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Dense Bus Networks of Diameter 2

J.-C. BERMOND, J. BOND AND S. DJELLOUL

ABSTRACT. In this paper we consider networks in which processors share a communication medium called a bus. These networks are called bus networks and are modeled by hypergraphs. The vertices of the hypergraph represent the processors of the network and edges represent buses. We give some techniques used to design dense bus interconnection networks of given maximum degree Δ, diameter D and maximum bus size r. In particular we construct a new family of bus interconnection networks of diameter 2 having a large number of processors.

1. Introduction

A bus interconnection network is a collection of processing elements (processors) and communication elements (buses). The processors produce and/or consume messages and the buses provide communication channels to exchange messages among the processors. Every bus provides a communication link between two or more processors.

For practical reasons, a processor may only be connected to a limited number of buses (this number is known as the processor degree) and a bus may only connect a limited number of processors (this number is known as the bus size). Therefore, messages may have to be relayed by a number of intermediate processors before arriving at their destinations, and thus the message transmission time becomes a function of the distance (measured in terms of the number of buses traversed by a message) between processors. The maximum distance over all pairs of processors is the network diameter.

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For given upper bounds on the processor degree Δ, bus size r, and network diameter D, the construction of bus networks with maximal number of processors is an important problem in the design of interconnection networks. We refer to the survey [**10**] for a state of art on this problem. Other design parameters such as network reliability, symmetry properties, ease of message routing, balanced message traffic throughout the network, implementation issues (algorithms and architecture) should also be taken into consideration.

Our aim is to give some new techniques to design dense bus connection networks of given maximum degree Δ, diameter D and maximum bus size r. In particular we construct a new family of bus connection networks of diameter 2 having a large number of processors.

This paper is organized as follows: In section 2 we give the necessary definitions and notation both from hypergraph theory and from design theory. We also state the (Δ, D, r)-hypergraph problem and give a Moore like upper bound. In section 3 we recall some earlier results obtained in the case of diameter 2. In section 4 we give our main theorem and deduce as corollary some lower bounds and construction of dense bus connection networks of diameter 2.

2. Definitions and notation

2.1. (Δ, D, r)-hypergraph problem. An *(undirected) hypergraph* H is a pair $H = V(H), E(H)$ where $V(H)$ is a non-empty set of elements, called *vertices*, and $E(H)$ is a finite set of subsets of $V(H)$ called *edges*. The number of vertices in the hypergraph is $n(H) = |V(H)|$ and the number of edges is $m(H) = |E(H)|$ where the vertical bars denote the cardinality of the set. The *degree* of a vertex v is the number of edges containing it and is denoted by $d_H(v)$. The *maximum degree* over all of the vertices in H is denoted by $\Delta(H)$. The *size* of an edge $E \in E(H)$ is its cardinality, and is denoted by $|E|$. The *rank* of H is the size of its largest edge, and is denoted by $r(H)$. A *path* in H from vertex u to vertex v is an alternating sequence of vertices and edges $u = v_0, E_1, v_1, \cdots, E_k, v_k = v$ such that $v_{i-1}, v_i \subseteq E_i$ for all $1 \leq i \leq k$. The *length* of a path is the number of edges in it. The *distance* between two vertices u and v is the length of a shortest path between them. The *diameter* of H is the maximum of the distances over all pairs of vertices, and is denoted by $D(H)$.

We call a hypergraph with maximum degree Δ, diameter D, and rank r, a (Δ, D, r)-*hypergraph*. The problem on bus networks we considered in the introduction is known as the (Δ, D, r)-*hypergraph problem* and consists of finding (Δ, D, r)-hypergraphs with the maximum number of vertices or finding large (Δ, D, r)-hypergraphs. We will denote by $n(\Delta, D, r)$ the maximum number of vertices of any (Δ, D, r)-hypergraph.

In the case $r = 2$ (graph case), this problem has been extensively studied and is known as the (Δ, D)-graph problem (see for example [**8**], [**9**]), and the maximum number of vertices in any (Δ, D)-graph is denoted by $n(\Delta, D)$.

Note that parts of this problem have been studied in other contexts with different notation. For example d or r is used for maximum degree, k or d is used for diameter, and b or k is used for rank. (In the notation of Design Theory r and k are used for maximum degree and rank, respectively.) We follow the notation of Hypergraph Theory [**3**].

Finally, let us mention that the drawing of hypergraphs can be very complex and therefore it is useful to represent a hypergraph H with a bipartite graph,

$$R(H) = (V_1(R) \cup V_2(R), E(R))$$

called the *bipartite representation graph*. Every vertex v_i in $V(H)$ is represented by a vertex v_i in $V_1(R)$ and every edge E_j in $E(H)$ is represented by a vertex e_j in $V_2(R)$. We draw an edge between $v_i \in V_1(R)$ and $e_j \in V_2(R)$ if and only if $v_i \in E_j$ in H.

If H is a (Δ, D, r)-hypergraph and $R(H)$ is its bipartite representation graph, then the maximum degrees in $V_1(R)$ and in $V_2(R)$ are Δ and r, respectively. The distance between two vertices of $V_1(R)$ is at most $2D$, but the diameter of $R(H)$ can be $2D$, $2D+1$ or $2D+2$ as the vertices of $V_1(R)$ and $V_2(R)$ do not play the same role. So, the (Δ, D, r)-hypergraph problem is partly related but different from the $(\Delta_1, \Delta_2; D')$-bipartite graph problem, i.e. finding large bipartite graphs with maximum vertex degrees Δ_1, Δ_2 and diameter D' (for details of this problem see [**13**]).

2.2. Duality tools. The *dual* of a hypergraph $H = (V(H), E(H))$ is the hypergraph $H^* = (V(H^*), E(H^*))$ where the vertices of H^* correspond to the edges of H, and the edges of H^* correspond to the vertices of H. A vertex e_j^* is a member of an edge V_i^* in H^* if and only if the vertex v_i is a member of E_j in H.

Bermond, Bond and Peyrat [**4**] observed the following relationship between a hypergraph and its dual:

Proposition 2.1. [*J.-C. Bermond, J. Bond, C. Peyrat*] *If H is a (Δ, D, r)-hypergraph then its dual hypergraph H^* is a (r, D^*, Δ)-hypergraph where $D-1 \leq D^* \leq D+1$.*

Note that, if G is a graph of maximum degree Δ and diameter D then its dual is a $(2, D^*, \Delta)$-hypergraph. Furthermore,

Proposition 2.2. [*J.-C. Bermond, J. Bond, C. Peyrat*] *If G is a bipartite (Δ, D)-graph then its dual hypergraph H^* is a $(2, D^*, \Delta)$-hypergraph where $D^* \leq D$.*

In what follows we will call the diameter of the dual of H the *line diameter* of H. In particular if a hypergraph H has line diameter 2 it means that for any pair of edges E and F either $E \cap F \neq \emptyset$ or there exists an edge I such that $E \cap I \neq \emptyset$ and $I \cap F \neq \emptyset$.

2.3. Design Theory. We will need some concepts of design theory (for more details and results see [**17**] and [**18**]). In particular, we need the concept of transversal design.

A *transversal design* $T[\Delta, \lambda; k]$ is a triple $(X, \mathcal{G}, \mathcal{P})$ where X is a finite set of elements, $\mathcal{G}$ a partition of X into Δ classes G_i, $i = 1, 2, \ldots, \Delta$, called *groups* or *classes* each containing exactly k elements and $\mathcal{P}$ a family of subsets, called *blocks* of X such that :

(i) For any block P of $\mathcal{P}$ and any group G_i $|P \cap G_i| = 1$.
(ii) Each pair $\{x, y\}$ of elements of X where x and y belong to distinct groups is contained in exactly λ blocks of $\mathcal{P}$.

A transversal design $T[\Delta, \lambda; k]$ has exactly Δk elements. Each block contains exactly Δ elements. Easy countings show that each element belongs to exactly $k\lambda$ blocks of $\mathcal{P}$ and that the number of blocks is $k^2\lambda$.

The case $\lambda = 1$ can be viewed as a partition of the edges of a complete Δ-partite graph into cliques of size Δ. It is known that there exists a $T[\Delta, 1; k]$ if and only if there exist $\Delta - 2$ mutually orthogonal latin squares of order k (see [**17**]). In particular $T[3, 1; k]$ transversal designs always exist and $T[4, 1; k]$ transversal designs exist if and only if $k \neq 2, 6$. For a general Δ, a $T[\Delta, \lambda; k]$ exists if k is large enough by a result of Wilson [**22**].

2.4. Moore-like bounds. As for graphs, we can determine an upper bound on the maximum number of vertices of a (Δ, D, r)-hypergraph . Indeed, by computing the number of vertices at distance $0, 1, \ldots, D$ from a given vertex, we find at most one vertex at distance 0, $\Delta(r-1)$ vertices at distance 1, $\Delta(\Delta - 1)(r-1)^2$ vertices at distance 2, $\Delta(\Delta-1)^2(r-1)^3$ vertices at distance 3, ..., $\Delta(\Delta-1)^{D-1}(r-1)^D$ vertices at distance D. Therefore, we obtain the Moore-like bound given by :

Proposition 2.3. *The maximum number $n(\Delta, D, r)$ of vertices of a (Δ, D, r)-hypergraph satisfies :*

$$n(\Delta, D, r) \leq 1 + \Delta(r-1)\Sigma_{i=0}^{i=D-1}(\Delta-1)^i(r-1)^i.$$

The topologies whose number of vertices achieves the Moore bound are called *Moore geometries*. Several authors have tried to know for what values of Δ, D and r Moore geometries exist. Now, it is established that the Moore bound may be achieved only in case $r = 2$ or D=1 or D=2 with $r > 3$ (see [**5**] for references).

In case $D = 1$, the Moore bound is $1 + \Delta(r-1)$. We have the following result [**6**] :

Proposition 2.4. [*J.-C. Bermond, J. Bond, J.-F. Saclé*] *The maximum number $n(\Delta, 1, r)$ of vertices of a $(\Delta, 1, r)$-hypergraph is equal to the Moore bound if and only if there exists a $(v, r, 1)$-BIBD with $v = 1 + \Delta(r-1)$.*

The problem of existence of $(v, r, 1)$-BIBD is itself a large domain of research and the results are complete only for $r \leq 6$.

3. Earlier results

Many results have been obtained on the construction of "good" (Δ, D, r) hypergraphs (see the survey [**10**]) in particular in the case $D = 1$ or $\Delta = 2$. Let us recall what is known in the case of diameter 2. The Moore bound is $(\Delta^2 - \Delta)r^2 - (2\Delta^2 - 3\Delta)r + (\Delta - 1)^2$. The case $D = 2$ and $\Delta = 2$ was studied in [**4**] and will be recalled in paragraph 4.

For $D = 2$ and $\Delta \geq 3$ one construction is based on the following propositions:

PROPOSITION 3.1. [*J. Bond*] *For any $\Delta \geq 2$, any $D \geq 2$ and any $r \geq 2$,*

$$n(\Delta, D, r) \geq rn(\Delta - 1, D - 1, r).$$

PROOF. Take r copies of a $(\Delta - 1, D - 1, r)$ hypergraph and join the r vertices having the same label in each copy with a hyperedge. Therefore we obtain a (Δ, D, r) hypergraph [**12**]. □

PROPOSITION 3.2. [*J.-C. Bermond, J. Bond, J.-F. Saclé*] *If a projective plane of order q exists then, for $\Delta = q + 1$ and $r \geq \Delta$, there exists a $(\Delta, 1, r)$-hypergraph with $\Delta r - (\Delta - 1)\lceil \frac{r}{\Delta} \rceil$ vertices and then, in this case, $n(\Delta, 1, r) \geq \Delta r - (\Delta - 1)\lceil \frac{r}{\Delta} \rceil$.*

PROOF. see [**6**]. □

PROPOSITION 3.3. *For any $\Delta \geq 3$ such that $\Delta - 1 = 1 + q$ where q is a prime power and for any $r \geq \Delta - 1$,*

$$n(\Delta, 2, r) \geq (\Delta - 1)r^2 - (\Delta - 2)r\lceil \frac{r}{\Delta - 1} \rceil.$$

PROOF. According to proposition 3.1, $n(\Delta, 2, r) \geq rn(\Delta - 1, 1, r)$. On the other hand, there exists a projective plane of order q and therefore a $(q + 1, 1, r)$ hypergraph for any prime power q. Then, using proposition 3.2, we have, for any $r \geq \Delta - 1$,
$n(\Delta - 1, 1, r) \geq (\Delta - 1)r - (\Delta - 2)\lceil \frac{r}{\Delta - 1} \rceil$. □

REMARK 3.1. *For any $\Delta \geq 3$ such that $\Delta - 1 = q + 1$ where q is a prime power,*

$$n(\Delta, 2, r) \geq f(\Delta) \text{ with } f(\Delta) = \frac{\Delta^2 - 3\Delta + 3}{\Delta - 1} r^2 + O(r).$$

We give here for $\Delta = 3, 4$ the lower bounds $f(\Delta)$ on $n(\Delta, 2, r)$, deduced from proposition 3.3 :

Δ	$f(\Delta)$	Moore bound
3	$\frac{3}{2}r^2 + O(r)$	$6r^2 - 9r + 4$
4	$\frac{7}{3}r^2 + O(r)$	$12r^2 - 20r + 9$

Table 1.

When $D = 2$ and $\Delta \geq 16$, Δ and r even, a better construction is obtained by considering the de Bruijn bus networks which have $\frac{\Delta^2}{16} r^2$ vertices [**7**].

REMARK 3.2. *In the particular case where r is a multiple of Δ, the lower bounds of table 1 will be improved by the technique of construction we shall propose in section 4.*

4. A new technique to design dense bus connection networks of diameter 2

4.1. Case $\Delta = 2$. The technique presented below is inspired from the construction presented in [**4**] for the case $D = 2$, $\Delta = 2$. We repeat this construction:

Step 1 : Consider the graph $G_0 = C_5$, the cycle of length 5. Note that C_5 has line-diameter 2.

Step 2 : Replace each vertex x_i of C_5 by a set X_i of k vertices and each edge $x_i x_j$ of C_5 by the edges of the complete bipartite graph constructed on $X_i \cup X_j$. We therefore obtain a graph G regular of degree $2k$, on $5k$ vertices and having $5k^2$ edges. One can check that G has still line-diameter 2.

Step 3 : Take the dual G^* of G. G^* is a $(2, 2, r) - hypergraph$ with $r = 2k$ and $N = 5k^2 = \frac{5r^2}{4}$ vertices.

So we have shown that $n(2, 2, r) \geq \frac{5r^2}{4}$ when r is even. In fact Kleitman [**19**] (see also [**21**] and [**14**]) has proved that $n(2, 2, r) \leq \frac{5r^2}{4}$ so

THEOREM 4.1. *If r is even, then $n(2, 2, r) = \frac{5r^2}{4}$.*

If r is odd, $r = 2k+1$ we have to modify slightly the construction by replacing two adjacent vertices x_1 and x_2 of the C_5 each by a set of $k + 1 = \lceil \frac{r}{2} \rceil$ vertices and the 3 other vertices by a set of $k = \lfloor \frac{r}{2} \rfloor$ vertices. The graph G obtained has maximum degree r and has $\frac{5r^2-2r+1}{4}$ edges. By taking its dual we obtain $n(2, 2, r) \geq \frac{5r^2-2r+1}{4}$. In [**21**] and [**14**] they have also proved that there is in fact equality. So

THEOREM 4.2. *If r is odd, then $n(2, 2, r) = \frac{5r^2-2r+1}{4}$.*

Note that the value given in [**4**] for r odd is incorrect.

4.2. General case : Main theorem. For $D = 2$ and $\Delta \geq 2$ we generalize the construction given before :

Step 1 : We construct a hypergraph H_0, with Δ-uniform rank and with line-diameter 2 (in the case $\Delta = 2$, H_0 was choosen as C_5). Let d be the maximum degree, n the number of vertices and m the number of edges of H_0.

Step 2 : Replace each vertex x_i of H_0 by a set X_i of k vertices and each edge $E = (x_1, x_2, \dots, x_\Delta)$ of H_0 by the edges (blocks) of a transversal design $T[\Delta, 1, k]$ constructed on the set $X = \cup X_i$, with classes(groups) X_i, $(i = 1, 2, ..., \Delta)$ corresponding to the vertices x_i of E. If such a transversal design exists it follows that the hypergraph H obtained from H_0 has nk vertices, mk^2 edges and maximum degree dk. Figure 1 shows an example with $\Delta = 4$, $k = 3$ and H_0 reduced to an edge $\{a, b, c, d\}$.

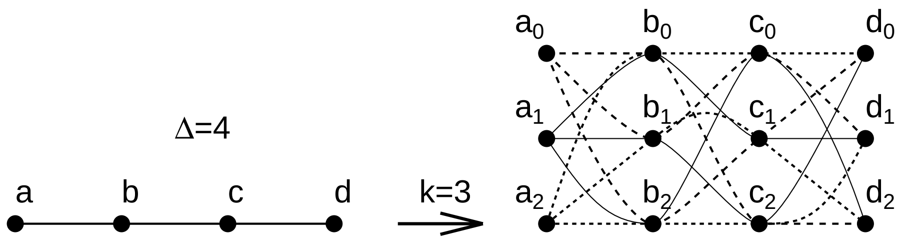

FIGURE 1. H_0 H_1

Furthermore we will prove that

LEMMA 4.1. *If there exists a transversal design $T[\Delta, 1, k]$ then the hypergraph H constructed from H_0 as described above has line-diameter 2.*

(Note that for $\Delta = 2$ we use a $T[2, 1, k]$ transversal design which is nothing else than a bipartite complete graph).

Step 3 : Take the dual H^* of H. By step 2 and the lemma H^* is a $(\Delta, 2, r)$-hypergraph with $r = dk$, $N = mk^2 = \frac{mr^2}{d^2}$ vertices. So we have

THEOREM 4.3. *If there exists a Δ-uniform hypergraph of line-diameter 2 with maximum degree d and m edges and if there exists a transversal design $T[\Delta, 1, k]$ then $n(\Delta, 2, dk) \geq mk^2$.*

PROOF. *(of lemma 4.1).* Let P and Q be two distinct edges of H. P (resp. Q) corresponds to some block of a transversal design $T[\Delta, 1, k]$associated to an edge E (resp. F) of H_0.

Case 1 : $E = F = (x_1, x_2, \dots, x_\Delta)$. Let p be a vertex of P which does not belong to Q; p belongs to some set X_i. Let q be a vertex of Q not in X_i. By the definition of a tranversal design there exists an edge R containing p and q. R intersects both P and Q and so the distance between P and Q is at most 2.

Case 2 : $E \neq F$, $E \cap F \neq \emptyset$. Let $x_i \in E \cap F$ and let $p = P \cap X_i$ (by definition $P \cap X_i$ is not empty) and $q = Q \cap X_i$. If $p = q$, P and Q are at distance 1. If $p \neq q$, let p' be an element of P different from p and let R be the edge of the transversal design associated to E containing p' and q. R intersects both P and Q and so the distance between P and Q is at most 2.

Case 3 : $E \cap F = \emptyset$. As H_0 is of line-diameter 2 there exist an edge E' intersecting both E end F. Let $x_i \in E \cap E'$ and $x_j \in F \cap E'$. Let $p = P \cap X_i$ and $q = Q \cap X_j$. By definition there exists in the transversal design associated to E' an edge R containing both p and q. So R intersects both P and Q and again the distance between P and Q is at most 2 (in fact exactly 2 in that case). □

REMARK 4.1. *This theorem can easily be generalized for a line-diameter D giving a lower bound for $n(\Delta, D, dk)$. Unfortunately this bound is of order mk^2 and therefore cannot be good asymptotically.*

4.3. Results. Note that to apply theorem 4.3 we need the existence of a transversal design $T[\Delta, 1, k]$ (see some known results in 2.3) and the construction of a Δ-uniform hypergraph H_0 of line-diameter 2. We can have many possible choices. For a given degree d of H_0 we will try to maximize m. For a given r we will choose the hypergraph H_0 optimizing $\frac{m}{d^2}$, with d divisor of r. (We can also in that case use a solution for a value near for r like we did for r odd in the case $\Delta = 2$.)

So all the difficulty consists in finding optimal or good hypergraphs H_0. The line-diameter of H_0 correspond to half the maximum distance between vertices belonging to the class $V_2(R)$ associated to the buses. As we want a line-diameter 2, we have to consider bipartite graphs of diameter 4, 5 or 6. In the literature (see for example [**13**]) has been considered the case of regular graphs (in that case $\Delta = d$). Using these results for diameter 5 we have a good family of hypergraphs.

Let us denote by $b'(\Delta, 5)$ the maximum number of vertices of a biregular bipartite graph with degree Δ and diameter 5, then we have a family of hypergraphs H_0 with $m = \frac{b'(\Delta,5)}{2}$ and $d = \Delta$. So we obtain:

THEOREM 4.4. *Let $\Delta \geq 3$ and r be a multiple of Δ, $r = \Delta k$, then if there exists a transversal design $T[\Delta, 1, k]$*

$$n(\Delta, 2, r) \geq \frac{b'(\Delta,5) r^2}{2\Delta^2}.$$

For example, in the case $\Delta = 3$, we have $b'(3,5) \geq 56$. So as there exists a $T[\Delta, 1, k]$ for any k, $n(3, 2, r) \geq \frac{28}{9} r^2$ for r multiple of 3.

In the case $\Delta = 4$, we have $b'(4,5) \geq 144$. So as there exists a $T[\Delta, 1, k]$ for any $k \neq 2, 6$, $n(4, 2, r) \geq \frac{9}{2} r^2$ for r multiple of 4, $r \neq 8, 24$.

Asymptotically we have a family better than those obtained before. In the case $\Delta = 3$, recall that $n(3, 2, r)$ was of order $\frac{3}{2} r^2$ and for $\Delta = 4$, $n(4, 2, r)$ was of order $\frac{7}{3} r^2$.

If one is particularly interested in some values of r not multiple of Δ, we can either use a d different from Δ (but in that case we have to construct "good" (d, Δ, D)-bipartite biregular graphs) or modifiy the construction by starting from a solution with a slightly different r.

5. Conclusion

The technique we have proposed in this paper provides a new family of dense bus connection networks of parameters $\Delta \geq 3$, diameter 2 and edge size r. Improvement in the case where r is not a multiple of Δ could be obtained by finding new (d, Δ, D)-bipartite biregular graphs. Using these results one can also improve, via compound techniques, some of the known values for $D \geq 3$. Unfortunately we are still far from the Moore bound and so new ideas have to be proposed in that case.

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I3S, CNRS, U.R.A. 1376, Université de Nice Sophia-Antipolis, 930 Route des Colles, B.P. 145, 06903 Sophia-Antipolis Cedex, France
E-mail address: bermond@unice.fr

I3S, CNRS, U.R.A. 1376, Université de Nice Sophia-Antipolis, 930 Route des Colles, B.P. 145, 06903 Sophia-Antipolis Cedex, France
E-mail address: bond@unice.fr

LRI, UA 410 CNRS, Université de Paris-Sud, 91405 Orsay Cedex, France
E-mail address: djelloul@lri.fr