C}_n$ or a bidirectional ring C_n^*. We are given also a traffic matrix, that is a family of connection requests represented by a multi-digraph I (the number of arcs from i to j corresponding to the number of requests from i to j). An interesting case is when there is exactly one request from i to j; then $I = K_n^*$. Satisfying a request from i to j consists in finding a route (dipath) in G and assigning it a wavelength. The grooming factor, g, means that a request uses only $1/g$ of the bandwidth available on a wavelength along its route. Said otherwise, for each arc e of G and for each wavelength w, there are at most g dipaths with wavelength w which contain the arc e.

During the 90's, a lot of research has concentrated in minimizing the number of wavelengths used in the network. In the mean time, the bandwidth of each wavelength (> 10 Gbit/s), the number of wavelengths per fiber (> 100) and the number of fibers per cable (> 100) have exploded, thus reporting the operational cost of a wavelength into the terminal equipment cost: filters, optical cross-connect, add/drop multiplexer (ADM),.... For example, in a node of an optical network we place an ADM only for those wavelengths carrying a request which has to be added or dropped in this node. So nowadays the objective is to minimize the total number of ADMs in the network, which is a challenging issue.

1.2 **Definition** [5] Grooming problem: Given a digraph G (network), a digraph I (set of requests) and a grooming factor g, find for each arc $r \in I$ a path $P(r)$ in G, and a partition of the arcs of I into subgraphs I_w, $1 \leq w \leq W$, such that $\forall e \in E(G)$, $\text{load}(I_w, e) = |\{P(r); r \in E(I_w); e \in P(r)\}| \leq g$. The objective is to minimize $\sum_{w=1}^{W} |V(I_w)|$, and this minimum is denoted by $A(G, I, g)$.

1.3 **Definition** TT_n is a transitive tournament on n vertices, that is the digraph with arcs $\{(i,j) \mid 1 \leq i < j \leq n\}$. We denote $\{a,b,c\}$ the TT_3 with arcs $\{a,b\}$, $\{b,c\}$, and $\{a,c\}$.

1.4 **Remark** When $G = P_n^*$, the shortest path from i to j is unique, and we can split the requests into two classes, those with $i < j$ and those with $i > j$. Therefore the grooming problem for P_n^* can be reduced to two distinct problems on $\vec{P}_n$. In particular we have $A(P_n^*, K_n^*, g) = 2A(\vec{P}_n, TT_n, g)$.

1.5 **Example** $A(\vec{P}_7, TT_7, 2) = 20$, and the partition consists of 6 subgraphs, the 5 TT_5 {2,4,5}, {3,4,6}, {1,5,6}, {2,6,7}, and {1,4,7}, plus the union of two TT_3 {1,2,3}+{3,5,7}.

1.6 Theorem [1] When n is odd, $A(\vec{P}_n, TT_n, 2) = \lceil (11n^2 - 8n - 3)/24 \rceil$; When n is even, $A(\vec{P}_n, K_n, 2) = (11n^2 - 4n)/24 + \varepsilon(n)$, where $\varepsilon(n) = 1/2$ when $n \equiv 2, 6 \pmod{12}$, $\varepsilon(n) = 1/3$ when $n \equiv 4 \pmod{12}$, $\varepsilon(n) = 5/6$ when $n \equiv 10 \pmod{12}$, and $\varepsilon(n) = 0$ when $n \equiv 0, 8 \pmod{12}$.

1.7 Remark In a unidirectional cycle $\vec{C}_n$, the path from i to j is unique. Wlog we can assign the same wavelength to the two requests (i, j) and (j, i), then the two associated paths contain each arc of $\vec{C}_n$. Therefore the load condition becomes $|E(I_w)| \leq g$ and the grooming problem becomes:

1.8 Definition [5] Grooming problem for $G = \vec{C}_n$: given n and g, find a partition of I into subgraphs B_w, $1 \leq i \leq W$, such that $|E(B_w)| \leq g$, which minimizes $\sum_{w=1}^{W} |V(B_w)|$. The minimum value is $A(\vec{C}_n, I, g)$.

1.9 Remark The partition of Definition 1.2 is obtained by associating to each B_w of the partition of Definition 1.8 its symmetric digraph B_w^* and letting $I_w = B_w^*$.

1.10 Example $A(\vec{C}_4, K_4^*, 3) = 7$, using a partition of K_4 consisting of the K_3 $\{1, 2, 3\}$ and the $K_{1,3}$ with edges $\{1, 4\}$, $\{2, 4\}$, and $\{3, 4\}$. $A(\vec{C}_7, K_7^*, 3) = 21$ using a (7,3,1) design (steiner triple system) and $A(\vec{C}_{13}, K_{13}^*, 6) = 52$ using a (13,4,1) design.

1.11 Theorem [2] $A(\vec{C}_n, K_n^*, 3) = n(n-1)/2 + \varepsilon_3(n)$, where $\varepsilon_3(n) = 0$ when $n \equiv 1, 3 \pmod 6$, $\varepsilon_3(n) = 2$ when $n \equiv 5 \pmod 6$, $\varepsilon_3(n) = \lceil n/4 \rceil + 1$ when $n \equiv 8 \pmod{12}$, and $\varepsilon_3(n) = \lceil n/4 \rceil$ otherwise.

1.12 Theorem [10] $A(\vec{C}_n, K_n^*, 4) = n(n-1)/2$.

1.13 Theorem [4] $A(\vec{C}_n, K_n^*, 5) = 4 \lfloor n(n-1)/10 \rfloor + \varepsilon_5(n)$, where $\varepsilon_5(n) = 0$ when $n \equiv 0, 1 \pmod 5$, $n \neq 5$, $\varepsilon_5(5) = 1$, $\varepsilon_5(n) = 2$ when $n \equiv 2, 4 \pmod 5$, $n \neq 7$, $\varepsilon_5(7) = 3$, $\varepsilon_5(n) = 3$ when $n \equiv 3 \pmod 5$, $n \neq 8$, and $\varepsilon_5(8) = 4$.

1.14 Theorem [3]

When $n \equiv 0 \pmod 3$, then $A(\vec{C}_n, K_n^*, 6) = \lceil n(3n-1)/9 \rceil + \varepsilon_6(n)$, where $\varepsilon_6(n) = 1$ when $n \equiv 18, 27 \pmod{36}$, and $\varepsilon_6(n) = 0$ otherwise, except for $n \in \{9, 12\}$ and some possible exceptions when $n \leq 2580$.

When $n \equiv 1 \pmod 3$, $A(\vec{C}_n, K_n^*, 6) = \lceil n(n-1)/3 \rceil + \varepsilon_6(n)$, where $\varepsilon_6(n) = 2$ when $n \equiv 7, 10 \pmod{12}$, and 0 otherwise, except for $A(\vec{C}_7, K_7^*, 6) = 17$, $A(\vec{C}_{10}, K_{10}^*, 6) = 34$, and $A(\vec{C}_{19}, K_{19}^*, 6) = 119$.

When $n \equiv 2 \pmod 3$, then $A(\vec{C}_n, K_n^*, 6) = (n^2 + 2)/3$, except possibly for $n = 17$.

1.15 Remark In another grooming problem (see [6]), the requests can be routed via different pipes. Each pipe contains at most g requests, and the objective is to minimize the total number of pipes (as equipments are placed only at the terminal nodes of the pipe). Thus, given a digraph I (requests) and a grooming factor g, the problem consists in finding a virtual multi-digraph H and, for each arc $r \in I$, a path $P(r)$ in H such that $\forall e \in E(H)$, $\mathrm{load}(I, e) \leq g$. The objective is to minimize $|E(H)|$, and the minimum is denoted by $T(I, g)$.

1.16 Example When $I = K_4^*$ and $C = 2$, then $H = C_4^*$. Requests $(i, i+1)$ (resp. $(i, i-1)$) are routed via arc $(i, i+1)$ (resp. $(i, i-1)$), requests $(1, 3)$ and $(3, 1)$ are routed clockwise, and $(2, 4)$ and $(4, 2)$ counterclockwise.

1.17 Remark For $C = 2$ the problem can be reduced to a partition of K_n^* or $(K_n - e)^*$ in TT_3 (See Directed Design or Mendelsohn's Designs). For $C = 3$ the result follows from the existence of a $PBD(n, \{3, 4, 5\})$ for $n \neq 6, 8$ (see chapter PBD).

1.18 Theorem [6] $T(K_n^*, 2) = \lceil 2n(n-1)/3 \rceil$ and $T(K_n^*, 3) = n(n-1)/2$.

1.2 See Also

§???	Directed designs.
§???	Graph decompositions
§???	Mendelsohn designs

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