Construction of Large Packet Radio Networks [1]

Jean-Claude Bermond, Pavol Hell, and Jean-Jacques Quisquater

Abstract.

We outline constructions of packet radio networks (with time division multiplexing) that achieve much better parameters than those previously proposed. Given the desired diameter and number of slots per time frame, our networks seek to maximize the possible number of users. We model this as a problem of constructing large graphs with given diameter and chromatic index, and relate it to extant work on large graphs with given maximum degree and diameter.

Index terms.

Arc-coloring, chromatic index, computer and communication networks, degree, diameter, edge-coloring, interconnection networks, packet radio networks, time division multiplexing.

[1] J-C. Bermond is with CNRS, Université de Nice-Sophia Antipolis, France.

P. Hell is with Simon Fraser University, Vancouver, Canada.

J-J. Quisquater is with Philips Research Laboratory, Louvain-la-Neuve, Belgium.

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1 Introduction

In a recent article, Prohazka [21] considered the problem of designing packet radio networks which use time division multiplexing and have a diameter constraint. In particular, he investigated the maximum possible number of users of such a network, with diameter D and with f time slots per frame. Since a user cannot transmit and/or receive more than one packet at a time, this amounts to assigning time slots to channels (one channel for each ordered pair of communicating users) in such a way that the channels involving a particular user in either transmission or reception are all assigned different time slots. (Frequency division multiplexing was also used to prevent interference between users, but the number of bands was assumed sufficiently large to assign different frequency bands to all channels operating in the same time slot.) Other work on packet radio networks may be found in [3], [19], [25].

The above problem can be modeled by directed graphs: the users correspond to vertices, the channels to arcs, and assigning time slots corresponds to coloring the arcs so that incident arcs have different colors. Thus the problem is to determine the maximum number of vertices of a digraph with diameter D whose arcs can be colored with f colors.

Prohazka offered some general constructions of such digraphs. The number of vertices of his digraphs grows asymptotically as, roughly, $(f/2)^{D/2}$. He also compiled a table of such digraphs with values of D up to ten and f up to twenty.

Prohazka's constructions all have an additional property. Namely, the digraphs constructed are all symmetric, i.e., all channels are bidirectional. This can be stated as a separate problem: Find the maximum number of vertices of a graph with diameter D whose edges can be colored with q colors in such a way that any two adjacent edges have different colors.

Our purpose in this paper is to point out the similarity of these problems with those of constructing large digraphs with diameter D and maximum in- and out-degree d, and large graphs with diameter D and maximum degree Δ. Since there is an extensive literature on these subjects, we can exploit this similarity to derive substantial improvements of the results of [21].

Indeed according to a well known theorem of Vizing, (see [10]), the edges of a graph of maximum degree Δ can be colored with at most $\Delta + 1$ colors (and clearly Δ colors are needed). Thus we can use any of the known constructions, e.g., from [2], [4], to considerably improve the bounds in [21]. Furthermore, it turns out that in many cases the edges of the largest known graphs with degree Δ and diameter D can be colored with Δ colors. This is a further improvement of the constructions, and one which, we shall argue, will be very difficult to better.

We shall point out that while the graph constructions are more natural, and the bidirectional networks perhaps more desirable, substantially larger numbers of users can be achieved with digraphs. A particularly useful class of digraphs in this context turns out to be the class of balanced digraphs – namely those in which neither the indegree nor the outdegree of any vertex exceeds half of the number of available colors. We shall use a variant of Vizing's theorem to conclude that a digraph with maximum in- and outdegree d admits an arc-coloring with $2d+2$ colors. This allows us to use known constructions of digraphs with maximum in- and -out- degree d and diameter D. Also in this case it is often true that for the largest known constructions of digraphs with maximum in- and out- degree d and diameter D, there is an arc-coloring with $2d$ colors (and clearly no fewer colors can suffice). For instance, $2d$ colors are sufficient for the de Bruijn and Kautz networks, which have the distinction of being the largest known general family of digraphs with maximum in- and out- degree d and diameter D. As a consequence we shall obtain digraphs with $(f/2)^D + (f/2)^{D-1}$ vertices and diameter D, whose arcs can be colored with f colors.

2 Definitions

The *packet radio network* is modeled by a *digraph* $G = (V, E)$ in which the *vertices* (elements of the set V) represent the users, and there is an *arc* (or directed edge, i.e., an element of the set E) xy from vertex x to vertex y to indicate that user x can transmit to user y. Thus the arcs represent unidirectional channels; a bidirectional channel joining x and y may be represented by two arcs xy and yx. If all channels are bidirectional, the resulting digraph is *symmetric*, i.e., it has with each arc xy also the arc yx. In this case we may consider instead an undirected *graph* with an edge $\{x, y\}$ replacing the two arcs xy and yx.

We are only interested in digraphs which are *strongly connected*, i.e., digraphs in which there exists a directed path from any vertex x to any vertex y. The length (number of arcs) of a shortest directed path from x to

y is called the *distance* ¿from x to y and denoted by $d(x, y)$. The *diameter* of a digraph G is the maximum distance $d(x, y)$ over all vertices x and y of G. The *outdegree* of a vertex x in the digraph G is the number of arcs xy in G, and is denoted by $d^+(x)$. The *indegree* $d^-(x)$ is defined analogously. Thus $d^+(x)$ is the number of channels x can transmit to and $d^-(x)$ the number of channels x can receive from. An *arc-coloring* of a digraph G is a mapping assigning colors (labels) to the arcs of G in such a way that two arcs having a common vertex obtain different colors. The *arc-chromatic index* of G is the minimum number of colors which make an arc-coloring of G possible.

An $(f, D)_C$–digraph is a digraph with diameter at most D and arc-chromatic index at most f. We denote, by $n_C(f, D)$, the maximum number of vertices of an $(f, D)_C$–digraph. The problem mentioned in the introduction is to evaluate $n_C(f, D)$. (Our notation would be more consistent with the rest of the paper if we denoted $n_C(f, D)$ by $n^t_C(f, D)$; we shall not do this to remain consistent with the notation of [21], but it may be helpful to be aware of this.)

As mentioned above, all of the networks constructed in [21] are bidirectional, i.e., correspond to symmetric digraphs. The construction of symmetric $(f, D)_C$–digraphs may be considered as a separate problem. It is somewhat more convenient to discuss this problem in the context of undirected graphs, as explained above. All the above definitions have obvious analogues for undirected graphs: For a connected graph G, the distance $d(x, y)$ is the length of a shortest path from x to y, and the *diameter* of G is the greatest $d(x, y)$ for all pair of vertices x, y of G. The *maximum degree* of G is the largest number of edges incident with any vertex of G, and the *edge-chromatic index* of G is the minimum number of colors which can be assigned to the edges of G so that two edges having a common vertex obtain different colors.

A $(q, D)_C$–graph is a graph with diameter at most D and edge-chromatic index at most q. We denote by $n^\star_C(q, D)$ the maximum number of vertices in a $(q, D)_C$–graph.

3 Graphs

A (Δ, D)–graph is a graph with diameter at most D and maximum degree at most Δ. We denote by $n^\star(\Delta, D)$ the maximum number of vertices in a (Δ, D)–graph. We begin by exploring the relationship between (Δ, D)–graphs and $(q, D)_C$–graphs. Indeed, the construction of large (Δ, D)–graphs and the evaluation of $n^\star(\Delta, D)$ is a challenging problem that has been widely considered in recent years, see, for example, the survey [4], or the special issue [26]. It is clear that in any edge-coloring of a graph G, a vertex of degree Δ needs at least Δ colors for its incident edges. Thus the edge-chromatic number q of G is at least the maximum degree Δ. On the other hand, a theorem of Vizing (see [10], Chapter 6), ensures that the edge-chromatic index q is a most $\Delta + 1$ (in other words, any graph G with maximum degree $q - 1$ admits an edge-coloring with q colors). Thus we have the following observations:

Proposition 1. $n^\star(q - 1, D) \leq n^\star_C(q, D) \leq n^\star(q, D)$. $\square$

Proposition 1 is useful, on the one hand, to obtain an upper bound on $n^\star_C(q, D)$ by using the upper bound on $n^\star(\Delta, D)$ known as the Moore bound [8]. The Moore bound is obtained by noting that there are, in a (Δ, D)–graph, at most $\Delta(\Delta-1)^{k-1}$ vertices at distance k ¿from any fixed vertex. Thus,

$$n^\star(\Delta, D) \leq 1 + \Delta + \Delta(\Delta-1) + ... + \Delta(\Delta-1)^{D-1}.$$

We conclude that

$$n^\star_C(q, D) \leq 1 + q + q(q-1) + ... + q(q-1)^{D-1}.$$

On the other hand, we also use Proposition 1 by appealing to any known construction of large $(q-1, D)$–graphs to obtain a lower bound on $n^\star_C(q, D)$. This improves practically all the lower bounds of [21]. For instance, the value $n^\star_C(10, 10)$, bounded below by 206 660 in [21], is at least $n^\star(9, 10)$ according to Proposition 1. Thus we may use a construction due to Campbell [12], showing this value to be bounded below by 19 845 936.

Further improvement will occur with many constructions of large (Δ, D)–graphs in which we can show an edge-coloring with Δ (rather than $\Delta + 1$) colors. An example of a (Δ, D)–graph that is colorable with Δ colors appears in Figure 1, where a largest possible $(3, 3)$–graph on 20 vertices is edge-colored with 3 colors. (Cf. [4]).) Thus $n^\star_C(3, 3) = n^\star(3, 3) = 20$ (to be compared with the bound of 14 in [21], Figure 4).

(Figure 1 somewhere here)

A Δ–coloring always exists for a bipartite (Δ, D)–graph. It is well known that the maximum degree and the edge-chromatic index of a bipartite graph are equal [10]. Thus $n^\star_C(q,D)$ is at least as large as the number of vertices of any bipartite (q,D)–graph. This is often useful to know, as many well known network constructions are bipartite – such as the hypercube, or the star graph [1]. Taking large bipartite (q,D)–graphs from [13] or [14], we further improve the lower bounds on many $n^\star_C(q,D)$; for instance, we obtain $n^\star_C(10,10) \geq 47\,059\,200$.

It is also shown in [18] that there exist bipartite (q,D)–graphs with $2(q/2)^{D-1} + 2(q/2)^{D-3}$ vertices. Thus we can conclude that $n^\star_C(q,D) \geq 2(q/2)^{D-1} + 2(q/2)^{D-3}$. This is an improvement over the asymptotic bounds discussed in section IV of [21], which are roughly of the order $(q/2)^{D/2}$ as q tends to infinity. (Cf. also [13] for a slightly smaller improvement.)

It has been our experience that in most examples of the largest known (Δ,D)–graphs, an edge-coloring with Δ colors is possible. This is the case in particular for the undirected de Bruijn and Kautz graphs. The latter are $(2d,D)$-graphs with $d^D + d^{D-1}$ vertices, obtained by ignoring the arc-directions of the Kautz digraphs $K(d,D)$ defined in the next section. In particular, we conclude (cf. Corollary 5):

Corollary 2.
$(q/2)^D + (q/2)^{D-1} \leq n^*_C(q,D) \leq q(q-1)^{D-1} + q(q-1)^{D-2} + ... + q + 1.$ □

The lower bound in the above corollary is a substantial improvement of the constructions in [21]. Even though it appears quite far ¿from the theoretical upper bound (derived from the Moore bound), we are confident that, in full generality, further improvements will not be easy. The reason for this is that any improved constructions would in particular provide better bounds for the much studied parameter $n^*(\Delta, D)$, via Proposition 1. This does not mean, of course, that *particular* values of $n_C^*(q, D)$, or even infinite sequences of such values, could not be easily estimated better than the above lower, or upper, bounds. In all the cases below, the best known (Δ, D)—graphs have been shown colorable with Δ colors:

- the $(2k+1, 2m)$–graphs $(kgeq2)$ with $k^m(k+1)^m$ vertices, known as the *sequence graphs* (cf. [15]), [17]
- the $(p+1, 2)$–graphs with p^2+p+1 vertices, p a prime power, arising from projective planes (cf. [4]), [23]
- some of the best known $(3, D)$–graphs, in particular those with $D = 4$ and 38 vertices, $D = 5$ and 70 vertices, [17], $D = 6$ and 128 vertices, $D = 7$ and 184 vertices, and $D = 8$ and 320 vertices, [22].
- the $(3, D)$–graphs, for $D = 2^k - 2$, with $2 \cdot 2^D$ vertices obtained by substituting an edge in each vertex of the de Bruijn graph $B(2, D)$, cf. below (and similar (Delta, D)–graphs when Δ is one plus a power of two), [7].

We conclude this section by noting that it is *not* always the case that the best (Δ, D)–graphs are Δ–edge-colorable. The unique largest $(3, 2)$–graph is the (ten-vertex) Petersen graph, cf. [10]; it is known that it is not 3–edge-colorable. There is no graph on 9 vertices with all degrees equal to 3, and if a nine-vertex graph has a vertex of degree two or less, then counting like in the Moore bound, starting from this vertex, we see that it cannot have diameter 2, and hence $n^{\star}(3,2)$ is at most eight. In fact, there is a $(3,2)_C$–graph with eight vertices, obtained from the cycle 0,1, ...,7,0 by adding the edges {0,4}, {1,5}, {2,6}, {3,7}. Therefore $n^{\star}_C(3,2) = 8$.

4 Digraphs

Let us now consider the construction of $(f,D)_C$–digraphs, i.e., digraphs with diameter at most D and arc-chromatic number at most f.

Since graphs correspond to symmetric digraphs, we have already constructed large $(f,D)_C$–digraphs. Let $n^s_C(f,D)$ denote the maximum number of vertices in a *symmetric* $(f,D)_C$–digraph. Any $(q,D)_C$–graph yields a symmetric $(2q,D)_C$–digraph by replacing each edge $\{x,y\}$ by the arcs xy and yx. Moreover, an edge-coloring of the graph gives rise to an arc-coloring of the digraph with twice as many colors. Hence,

$n_C(2f,D) \geq n^s_C(2f,D) = n^{\star}_C(f,D)$.

Moreover, we expect to be able to construct even larger $(f,D)_C$–digraphs when the symmetry condition is removed. Note that we know from Corollary 2, that, roughly, $n^s_C(2f,D) \leq f^D$. It is our objective to construct (non-symmteric) $(f,D)_C$–digraphs larger than this upper bound for symmetric digraphs.

Consider first another special class of $(f,D)_C$–digraphs. We shall say that an $(f,D)_C$–digraph is *balanced* if both $d^+(x) \leq f/2$ and $d^-(x) \leq f/2$ for every vertex x. Let $n^b_C(f,D)$ be the maximum number of vertices in a balanced $(f,D)_C$–digraph. A *(d,D)–digraph* is a digraph G with diameter D and maximum in- and out- degree d. Note that while this notion is analogous with the notion of a (Δ, D)–graph, we don't have here the close analogy we had between (Δ, D)–graphs and $(\Delta, D)_C$–graphs. Specifically, the reader should be aware that a $(2d, D)$–digraph has maximum in- and out- degree $2d$, while an $(2d, D)_C$–digraph has, in particular, each vertex with $d^+ + d^-$ bounded by $2d$. Thus, in some sense, a (d, D)–digraph is automatically balanced, and we recognize this by writing, contrary to the usual notation, [4], $n^b(d, D)$ for the maximum number of vertices of a (d, D)–digraph.

We then have the following very close relationship of $n^b_C(f, D)$ and $n^b(d, D)$.

Proposition 3. $n^b(f-1, D) \leq n^b_C(2f, D) \leq n^b(f, D)$.

Proof. The first inequality follows, as before, from a version of Vizing's theorem [10]. In fact, any $(f-1, D)$–digraph with edge-directions ignored is an undirected multigraph with edge multiplicities at most two. The maximum degree in this multigraph is at most $2f-2$ and Vizing's theorem implies that the edges of the multigraph, and hence also the arcs of the digraph, can be colored with $2f$ colors. Thus any $(f-1, D)$–digraph is also a balanced $(2f, D)_C$–digraph. The second inequality is trivial. It simply says that in a balanced $(2f, D)_C$–digraph both in- and out- degrees are at most f. $\square$

In view of Proposition 3, we can again use the known constructions of large (d, D)–digraphs. The best *general* construction among these is the following (we assume $d \geq 2$): The *de Bruijn digraph* $B(d, D)$ has as its vertices all strings of length D over the alphabet $\{0, 1, ..., d-1\}$; there is an arc from a vertex $a_1a_2a_3...a_D$ to all vertices $a_2a_3...a_Da$ with a in $\{0, 1, ..., d-1\}$. The *Kautz digraph* $K(d, D)$ has as its vertices all those strings of length D over the alphabet $\{0, 1, ..., d\}$ in which consecutive characters are distinct;

there is an arc from a vertex $a_1a_2a_3...a_D$ to all vertices $a_2a_3...a_Da$ with a from $\{0,1,...,d\}$ and distinct from a_D. It is easy to see (cf. [5], [16], [24]) that both $B(d,D)$ and $K(d,D)$ are (d,D)–digraphs. Moreover, the digraph $B(d,D)$ has d^D vertices and the digraph $K(d,D)$ has d^D+d^{D-1} vertices. We immediately obtain, via the proof of Proposition 3, large (balanced) $(2d+2,D)_C$–digraphs. However, we were able to prove (see [6]) that $B(d,D)$ and $K(d,D)$ are in fact $(2d,D)_C$–digraphs:

Proposition 4. Both $B(d,D)$ and $K(d,D)$ (for $D \geq 2$) have arc-chromatic index $2d$. □

¿From this we find a good estimation of the maximum number of vertices in a balanced $(f,D)_C$–digraph: Consider the Moore bound for $n^b(d,D)$, obtained by noting that in a (d,D)–digraph there are at most d^k vertices of distance k from a fixed vertex. (It is known here that the Moore bound cannot be attained, and thus the upper bound given below is strict, unless d or D is equal to one, [11], [20]).

Corollary 5. $f^D+f^{D-1} \leq n_C^b(2f,D) \leq f^D+f^{D-1}+...+1.$ □

It follows, e.g., that the Kautz digraph $K(3,3)$ on 36 vertices (cf. Figure 2) yields $n_C(6,3) \geq 36$. This improves on the value 14 obtained by Prohazka and on the value 20 obtained for $n_C^*(3,3)$ in the preceding section.

(Figure 2 somewhere here)

Proposition 4 is a special case of a more general result on line digraphs. Using this general result and the family constructed in [16], J. Bond [9] has recently obtained a slight improvement of Corollary 5 in the case of $2d =$

4. (As always, this was done by finding a 4-arc-coloring of a large $(2, D)$-digraph.). In fact he has shown that if $D \geq 6$, then $n_C(4, D) \geq 25 \cdot 2^{D-4}$, rather that $\geq 24 \cdot 2^{D-4}$ implied by Corollary 5.

We now return to the general problem of large (not necessarily balanced) $(f, D)_C$–digraphs. This problem also has an analogous problem concerned with the degree; however, that problem does not seem to have been previously studied. Let $n^t(d, D)$ denote the maximum number of vertices of a digraph of diameter D such that the *total degree* $d^+(x) + d^-(x)$ of each vertex x is at most d.

Note that any (d, D)–digraph has maximum total degree bounded by $2d$. Moreover, as the digraph is strongly connected, if $d^+(x) + d^-(x) \leq 2d$ then $1 \leq d^+(x) \leq 2d - 1$ and $1 \leq d^-(x) \leq 2d - 1$. Therefore

$$n^b(d, D) \leq n^t(2d, D) \leq n^b(2d - 1, D).$$

The close relationship between $n^t(d, D)$ and $n_C(f, D)$ is made explicit in the following proposition, proved along the lines of the proof of Proposition 3.

Proposition 6. $n^t(f - 2, D) \leq n_C(f, D) \leq n^t(f, D)$. □

One can argue in a spirit similar to proving the Moore bound, that $n^t(2f, 2) \leq f^2 + f$. Thus combining Propositions 3 and 6, and Corollary 5 we obtain the precise value of the parameter $n_C(2f, 2)$:

Corollary 7. $n_C(2f, 2) = f^2 + f$. □

5 Conclusion

We described several constructions (and referred to many others) of large graphs with given diameter and maximum degree Δ which can be edge-colored with Δ colors. As a consequence we obtained the following lower bounds on $n^*_C(\Delta, D)$:

- $n^*_C(3,2) = 8$
- $n^*_C(3,3) = 20$, $n^*_C(3,4) \geq 32$, $n^*_C(3,5) \geq 56$, $n^*_C(3,6) \geq 128$, $n^*_C(3,7) \geq 184$ and $n^*_C(3,8) \geq 320$.
- $n^*_C(10,10) \geq 47\,059\,200$
- $n^*_C(3, 2^k - 2) \geq 2 \cdot 2^{(}2^k - 2)$
- $n^*_C(2f, D) \geq f^D + f^{D-1}$
- $n^*_C(2f+1, 2m) \geq f^m(f+1)^m$
- $n^*_C(p+1, 2) \geq p^2 + p + 1$

For the corresponding problem on digraphs we have $n_C(2f, D) \geq n^*_C(f, D)$, and in addition:

- $n_C(6,3) \geq 36$
- $n_C(2f, D) \geq f^D + f^{D-1}$
- $n_C(4, D) \geq 25 \cdot 2^{D-4}$
- $n_C(2f, 2) = f^2 + f$

In both cases we have also described upper bounds and general lower bounds relating these problems to those of finding large graphs (digraphs) with given maximum (in- and out-) degree and diameter.

¿From a practical perspective, we suggest the following technique to construct large $(q, D)_C$–graphs for given values of q and D. Consider first the largest known (q, D)–graphs. If any such a graph can be shown to admit an edge-coloring with q colors, one should use it. No larger graph is likely to be easily found, as it would also be an improvement on the extensively studied value $n^\star(q, D)$. Otherwise consider (q, D)–graphs for which a q–edge-coloring is known and which still have a relatively large number of vertices. In any case, we can use, for instance, the largest known bipartite (q, D)–graph. Similar comments apply in the case of $(f, D)_C$–digraphs.

Finally, we note that the best constructions we obtained for the case of general digraphs are better than the theoretical upper bounds for symmetric digraphs – compare the lower bound in Corollary 5 with the rough upper bound $n_C^s(2f, D) \leq f^D$ mentioned at the beginning of section 4. Thus there is a heavy penalty for requiring radio packet networks to be symmetric. On the other hand, it appears that requiring them to be balanced is not an obstacle, and in fact all of the best constructions happen to be balanced.

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Footnotes:

J-C. Bermond is with CNRS, Université de Nice - Sophia Antipolis, France.

P. Hell is with Simon Fraser University, Vancouver, Canada.

J.-J. Quisquater is with Philips Research Laboratory, Louvain-la-Neuve, Belgium.

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Index terms.

Arc-coloring, chromatic index, computer and communication networks, degree, diameter, edge-coloring, interconnection networks, packet radio networks, time division multiplexing.

Figure captions

Figure 1.

A largest $(3,3)_C$–graph, with a 3-coloring of its edges.

Figure 2.

The $(6,3)_C$–digraph $K(3,3)$, with a 6-coloring of the arcs

Preferred address:

Pavol Hell

School of Computing Science

Simon Fraser University

BURNABY, B.C., Canada V5A 1S6