Gathering in specific radio networks †

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In this paper, we address the problem of gathering information into a central node of a radio network, where interference constraints are present. We take into account the fact that, when a node transmits, it produces interference in an area larger than the area in which its message can actually be received. The network is modeled by a graph; a node is able to transmit one unit of information to the set of vertices at distance at most d_T in the graph, but when doing so it generates interference that does not allow nodes at distance up to d_I ($d_I \geq d_T$) to listen to other transmissions. Time is synchronous and divided into time-steps in each of which a round (set of non-interfering radio transmissions) is performed. Because this problem is hard to approximate in general graphs, we study *good* approximations in some specific topologies, like the path, balanced stars and the 2 dimensional grid. In all these cases we provide algorithms whose performance differs only by an additive constant from the theoretical minimum.

Keywords: gathering, radio networks, approximation, path, grid, stars, interference constraints.

1 Introduction

1.1 Background and motivation

In radio networks a set of radio devices communicate by using radio transmissions and is therefore subject to communication and interference constraints. This means that only certain transmissions can be performed simultaneously, hence the devices have to act cooperatively in order to achieve an effective flow of information in the network. In this context, we study a problem proposed by France Telecom, about "how to provide Internet to villages" [BBS05].

The houses of the village are equipped with radio devices and they want to access the *rest of the world* via Internet. For that purpose they have to send (and receive) information through a gateway where there is a *central* antenna. This creates a special *many-to-one* information flow demand in which the access to the gateway must be provided. Hence, we will consider a specific traffic pattern, similar to a single commodity flow with a distinguished node representing the gateway, called *sink* and denoted t.

Nodes are all equipped with a communication device that can be tuned either to transmit or to listen (exclusively). We call a slot a time interval during which the communication pattern stays unchanged (in particular the tuning status of the communication devices stays the same). We will assume that all the slots have the same duration, hence nodes communicate during synchronous and regular *time slots*. We will use the word *round* (or step) to indicate a given time slot. Furthermore, we will use as unit of traffic the amount of information that devices can transmit during a round.

Interference and communication constraints are widely modeled by associating to each node a *transmission area* in which it can transmit a message and an *interference area* in which it produces

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a strong noise. The communication from a node u to a node v is possible if v is in the transmission area of u, and no third node transmitting has v in its interference area.

We model the transmission area and the interference area as balls in the graph by introducing two parameters: d_T, the transmission radius and d_I, the interference radius and we suppose that $d_I \geq d_T$. The transmission area (resp. interference area) is then the ball of radius d_T (resp. d_I).

Even though the information transmitted by a node may become available to several nodes in its transmission area, when gathering, to have two copies of a message in different vertices does not help, so we can assume that when a sender transmits, it does so to a unique receiver.

Under this model, the problem raised by France Telecom consists of gathering information from each node of the network into the central node (the sink t).

Our objective is to find the minimum time (in number of rounds) needed to achieve gathering, which is called the Minimum Time Gathering Problem (MTG). Fig. 1 shows an optimal gathering protocol using 18 rounds for a path of 7 vertices (each having one piece of information), with $d_T = 1$, $d_I = 2$ and $t = 0$.

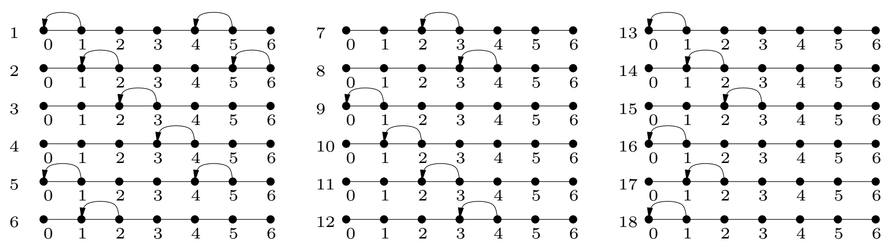

Fig. 1: A protocol that gathers into $t = 0$ one message from each vertex in the path when $d_T = 1, d_I = 2$.

1.2 Related work and Results

Basic communication problems for the dissemination of information (like gathering, broadcasting, gossiping) have been widely studied in classical interconnection networks [HKP+05]. The fact that a node cannot both send and receive in the same round is known as the *half-duplex* 1*-port* model and the unit message constraint is studied for example in [BGRV98].

The problem of finding the minimum time gathering protocol (MTG) in general graphs is NP-Hard when $d_I = d_T$ and does not admit a Fully Polynomial Time Approximation Scheme (FPTAS) if $d_I > d_T$ (that is: to find an approximation with quality $(1+\varepsilon)$ requires a time that is not polynomial in ε^{-1} unless P=NP). These two hardness results remain true even in the unitary case where each vertex has exactly one message to transmit to the sink [BGK+06]. The same paper gives a lower bound and a 4-approximation algorithm (independently of d_I, d_T) for the minimum gathering time of any graph. Nevertheless, these results hold only for general graphs, and the hardness for specific topologies (like the path, grids and stars) remains open.

The case where furthermore each node has a exactly one unit of information to transmit has also been studied in the case of specific graph topologies for certain values of d_I, d_T. In particular, the case $d_T = 1$ was studied in [BCY06] for the case where the graph is a path, and in [BP05] for the case where the graph is the 2 dimensional square grid.

In this paper we consider also the case of unitary messages for paths and grids, but for any value of d_T. Even though our protocols do not match the lower bounds, the gap is an additive constant that depends only on d_I, d_T. We also study the case of stars and show that the general lower bound of [BGK+06] is tight up to a constant that does not depend on the size of the network.

Due to space constraints, most of the proofs are omitted in this extended abstract.

2 The model: definitions and notation

An instance of the MTG problem is defined by a graph $G = (V, E)$ with a distinguished vertex $t \in V$, called the *sink*, two integers $d_I, d_T \in \mathbb{N}$, such that $d_I \geq d_T > 0$, and a function $w : V \rightarrow \mathbb{N}$, $w(u)$ being the number of information pieces (shortly messages) to transmit from vertex u to the sink t. d_I is the *interference distance*, d_T is the *transmission distance*, and $n = |V|$ is the *size of the network*. In this paper we address the unitary minimum time gathering (UMTG) where $w(u) = 1$ $(\forall u \in V)$ and write $m(u)$ for the message originated at vertex u.

The distance between two vertices u and v is the length of the shortest path from u to v and is denoted $d_G(u,v)$. For $u \in V$ and $h \in \mathbb{N}$, we define the *h-neighborhood of* u as $\Gamma_G^h(u) = \{v \in V : d_G(u,v) \leq h\}$. When the context is clear we will omit the index G.

A *call* is a couple (s,r), $s,r \in V$ with $0 < d(s,r) \leq d_T$ where s is the *sender* and r the *receiver*. The call (s,r) *interferes* with the call (s',r') if $d(s,r') \leq d_I$. We say that the two calls (s,r) and (s',r') are *compatible* if they do not interfere, that is both $d(s,r') > d_I$ and $d(s',r) > d_I$.

A *round* is a set of compatible calls. During a round, a sender transmits a new message if there is one available. A *gathering protocol* is an ordered sequence of rounds that allows to gather the information of the nodes into the sink.

We often specify protocols by giving simply the sequence of rounds, without specifying which message is sent, indeed that is irrelevant as long as each vertex can forward something new.

The goal of the Unitary Minimum Gathering Problem is to find a protocol that gathers all the messages into the sink and that takes a minimum number of rounds (denoted by $g_{d_I,d_T}(G,t)$).

3 Results

Table 1 shows the main results of the paper. The notation is the following:

LB (resp. UB) is a lower bound (resp. upper bound) on $g_{d_I,d_T}(G,t)$.

P_n is the path with n vertices $0,1,\dots,n-1$. Vertex i is connected with $i+1$ for any $i = 0,\dots,n-2$.

$S_{K,l}$ is the balanced star with K branches. $S_{K,l}$ consists of K copies of P_l (the branches) sharing a common extreme, the sink t.

$G^2(p,q)$ is the 2-dimensional grid, with vertex set $V = \{(i,j) : -p \leq i \leq p, -q \leq j \leq q\}$. So $n = (2p+1)(2q+1)$. $(x,y),(x',y') \in V$ are connected when $|x-x'|+|y-y'| = 1$. We assume that $p,q \geq d_I + d_T + 1$ and $t = (0,0)$.

$O(1)$ is a constant that depends on d_I, d_T but not on the size of the network n.

Topology (G)	LB	UB
P_n	$\frac{d_I+d_T+1}{d_T}\max[t,n-t] - O(1)^{1,2}$	$\frac{d_I+d_T+1}{d_T}\max[t,n-t] + O(1)$
$S_{K,l}$, $\lfloor d_I/d_T \rfloor$ odd	$\frac{1}{2}(1+\lfloor d_I/d_T \rfloor)n - O(1)$	$\frac{1}{2}(1+\lfloor d_I/d_T \rfloor)n + O(1)^3$
$S_{K,l}$, $\lfloor d_I/d_T \rfloor$ even	$\frac{1}{2}\lfloor d_I/d_T \rfloor n + \frac{n}{K} - O(1)^1$	$\frac{1}{2}\lfloor d_I/d_T \rfloor n + \frac{n}{K} + O(1)$
$G^2(p,q)$, $\lfloor d_I/d_T \rfloor$ odd	$\frac{1}{2}(1+\lfloor d_I/d_T \rfloor)n - O(1)$	$\frac{1}{2}(1+\lfloor d_I/d_T \rfloor)n + O(1)^3$
$G^2(p,q)$, $\lfloor d_I/d_T \rfloor$ even	$\frac{1}{2}\lfloor d_I/d_T \rfloor n + \frac{n}{4} - O(1)^{1,4}$	$\frac{1}{2}\lfloor d_I/d_T \rfloor n + \frac{n}{4} + O(1)^4$

Tab. 1: Approximation results for gathering in specific topologies. [1]Partially improves bounds from [BGK+06, BCY06]. [2]Matches bound from [BCY06] when $d_T = 1$ (optimal for $t = 0$). [3]Matches general lower bound up to an additive constant. [4]Derives from the results for the balanced star with 4 branches.

Further results for gathering into an extreme vertex of the path ($t = 0$)

When $n \leq d_I + 3$, only one call per round can be done without interference and because transmitting $m(i)$ to the sink requires at least $\left\lceil \frac{i}{d_T} \right\rceil$ rounds, we have $g_{d_I,d_T}(P_n,0) = \sum_{i=1}^{n-1} \left\lceil \frac{i}{d_T} \right\rceil$. Let $W = g_{d_I,d_T}(P_{d_I+2},0) = \sum_{i=1}^{d_I+2} \left\lceil \frac{i}{d_T} \right\rceil$.

When $n > d_I + 3$, we have two lower bounds $\text{LB}_1 = W + \left\lceil \frac{d_I+2}{d_T} \right\rceil (n-1-(d_I+2))$ (follows from the general lower bound in [BGK+06]) and $\text{LB}_2 = \left\lceil \frac{d_I+d_T+1}{d_T}(n-1) - \frac{(d_I+d_T+1)(d_I+d_T)}{2d_T} \right\rceil$.

To prove LB_2 we observe that inside $[0, d_I + d_T + 1]$ we have, during a round, either one call of length at most d_T or two calls whose sum of lengths is at most d_T (see Fig. 2).

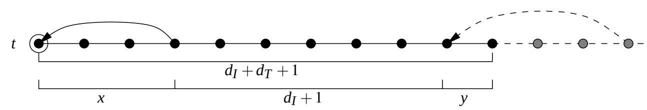

Fig. 2: The progress towards t inside $\Gamma^{d_I+d_T+1}(t)$ is at most d_T per round, because $x+y+d_I+1 \leq d_I+d_T+1 \Rightarrow x+y \leq d_T$. In this example, $d_I=5, d_T=4$, and $x+y=4$.

For a node i the sum of the lengths of the calls where $m(i)$ is transmitted within the interval $[0, d_I+d_T+1]$ is either i if $i \leq d_I+d_T$, or d_I+d_T+1 if $i \geq d_I+d_T+1$. It follows that $d_T g_{d_I,d_T}(P_n,0) \geq \sum_{i=1}^{d_I+d_T} i + (d_I+d_T+1)(n-1-d_I-d_T)$, and therefore $g_{d_I,d_T}(P_n,0) \geq \frac{d_I+d_T+1}{d_T}(n-1) - \frac{(d_I+d_T+1)(d_I+d_T)}{2d_T}$.

As upper bound, we have designed a protocol that spends $\mathrm{UB} = \frac{d_I+d_T+1}{d_T}(n-1) - f(d_I,d_T)$ rounds, where $f(d_I,d_T) \geq 0$ and does not depend on n.

To determine the exact value of $g_{d_I,d_T}(P_n,0)$ seems a difficult problem. However, we have been able to calculate $g_{d_I,d_T}(P_n,0)$ for some values of d_I and d_T.

If $d_I = a d_T - 1$ for some $a \in \mathbb{N}$, then $g_{d_I,d_T}(P_n,0) = \mathrm{LB}_1 = W + (a+1)(n-1-(d_I+2)) = \mathrm{UB}$.

If $d_T = 1$, then $g_{d_I,d_T}(P_n,0) = \mathrm{LB}_1 = \mathrm{LB}_2 = \mathrm{UB} = (d_I+2)(n-1) - \frac{(d_I+2)(d_I+1)}{2}$ (which matches the result in [BCY06]).

If $d_T = 2, d_I = 2k+1$, we are covered by the previous case. We have $\mathrm{LB}_2 < g_{d_I,d_T}(P_n,0) = \mathrm{LB}_1 = (k+2)(n-k-2) = \mathrm{UB}$.

If $d_T = 2, d_I = 2k$, then (a) $n \leq 3k+4 \Rightarrow g_{d_I,d_T}(P_n,0) = (k+1)(n-k-1)$ (LB_1 matches UB); or (b) $n \geq 3k+4 \Rightarrow g_{d_I,d_T}(P_n,0) = \left\lceil \frac{2k+3}{2}(n-k-2) \right\rceil$ (using a specialized protocol which matches LB_2).

4 Conclusions

This paper studied the Unitary Minimum Time Gathering Problem in some cases where the structure of the graph allows to find good solutions. For the path, balanced stars, and the 2D-grid, we are able to design protocols with a number which differs from the optimum by an additive constant. Moreover, this constant does not depend on the size of the network, but only on the values of the interference and transmission radius, d_I and d_T. However, this constant increases with these parameters.

Finally, we do not know what is the hardness of finding the minimum time gathering in these topologies and we conjecture it might be polynomial at least for paths.

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